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Temperature Fluctuation and the Specific Heat in Au+Au Collisions at Collision Energies from 5 to 200 GeV

LI Xiujun[1,2], SI fan[1,2], FU Zebang[1,2], ZHANG Yifei[1,2,†]

(*1. State Key Laboratory of Particle Detection and Electronics, University of Science and Technology of China, Hefei 230026, China;*
2. Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China)

Abstract: We report the results of the energy dependence of specific heat ($C_{\rm V}$) of hadronic matter in a multiphase transport (AMPT) model and compared with the experimental results from Ref. [PhysRevC.94.044901]. The temperature high order fluctuations in Au+Au collisions in AMPT model are also reported. $C_{\rm V}$ is a thermodynamic quantity that characterizes the equation of state of the system. For a system undergoing phase transition, $C_{\rm V}$ is expected to diverge at the critical point. Fluctuations of temperature are sensitive observables to probe the QCD critical point. The $C_{\rm V}$ is extracted by analyzing the data on event-by-event mean transverse momentum ($\langle p_{\rm T}\rangle$). The $\langle p_{\rm T}\rangle$ distributions in finite $p_{\rm T}$ ranges are converted to distributions of effective temperature ($T_{\rm eff}$). The $C_{\rm V}$ is extracted from the $T_{\rm eff}$ distributions. The fluctuations of temperature are measured by calculating the high order cumulants of the $T_{\rm eff}$ distributions. We find that both $C_{\rm V}$ and high order cumulants of the temperature show monotonic distributions in energy dependence, which is expected that there is no phase transition critical point in the AMPT model. At low energies, a sharp drop of $C_{\rm V}$ from the experimetal results is observed and it deviates from the AMPT results. The AMPT model can provide a non-critical background, which can provides a good reference for comparison with experimental results to search for the QCD critical point.
Key words: AMPT model; QCD critical point; heavy ion collision; specific heat; statistical fluctuation
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1 Introduction

Exploring the quantum chromodynamic (QCD) phase structure is one of the main goals of heavy ion collision experiments. The QCD critical point is the end point of the first-order phase boundary[1]. One of the major goals of beam energy scan II (BESII 2019-2021) program at RHIC is to search for the critical point in the QCD phase diagram. The experimental confirmation of the QCD critical point is essential in studying the QCD phase structure[2].

Lattice QCD calculations predicted a phase transition of confined partonic matter phase, called the Quark-Gluon Plasma (QGP) at extremely high temperature and energy. Critical point($\mu_{\rm B,c}$, $T_{\rm c}$) is located as the end point of the first order phase transition line. At higher temperature and lower baryon chemical potentials than the critical point the transition is no long first-order, but a cross-over[3–8]. Lattice QCD[9–11] and Dyson-Schwinger Equation[12–14] give serious theoretical prediction in a range about $115< T_{\rm c} <162$ MeV.

Specific heat, $C_{\rm V}$, is a thermodynamic quantity characterizing the equation of state of the system. It is a measure of the temperature fluctuation and its singularity behavior is relevant to the phase transition[15]. For a system undergoing phase transition, $C_{\rm V}$ is expected to diverge at the critical point[16]. Temperature fluctuation of the system provides an estimation of $C_{\rm V}$. Near the critical point, the specific heat is normally expressed in terms of a power law, $C_{\rm V} \propto| T-T_{\rm c} |^{-\alpha}$, where $T_{\rm c}$ is the critical temperature and is critical exponent[17]. Thus the $C_{\rm V}$ can be effectively used to probe the critical point.

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Biography: LI Xiujun(1992–), female, Jingzhou, Hubei, Postgraduate, working on the high energy nuclear physics; E-mail: lixiujun@mail.ustc.edu.cn
† **Corresponding author:** ZHANG Yifei, E-mail: ephy@ustc.edu.cn.

Furthermore, the fluctuations are very sensitive to the nature of the transition. Fluctuations of temperaure also serve as the sensitive observable in heavy-ion collisions to search for the QCD phase transition and critical point.

2 The distributions of $\langle p_{\rm T}\rangle$ and the effective temperature

It has been observed that the $\langle p_{\rm T}\rangle$ distributions are nicely described by using the gamma (Γ) distribution[18–19]:

$$f(x)=\frac{x^{\alpha-1}{\rm e}^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}}\ , \tag{1}$$

Here x represents the $\langle p_{\rm T}\rangle$. The mean (μ) and standard deviation (σ) of the distribution are related to the fit parameters α and β by $\mu=\alpha\beta$ and $\sigma=\sqrt{\alpha\beta^2}$.

The temperature of the system can be obtained from the particle transverse momentum ($p_{\rm T}$) . The $p_{\rm T}$ distribution can be fitted to an exponential Boltzmann-type distribution in which the temperature can be extracted event-by-event[15]:

$$F(p_{\rm T})=\frac{1}{p_{\rm T}}\frac{{\rm d}N}{{\rm d}p_{\rm T}}\approx A{\rm e}^{-p_{\rm T}/T_{\rm eff}}\ , \tag{2}$$

where A is a normalization factor and $T_{\rm eff}$ is the apparent or effective temperature of the system.

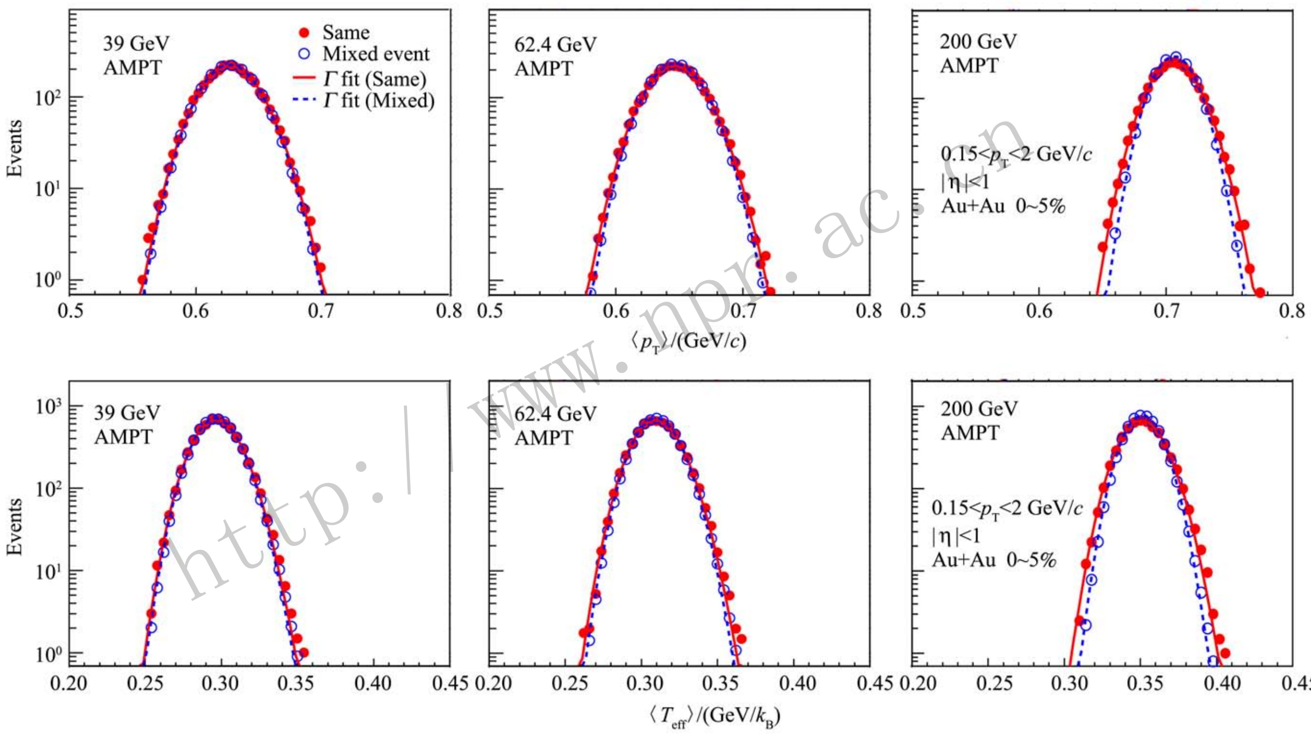

Fig. 1 (color online) Upper panels show event-by-event $\langle p_{\rm T}\rangle$ distributions of the charged particles for 0∼5% most central Au+Au collisions at collision energies $\sqrt{s_{\rm NN}}$ = 39, 62.4, and 200 GeV within $|\eta|$<1 and 0.15 <$p_{\rm T}$<2.0 GeV. The lower panels show the event-by-event extracted $T_{\rm eff}$ distributions. The solid and dashed lines show the fits of same events and mixed events with Γ functions.

In a system, for a range of $p_{\rm T}$ within a to b one can estimate $\langle p_{\rm T}\rangle$ via[19]:

$$\langle p_{\rm T}\rangle=\frac{\int_0^\infty p_{\rm T}^2F(p_{\rm T}){\rm d}p_{\rm T}}{\int_0^\infty p_{\rm T}F(p_{\rm T}){\rm d}p_{\rm T}}=\frac{2T_{\rm eff}^2+2m_0T_{\rm eff}+m_0^2}{m_0+T_{\rm eff}}\ , \tag{3}$$

where m_0 is the rest mass of the particle. Note that the integration for $p_{\rm T}$ is from 0 to ∞. But in reality the $p_{\rm T}$ window is finite. The expressions for $p_{\rm T}$ within a range from a to b, can be obtained using the following relation[19]:

$$\langle p_{\rm T}\rangle=\frac{\int_a^b p_{\rm T}^2F(p_{\rm T}){\rm d}p_{\rm T}}{\int_a^b p_{\rm T}F(p_T){\rm d}p_{\rm T}}=2T_{\rm eff}+\frac{a^2{\rm e}^{-a/T_{\rm eff}}-b^2{\rm e}^{-b/T_{\rm eff}}}{(a+T_{\rm eff}){\rm e}^{-a/T_{\rm eff}}-(b+T_{\rm eff}){\rm e}^{-b/T_{\rm eff}}}\ , \tag{4}$$

This equation links the value of $\langle p_{\rm T}\rangle$ within a specified range of $p_{\rm T}$ to $T_{\rm eff}$. The $T_{\rm eff}$ is extracted within the $p_{\rm T}$ range 0.15<$p_{\rm T}$<2.0 GeV.

The $p_{\rm T}$ and $T_{\rm eff}$ distributions have been studied with the string melting mode of a multiphase transport (AMPT) model[20] data. The radial flow may contribute to the temperature fluctuations but in this work we assume that the radial flow fluctuation is small and negligible. This will be studied in more details in future. In events generated from AMPT model, 0∼5% most central collisions are selected with the impact parameter cut $b<3.0$ fm. The results are shown within $|\eta|<1$ and $0.15<p_T<2.0$ GeV. Both the real and mixed event p_T and T_{eff} distributions are fitted with the Γ function and the fits are shown by the solid and dashed lines.

The results of 39, 62.4, 200 GeV of AMPT model are shown in Fig. 1. The solid points are the event-by-event $\langle p_T \rangle$ distributions or T_{eff} distributions from the AMPT model data(same event). The open circles are the results of mixed events. The mixed events are created by randomly selecting charged particles from different events. The mixed event distributions contain all the systematic effects include statistical fluctuations. The nonstatistical fluctuations can be extracted from the difference between the same event and mixed event distributions. In Figs. 2 and 3, we

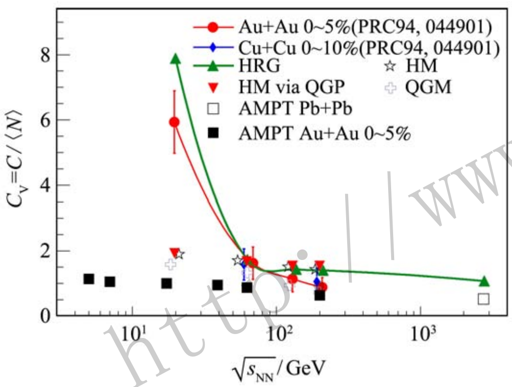

Fig. 2 (color online) Specific heat, C_V, of the charged particles as a function of collision energy from the AMPT model for 0∼5% most central Au+Au collisions. Except the Au+Au 0∼5% AMPT results, others are from Ref. [17] for comparison.

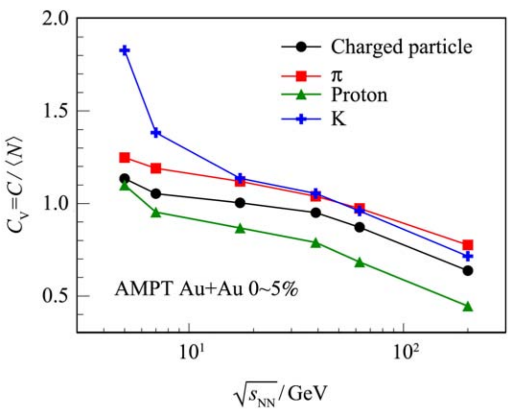

Fig. 3 (color online) Specific heat, C_V, of pions, protons, kaons, and the charged particles as a function of collision energy for 0∼5% most central Au+Au collisions at $\sqrt{s_{NN}}$=5 to 200 GeV from the AMPT model.

know that C_V is larger at a higher collision energy. Heavy-ion systems at higher collision energies require less energy for changing temperature, which results in larger temperature fluctuation.

3 Method to extract the specific heat

For a system in equilibrium, the event-by-event temperature fluctuation is controlled by the heat capacity[16, 21–22]:

$$P(T) \approx A\exp\left[-C\frac{(\Delta T)^2}{\langle T\rangle^2}\right] , \tag{5}$$

where A is a normalization factor and C is the heat capacity, $\langle T\rangle$ is the mean effective temperature and $\Delta T = T - \langle T\rangle$ is the variance in the effective temperature. The T_{eff} distribution is fitted with this equation. The heat capacity, C, is extracted from this fit.

The specific heat, C_V, is the heat capacity per unit phase space volume, $C_V=C/\varDelta$, where $\varDelta$ is an estimate of the phase space volume. In lattice calculations $\varDelta=VT^3$. However, in experiments it is simpler to measure the dimensionless quantity C/N where N is the particle multiplicity, and thus $\varDelta=N$[17].

We compare the AMPT model with other model calculations and former experimental results for the specific heat $C_V = C/N$. We also compare results of C_V of the pions, kaons, protons and the charged particles. In Fig. 2 and Fig. 3, C_V decreases with the increase of collision energy. In Fig. 2, the experimental results of Au+Au 0∼5% have a sharp drop at low energy, which hasn't been found in the Au+Au 0∼5% AMPT model results. The C_V of charged particles is smaller than that of pions due to proton contributions, since the C_V of protons is systematically smaller than that of pions over the collision energies.

4 The high order fluctuations of temperuture

We use cumulants to characterize the fluctuations of the effective temperture.The cumulants of event-by-event particle T_{eff} distributions can be expressed in terms of moments with the following relations:

$$C_1 = \langle N\rangle , \tag{6}$$

$$C_2 = \langle N^2\rangle - \langle N\rangle^2 , \tag{7}$$

$$C_3 = 2\langle N\rangle^3 - 3\langle N\rangle\langle N^2\rangle + \langle N^3\rangle , \tag{8}$$

$$C_4 = -6\langle N\rangle^4 + 12\langle N\rangle^2\langle N^2\rangle - 3\langle N^2\rangle^2 - 4\langle N\rangle\langle N\rangle^3 + \langle N\rangle^4 , \tag{9}$$

where the $\langle N^n\rangle$ is the n^{th} order moments[23–25].

The variance, skewness and kurtosis can be calculated by cumulants, respectively[23–25]:

$$\sigma^2 = C_2, \quad S = \frac{C_3}{(C_2)^{3/2}}, \quad \kappa = \frac{C_4}{(C_2)^2} \ . \tag{10}$$

Then, the moment products $S\sigma$ and $\kappa\sigma^2$ can be expressed by:

$$S\sigma = \frac{C_3}{C_2}, \quad \kappa\sigma^2 = \frac{C_4}{C_2} \ . \tag{11}$$

The errors of the moment products $S\sigma$ and $\kappa\sigma^2$ can be calculated according to the delta theorem in statistics[23].

The $T_{\rm eff}$ distributions are nicely described by using the Γ distribution $f(x) = \frac{x^{\alpha-1}{\rm e}^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}}$. The moment can be expressed as:

$$\begin{aligned}\left\langle N^k \right\rangle &= \frac{1}{\Gamma(\alpha)\beta_\alpha}\int_0^\infty x^k x^{\alpha-1}{\rm e}^{-x/\beta}{\rm d}x \ , \\ &= \frac{\beta^k}{\Gamma(\alpha)}\int_0^\infty t^{\alpha+k-1}{\rm e}^{-t}{\rm d}t = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}\beta^k \ , \\ &= \alpha(\alpha+1)\dots(\alpha+k-1)\beta^k \ . \end{aligned} \tag{12}$$

The central moment can be expressed as:

$$\left\langle (N-\langle N\rangle)^k \right\rangle = \sum_{i=0}^{k} C_k^i(-\langle N\rangle)^i \left\langle N^{k-i} \right\rangle \ . \tag{13}$$

Then we can obtain that:

$$S\sigma = 2\beta, \quad \kappa\sigma^2 = 6\beta^2 \ . \tag{14}$$

In both Fig. 4 and Fig. 5, the $S\sigma$ and $\kappa\sigma^2$ decrease with the increase of collision energy. Both of them has a sharp drop at low energy. The $S\sigma$ and $\kappa\sigma^2$ of π are larger than those of charged particles.

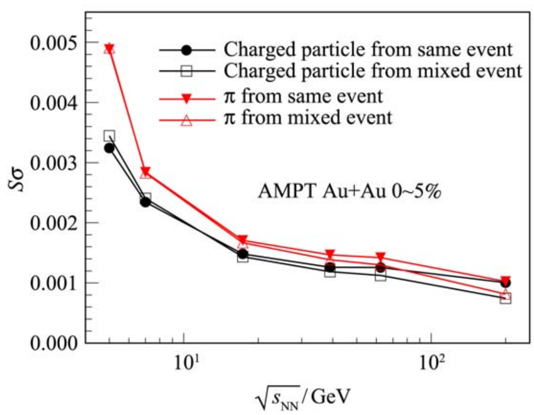

Fig. 4 (color online) Collision energy dependence of the $S\sigma$ in 0∼5% most central Au+Au collisions at $\sqrt{s_{\rm NN}}$ =5 to 200 GeV from AMPT model.

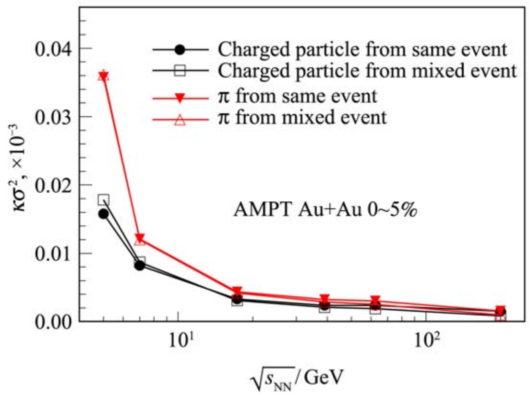

Fig. 5 (color online) Collision energy dependence of the $\kappa\sigma^2$ in 0∼5% most central Au+Au collisions at $\sqrt{s_{\rm NN}}$ =5 to 200 GeV from AMPT model.

5 Summary

In the simulations for 0∼5% most central Au+Au collisions at energies from 5 to 200 GeV, we have reported the calculation results of the specific heat of the charged particles, pions, kaons, and protons from the AMPT model. And we compared the specific heat of charged particles with other model calculations and former experimental results to investigate the system energy dependence. We have also reported the high order flutuations of temperature distributions in AMPT model by calculating the moment products $\kappa\sigma^2$ and $S\sigma$.

By studying the specific heat and temperature flutuations in AMPT model for heavy ion collisions, we find the results of $C_{\rm V}$, $\kappa\sigma^2$ and $S\sigma$ show monotonic distributions in energy dependence, which is expected that there is no phase transition critical point in the AMPT model. This provides a good reference for comparison with experimental data to search for the signal of critical point. At low energies, a sharp drop of $C_{\rm V}$ from the experimental results is observed and it deviates from the AMPT results. Further, more results with the coming STAR experimental data at different energies will offer further exploration of the critical point.

References:

[1] GAVAI R V, GUPTA S. Phys Rev D, 2005, **72**: 054006.

[2] LUO X. EPJ Web of Conferences, 2017, **141**: 04001.

[3] AOKI Y, ENDRODI G, FODOR Z, *et al.* Nature, 2006, **443**: 675.

[4] HARRIS J W, MULLER B. Annu Rev Nucl Part Sci, 1996, **46**: 71.

[5] KARSCH F. Lect Notes Phys, 2001, **583**: 209.

[6] AOKI Y, FODOR Z, KATZ S D, *et al.* Phys Lett B, 2006, **643**(1): 46.

[7] METER-ORTMANNS H. Reviews of Modern Physics, 1996, **68(2)**: 473.

[8] GOCKSCH A. Phys Rev D, 1988, **37**(4): 1014.

[9] FODOR Z, KATZ S D. Journal of High Energy Physics, 2004, **2004**: 050.

[10] DATTA S, GAVAI R V, GUPTA S. Nucl Phys A, 2013, **904-905**: 883c.

[11] KARSCH F, BAZAVOV B, DING H, *et al.* Nucl Phys A, 2016, **956**: 352.

[12] XIN X, QIN S, LIU X. Phys Rev D, 2014, **90**: 076006.

[13] SHI C, WANG Y L, JIANG Y, *et al.* Journal of High Energy Physics, 2014, **2014**: 14.

[14] FISCHER C S, LUECKER J, WELZBACHER C A. Phys Rev D, 2014, **90**(3): 034022.

[15] SA B, LI X, HU S, *et al.* Phys Rev C, 2007, **75**: 054912.

[16] HEISELBERG H. Physics Reports, 2001, **351**(3): 161.

[17] BASU S, CHATTERJEE S, CHATTERJEE R, *et al.* Phys Rev C, 2016, **94**: 044901.

[18] ADAMS J, AGGARWAL M M, AHAMMED Z, *et al.* Phys Rev C, 2005, **72**: 044902.

[19] ADAMCZYK L, ADKINS J K, AGAKISHIEV G, *et al.* Phys Rev C, 2013, **87**: 064902.

[20] LIN Z W, KO C M, LI B A, *et al.* Phy Rev C, 2005, **72**: 064901.

[21] LAUDAU L D, LIFSCHITZ I M. Course of Theoretical Physics, Statistical Physics. 3th Ed. Beijing: World Press, 1999: 338.

[22] MA G L, MA Y G, SA B H, *et al.* High Energy Physics and Nuclear Physics, 2004, **28**(4): 398. (in Chinese)
(马国亮, 马余刚, 萨本豪, 等. 高能物理与核物理, 2004, **28**(4): 398.)

[23] LUO X. J Phys G: Nucl Part Phys, 2012, **39**: 025008.

[24] LUO X, XU J, MOHANTY B, *et al.* J Phys G: Nucl Part Phys, 2013, **40**: 105104.

[25] LUO X, XU N. Nucl Sci Tech, 2017, **28**: 112.

Au+Au重离子碰撞中5~200 GeV碰撞能量下的温度涨落与比热

李秀君[1,2]，司 凡[1,2]，付泽邦[1,2]，张一飞[1,2,†]

(1. 核探测与核电子学国家重点实验室，中国科学技术大学，合肥　230026;
2. 中国科学技术大学近代物理系，合肥　230026)

摘要： 使用多相输运(AMPT)模型来研究相对论重离子碰撞中强子物质的比热($C_{\rm V}$)与对撞能量的关系以及温度的高阶涨落，并将之与文献[PhysRevC.94.044901]实验数据的比热结果进行了比较。对经历相变的系统，比热($C_{\rm V}$)作为表征系统状态方程的热力学量，其值预期在临界点发散。而温度的高阶涨落对相变敏感，比热($C_{\rm V}$)和温度的高阶涨落都是适于探测QCD相变和临界点的敏感探针。通过逐个事例的平均横动量($\langle p_{\rm T}\rangle$)来提取有效温度$T_{\rm eff}$，再通过粒子的有效温度$T_{\rm eff}$的分布提取出了相应粒子的热容。通过有效温度($T_{\rm eff}$)的分布的高阶矩来计算温度的高阶涨落。发现AMPT模型中比热和温度的高阶矩都随温度单调递减。同时还发现在低碰撞能量时，实验数据的比热结果有随能量增加而有一个急速下降，与AMPT模型的走势显著不同。AMPT模型中没有QCD临界点，提供了一个无临界点的参考背景。AMPT模型的计算结果可与实验结果比较作为实验上寻找QCD临界点的参考。

关键词： AMPT模型；QCD临界点；重离子碰撞；比热；统计涨落

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† **通信作者：** 张一飞，E-mail: ephy@ustc.edu.cn。