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# Theoretical Studies of α Condensates in Heavy Self-conjugate Nuclei

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**Abstract:** α condensates are exotic states in nuclear many-body systems, and can be viewed as the generalization of the Bose-Einstein condensate in nuclear physics. It is widely believed that, α condensates exist not only in $^{12}$C, but also in heavier self-conjugate nuclei such as $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, *etc.* It is important to understand the physical properties of these α condensates in heavy self-conjugate nuclei from the theoretical perspective, and the theoretical results could be a useful reference for the experimental studies. This work reviews the basic frameworks to study α condensates, including the Tohsaki-Horiuchi-Schuck-Röpke wave function, the Yamada-Schuck model, and the recently proposed semi-analytic approximation. The impacts of the four-body interactions of α particles on the physical properties of α condensates are reported. The breakup of α condensates and the one-dimensional α condensates are discussed briefly as the possible future directions in this field.

**Key words:** α condensate; self-conjugate nucleus; α clustering
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## 1 Introduction

α condensates are exotic states in nuclear many-body systems, characterized by having α clusters moving in the lowest $0S$ orbit of their common mean field as the building blocks of the states. Proposed by Tohsaki, Horiuchi, Schuck, and Röpke (THSR) in 2001[1], α condensates have been investigated intensively from both the theoretical and experimental perspectives. It is now widely believed that the second $0^+$ state of $^{12}$C at 7.65 MeV, which is also known as the Hoyle state[2], could be an example of the α condensate[3–4]. Indeed, microscopic calculations in the 1970s have already shown that the Hoyle state could be viewed, to the good approximation, as a loose assembly of three α clusters interacting with each other predominantly in the relative $S$ wave, which agrees with the modern concept of α condensates[5–10]. When translated into mathematics, the physical picture of α condensates inspires THSR to propose their famous wavefunction, which, along with its extensions based on the container picture, could provide elegant theoretical descriptions for not only gas-like α condensates but also more compact cluster structures in light nuclei[11–17].

Besides the Hoyle state of $^{12}$C, it is conjectured that α condensates could also be found in heavier self-conjugate nuclei such as $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, *etc.* In Ref. [1] THSR have already pointed out that the $0^+$ excited state of $^{16}$O in the vicinity of the 4α disintegration threshold could be another example of α condensates. This is convinced by later theoretical studies based on the OCM (orthogonality condition model)[18] and THSR wave function[19], which predict that the $0^+_6$ state of $^{16}$O with the excitation energy $E_x = 15.1$ MeV could be the α condensate. The breakup of the excited states of $^{16}$O into many α particles is studied carefully by a recent experiment using the thick target inverse kinematics, which shows that a structure at around 15.2 MeV could be the α condensate analogous to the Hoyle state, This is consistent with theoretical estimations[20].

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Compared with the α condensates in $^{12}$C and $^{16}$O, α condensates in heavier self-conjugate nuclei are less studied in literature. At present, their physical properties are less clear both theoretically and experimentally. The main purpose of this work is to give a brief review of the previous theoretical attempts to fill this gap, and give some comments on several future directions in this field. The following parts of this work are organized as follows: In Section 2, we review several theoretical frameworks proposed in literature to study the α condensates in heavy self-conjugate nuclei, starting with the THSR wavefunction, which is the standard theory for α condensates and initiates the whole field seventeen years ago. In Section 3, we give a brief introduction to several open directions in the field and comment briefly on the possible approaches to study these problems. In Section 4, the conclusions are given.

## 2 Theoretical frameworks

### 2.1 THSR wave function

The THSR wave function is the standard theory for the theoretical study of α condensates and is the starting point of all the honors and glories in the field. It has been applied successfully to study the α condensate in $^{8}$Be, $^{12}$C, and $^{16}$O, and gives theoretical results that agree well with the experimental data. The THSR wave function for the $n\alpha$ system with deformations is given by

$$\begin{aligned}\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z)=&\int \mathrm{d}^3R_1\cdots\mathrm{d}^3R_n\times\\ &\exp\left\{-\sum_{i=1}^{n}\left(\frac{R_{ix}^2}{\beta_x^2}+\frac{R_{iy}^2}{\beta_y^2}+\frac{R_{iz}^2}{\beta_z^2}\right)\right\}\times\\ &\Phi_B(\boldsymbol{R}_1,\cdots,\boldsymbol{R}_n),\end{aligned}\tag{1}$$

$$\begin{aligned}\Phi_B(\boldsymbol{R}_1,\cdots,\boldsymbol{R}_n)=&\det\{\varphi_{0s}(\mathbf{r}_1-\boldsymbol{R}_1)\chi_{\sigma_1\tau_1}\cdots\times\\ &\varphi_{0s}(\mathbf{r}_{4n}-\boldsymbol{R}_n)\chi_{\sigma_{4n}\tau_{4n}}\}.\end{aligned}\tag{2}$$

Here and in the following, the physical coordinates of the nucleons $\{\boldsymbol{r}_i\}$ are often suppressed for convenience. $\varphi_{0s}(\boldsymbol{r})=(\pi b^2)^{-3/4}\exp\left(-\frac{r^2}{2b^2}\right)$ is the coordinate component of the single-nucleon wave function, $\chi_{\sigma\tau}$ is the spin-isospin component of the single-nucleon wave function, and $\Phi_B$ is the famous Brink wave function[21]. With the help of the relation

$$\begin{aligned}&\frac{1}{\sqrt{4!}}\det\{\varphi_{0s}(\boldsymbol{r}_1-\boldsymbol{R})\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}(\boldsymbol{r}_4-\boldsymbol{R})\chi_{\sigma_4\tau_4}\}\\ &=\left(\frac{4}{\pi b^2}\right)^{3/4}\exp\left\{-\frac{2}{b^2}(\boldsymbol{X}-\boldsymbol{R})^2\right\}\phi(\alpha),\end{aligned}\tag{3}$$

and the separation of the center-of-mass wave function, the THSR wave function in Eq. (1) can be simplified further to be

$$\begin{aligned}\Phi_{\mathrm{n}\alpha}\propto\hat{\Phi}_{\mathrm{n}\alpha}\equiv\mathcal{A}\Bigg[&\exp\left\{-\sum_{i=1}^{n}\sum_{k=x,y,z}\frac{2}{B_k^2}(X_{ik}-X_{Gk})^2\right\}\times\\ &\phi(\alpha_1)\cdots\phi(\alpha_n)\Bigg].\end{aligned}\tag{4}$$

Here, $\phi(\alpha)$ is the intrinsic wave function of the α cluster, the parameter $B_{x,y,z}^2=b^2+2\beta_{x,y,z}^2$, $\boldsymbol{X}_i=\frac{1}{4}\sum_{m=1}^{4}\boldsymbol{r}_{im}$ is the center-of-mass coordinate of the $i$th α cluster, and $\boldsymbol{X}_G=\frac{1}{n}\sum_{i=1}^{n}\boldsymbol{X}_i$ is the center-of-mass coordinate of the nα system. $\hat{\Phi}_{n\alpha}$ is the intrinsic wave function of the nα system. Eq. (4) allows an elegant interpretation of the THSR wave function as to describe the motion of nα clusters in a big container centered at the center-of-mass of the $n\alpha$ system with its sizes characterized by $B_x$, $B_y$, and $B_z$ in the $x$, $y$, and $z$ directions.

The THSR wave function can be used in two different ways. The first is to treat the THSR wave function as a trial wave function for the nα system. Suppose the THSR intrinsic wave function with the good angular momentum $J$ is given by $\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})$, with $\boldsymbol{B}$ standing for $\{B_x,B_y,B_z\}$. The free parameters $\boldsymbol{B}$ are determined by surveying the landscape of the energy functional to determine its minimal and saddle points,

$$E_J(\boldsymbol{B})=\frac{\langle\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})|H|\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})\rangle}{\langle\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})|\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})\rangle}.\tag{5}$$

Previous studies show that, the ground state of $^{12}$C is described by the minimal point of $E_{J=0}(\boldsymbol{B})$, while the Hoyle state, *i.e.*, the α-condensate state in $^{12}$C, is described by the first saddle point of $E_{J=0}(\boldsymbol{B})$. The other way to use the THSR wave function is to regard it as another basis to be used to determine the real wave function of the nα system according to the philosophy of the GCM (Generator Coordinate Method). Explicitly, the real wave function $\Psi^J_{\mathrm{n}\alpha}$ of the nα system could be written as

$$\Psi^J_{\mathrm{n}\alpha}=\sum_i f_J(\boldsymbol{B}_i)\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B}_i).\tag{6}$$

Here, $\{\boldsymbol{B}_i\}$ is some discretized representation of the continuum parameter space $\{\boldsymbol{B}\}$. $\{f_J(\boldsymbol{B}_i)\}$ are the superposition coefficients of the THSR basis, and can be determined by solving the Hill-Wheeler equation

$$\sum_j\langle\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B}_i)|H-E|\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B}_j)\rangle f_J(\boldsymbol{B}_j)=0,\tag{7}$$

which is mathematically equivalent to the diagonalization of the Hamiltonian in the nonorthogonal basis $\{\hat{\Phi}^J_{\mathrm{n}\alpha}(\boldsymbol{B})\}$. Both technical routines mentioned above

have been applied to study the Hoyle state in $^{12}$C, with the nucleon-nucleon effective interactions taken to be, *e.g.*, the Tohsaki F1 force[22], the modified Volkov force[23], *etc.* The predicted energy of the Hoyle state agrees well with the experimental data, and the radius of the Hoyle state is found to be around 4.29 fm, which is much larger than the ground-state radius which is found experimentally to be 2.65 fm[1].

In principle, the THSR wave function could also be used to study α condensates in heavier self-conjugate nuclei. In Ref. [19] the α-condensate state in $^{16}$O is studied with the (extended) THSR wave function. As far as we known, at present the theoretical studies on α condensates in self-conjugate nuclei heavier than $^{16}$O are still not available. The possible reasons might be related to the increasing demand on the computational resources, as well as some difficulties in choosing the appropriate effective nuclear forces for heavier systems[24].

### 2.2 Yamada-Schuck model

Concerning the α condensates in heavy self-conjugate nuclei, an important progress was made by Yamada and Schuck in 2004[25]. The aim of their study is to try to figure out how many α clusters could be put into a quasistable α-condensate state. Here, by "quasistable", we mean that the α clusters are bounded by a nonvanishing Coulomb barrier. This question is not only interesting from the theoretical side but also important for the experimental studies. As mentioned before, carrying out a microscopic study on α condensates in self-conjugate nuclei heavier than $^{16}$O is not easy. As a result, in the work by Yamada and Schuck, the α clusters are treated as a point-like bosonic particle. This simplification is not unreasonable if the α-condensate states have large sizes, which could decrease the impacts of the Pauli exclusion principle. Mathematically, the wave function of the $n\alpha$ condensate is given by

$$\Psi_{\mathrm{n}\alpha}(\{\boldsymbol{r}_i\})=\prod_{i=1}^{n}\psi(\boldsymbol{r}_i), \tag{8}$$

where $\psi$ is the normalized single-α wave function and $\boldsymbol{r}_i$ is the coordinate of the $i$th α cluster. Then, the dynamics of the nα system could be described by the Gross-Pitaevskii equation

$$-\frac{\hbar^2}{2m_\alpha}\left(1-\frac{1}{n}\right)\nabla^2\psi(\boldsymbol{r})+U(\boldsymbol{r})\psi(\boldsymbol{r})=\varepsilon\psi(\boldsymbol{r}), \tag{9}$$

$$U(\boldsymbol{r})=(n-1)\int \mathrm{d}\boldsymbol{r}'|\psi(\boldsymbol{r}')|^2 v_2(\boldsymbol{r}',\boldsymbol{r})+\frac{1}{2}(n-1)(n-2)\times\int \mathrm{d}\boldsymbol{r}''\boldsymbol{r}'|\psi(\boldsymbol{r}'')|^2|\psi(\boldsymbol{r}')|^2 v_3(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}''). \tag{10}$$

Here, $m_\alpha$ is the mass of the α particle, $U$ is the common mean field within which all the α clusters move, $v_2$ and $v_3$ are the two-body and three-body interactions of α particles, and $\varepsilon$ is the eigenenergy of the Gross-Pitaevskii equation. The factor $\left(1-\frac{1}{n}\right)$ in the kinematic term of Eq. (9) corresponds to the center-of-mass corrections. Given the two-body and three-body interactions of the α particles, the Gross-Pitaevskii equation Eq. (9) could be solved iteratively, and the physical observables such as the energy and the root-mean-square radius of the nα system could be calculated.

In Ref. [25], the following effective interactions of α clusters are adopted,

$$v_2(\boldsymbol{r},\boldsymbol{r}')=50\exp[-0.4^2(\boldsymbol{r}-\boldsymbol{r}')]-34.101\exp[-0.3^2(\boldsymbol{r}-\boldsymbol{r}')^2]+v_{\mathrm{Coul}}(\boldsymbol{r},\boldsymbol{r}'), \tag{11}$$

$$v_3(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{r}'')=151.5\exp\left\{-0.15[(\boldsymbol{r}-\boldsymbol{r}')^2+(\boldsymbol{r}'-\boldsymbol{r}'')^2+(\boldsymbol{r}''-\boldsymbol{r})^2]\right\}, \tag{12}$$

$$v_{\mathrm{Coul}}(\boldsymbol{r},\boldsymbol{r}')=\frac{4e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{erf}(a|\boldsymbol{r}-\boldsymbol{r}'|). \tag{13}$$

The numerical results for various physical observables could be found in Table 1. It is found that, by adopting the effective interactions Eq. (11)∼(13), for self-conjugate nuclei heavier than $^{44}$Ti, there is no quasi-stable nα condensate that is bounded by the Coulomb barrier. In other words, the quasi-stable α condensate could exist in self-conjugate nuclei from $^{8}$Be to around $^{44}$Ti. This puts an upper limits for the number of α clusters allowed in a quasi-stable α condensate.

Table 1　The energies and root-mean-square radii of the α-condensate states in self-conjugate nuclei from $^{12}$C to $^{44}$Ti given by the Yamada-Schuck model and the semi-analytic approximation.

| $n$ | Nucleus | $E_{\mathrm{YS}}$ /MeV | $\sqrt{\langle r_N^2\rangle}_{\mathrm{YS}}$ /fm | $E_{\mathrm{SAA}}$ /MeV | $\sqrt{\langle r_N^2\rangle}_{\mathrm{SAA}}$ /fm |
|---|---|---|---|---|---|
| 3 | $^{12}$C | 0.98 | 4.87 | 0.99 | 4.88 |
| 4 | $^{16}$O | 1.84 | 5.23 | 1.86 | 5.34 |
| 5 | $^{20}$Ne | 3.04 | 5.55 | 3.09 | 5.74 |
| 6 | $^{24}$Mg | 4.63 | 5.85 | 4.70 | 6.10 |
| 7 | $^{28}$Si | 6.61 | 6.13 | 6.70 | 6.44 |
| 8 | $^{32}$S | 8.99 | 6.40 | 9.10 | 6.77 |
| 9 | $^{36}$Ar | 11.8 | 6.68 | 11.88 | 7.10 |
| 10 | $^{40}$Ca | 15.0 | 6.95 | 15.05 | 7.46 |
| 11 | $^{44}$Ti | 18.6 | 7.24 | 18.58 | 7.86 |

### 2.3 Semi-analytic approximation and four-body interaction of α particles

In Ref. [26], the authors propose a semi-analytic approximation to the Yamada-Schuck model, and use this approximation to study the impacts of the four-body interaction of α particles on the physical properties of nα condensates. In the semi-analytic approximation, the realistic single-α wave function $\psi(\boldsymbol{r})$ is approximated by the ground-state harmonic-oscillator (HO) wave function

$$\psi(\boldsymbol{r})=\left(\frac{2}{\pi B^2}\right)^{3/4}\exp\left(-\frac{r^2}{B^2}\right). \tag{14}$$

Compared with the THSR wave function and the container picture in Section 2.1, the HO parameter $B$ could be identified loosely with the container-size parameter $B$ of Eq. (4) in the spherical THSR wave function. Therefore, to certain extent, the trial wave function in Eq. (27) could be treated as the bosonic correspondence of the spherical THSR wave function. Microscopic calculations have shown that a single THSR wave function is an excellent approximation to the realistic many-body wave function of the α-condensate systems. It is nature to guess that the single trial function in Eq. (27) could also be a good approximation of the realistic wave function in the Yamada-Schuck model. Our study shows that this is indeed the case. Adopting the effective interactions Eq. (11)∼(13), the free parameter $B$ could be determined by the variational method, and the physical observables could be calculated correspondingly.

The semi-analytic approximation allows us to study further the impacts of the four-body interaction of α particles without increasing significantly the computational burden. Previous theoretical studies show that α particles have not only the two-body and three-body interactions but also the four-body ones[18]. However, it is fair to say that, at present, the physical properties of the four-body interactions are less known both from the theoretical side and from the experimental side. Therefore, we consider the following ansatz for the four-body interaction

$$\begin{aligned}v_4(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3,\boldsymbol{r}_4)=U_4\exp\Big\{-\mu_v^2\big[&(\boldsymbol{r}_1-\boldsymbol{r}_2)^2+\\&(\boldsymbol{r}_1-\boldsymbol{r}_3)^2+(\boldsymbol{r}_1-\boldsymbol{r}_4)^2+(\boldsymbol{r}_2-\boldsymbol{r}_3)^2+\\&(\boldsymbol{r}_2-\boldsymbol{r}_4)^2+(\boldsymbol{r}_3-\boldsymbol{r}_4)^2\big]\Big\}.\end{aligned} \tag{15}$$

Here, we only consider the possibility for the four-body interaction to be repulsive ($U_4>0$) to avoid the appearance of the so-called collapsed state as mentioned in Ref. [25]. With the parameter $U_4$ and $\mu_v$ taking some specific values, the values for the parameter $B$ could be found by locating the position of the energy pocket of the energy functional $E_{n\alpha}(B)$ through variational calculations, and the results for $^{28}$Si, $^{32}$S, $^{36}$Ar, and $^{40}$Ca could be found in Fig. 1, along with the corresponding values for the parameter $\widetilde{B}$ for the Coulomb barrier. It is found that, when the strength of the four-

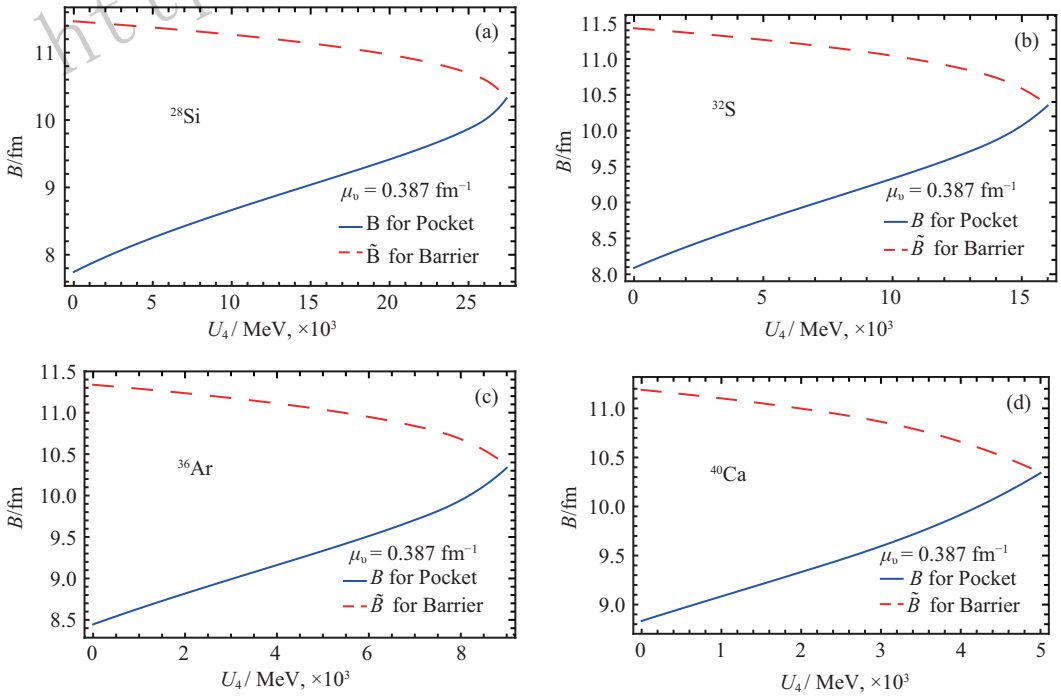


Fig. 1 (color online) The parameter $B$ for the energy pocket and the parameter $\widetilde{B}$ for the energy barrier versus the four-body interaction strength $U_4$ for the self-conjugate nucleus $^{32}$S, with the parameter $\mu_v=0.387$ fm$^{-1}$.

body interaction increases, the parameter $B$ for the energy pocket also increases gradually, which means that the repulsive four-body interactions tend to increase the size of the α condensate, while the parameter $\widetilde{B}$ for the energy barrier decreases gradually. At some critical value, the parameter $B$ coincides with the parameter $\widetilde{B}$, which means that the quasi-stable nα condensate no longer exist for the four-body interaction exceeding the critical strength. For instance, for $^{32}$S, this critical strength is found to be $U_4 \approx 16000$ MeV when $\mu_v = 0.387$ fm$^{-1}$[18]. In other words, depending on the size of the four-body interaction, the critical self-conjugate nucleus that allows the existence of the quasi-stable α condensate with the maximal number of α clusters may be (much) lighter than $^{44}$Ti[26]. This result could be important for the experimental research on the α condensate in heavy self-conjugate nuclei.

## 3 Future directions

In this section, we would like to discuss some future directions in studying α condensates in heavy self-conjugate nuclei, along with comments on some basic ideas of our group to study these problems.

### 3.1 Breakups of α condensates

The breakups could provide value information on the physical properties of the α condensate and have been used to study the α-condensate state in $^{12}$C and $^{16}$O experimentally[20, 27–28]. In Ref. [27–28], stringent upper bounds are placed for the 3α direct decays from the Hoyle state of $^{12}$C, whose theoretical explanations are now an important open question to the field of α condensates. Recently, the breakup reactions of $^{16}$O into $^{8}$Be + $^{8}$Be and $\alpha + {}^{12}$C have been studied experimentally for the sixth $0^+$ excited state at $E_x = 15.2$ MeV. Although the statistical errors are still not small enough to put the final words on the nature of this $0^+_6$ state, the experimental results are encouragingly consistent with the assumption of the α condensate[20]. On the theoretical side, there are also important results, among which we would like to mention the works of Ref. [29] that, for a given α condensate different partitions involving the α-condensed subsystems should be populated with equal probabilities, as in the α condensate all the α clusters occupy approximately the same lowest 0S orbit of their common mean field. Take the α-condensate state of $^{24}$Mg as an example, which consists of six α clusters. In this case, we could have the following breakup patterns

$$^{24}\mathrm{Mg}^* \to {}^{20}\mathrm{Ne}^* + \alpha, \quad (16)$$

$$\to {}^{16}\mathrm{O}^* + {}^{8}\mathrm{Be}, \quad (17)$$

$$\to {}^{12}\mathrm{C}^* + {}^{12}\mathrm{C}^*, \quad (18)$$

$$\to {}^{16}\mathrm{O}^* + \alpha + \alpha, \quad (19)$$

$$\to {}^{12}\mathrm{C}^* + {}^{8}\mathrm{Be} + \alpha, \quad (20)$$

$$\to {}^{8}\mathrm{Be} + {}^{8}\mathrm{Be} + {}^{8}\mathrm{Be}, \quad (21)$$

$$\to {}^{12}\mathrm{C}^* + \alpha + \alpha + \alpha, \quad (22)$$

$$\to {}^{8}\mathrm{Be} + \alpha + \alpha + \alpha + \alpha, \quad (23)$$

$$\to \alpha + \alpha + \alpha + \alpha + \alpha + \alpha. \quad (24)$$

Here, by the superscript $*$ we refer to the α-condensate state in the corresponding nucleus. Then, according to the equal population approximation, the formation probabilities of all the above partitions from the parent α condensate should be equal to each other. The equal population approximation is of special interest to nuclear experimentalists. Recently, it has been adopted by the authors of Ref. [28] and Ref. [20] in their computation of the theoretical reference values or the physical interpretation of the experimental results.

Concerning the breakups of α condensates, at present and in the near future, the authors are of special interest in the problem to carry out the microscopic studies of the breakup reactions of α condensates in heavy self-conjugate nuclei, especially the the nα direct decays. For the Hoyle state, its breakup into $^{8}$Be + α has been studied by using the resonating group method[7, 9] and the THSR wave function[30–32]. Similar microscopic calculations could also be found for the breakup of the α-condensate state of $^{16}$O into $^{12}\mathrm{C} + \alpha$[31]. However, at present, there are few microscopic studies on other possible breakup channels of the α-condensate states, and it is important to fill this blank. These theoretical results could also be helpful for further comparisons between experimental data and theoretical results. A byproduct of these studies is to check explicitly the validity of the equal population approximation, which is still not available at present. Concerning its current role in interpreting experimental results, it is necessary to have a detailed check of its validity.

### 3.2 One-dimensional α condensates

Another important direction is the study of one-dimensional α-condensate states of self-conjugate nuclei. The possibility of the linear chain state made of α clusters was first proposed by Morinaga in the 1950s[33–34]. Since then, the α-chain state and its generalizations have been studies by various authors using different methods. For instance, the AMD (antisymmetrized molecular dynamics) and FMD (fermionic molecular dynamics) calculations suggest that the $0^+$ state at 10.3 MeV of $^{12}$C could be a 3α-chain state[35–36]. For the 4α-chain states in $^{16}$O, Ref. [37] re-

ports the observation of the $2^+$, $4^+$, and $6^+$ states with the excitation energies around 20 MeV, which could be identified as members of the rotational band of the 4α-chain state. The properties of these excited states are also studied by Ref. [38–39] and similar results are obtained. Recently, it is pointed by Ref. [40] that the physical properties of the α-chain states in $^{12}$C and $^{16}$O could be captured elegantly by the extremely deformed THSR wave function, suggesting that these α-chain states correspond to one-dimensional α condensates. Explicitly, given the nα THSR wave function Eq. (4), the α-chain states are realized by putting the constraint

$$\beta_x = \beta_y = 0 \quad \Longrightarrow \quad B_x = B_y \approx b. \tag{25}$$

The value of the parameter $b$ depends on the choice of nucleon-nucleon forces and is typically found to be around 1.36 fm. With such an ansatz, it is obtained in Ref. [40] that the single extremely deformed THSR wave functions have the squared overlaps of 99% and 94% with respect to the GCM wave function.

It is interesting and important to study the physical properties of one-dimensional α condensates in self-conjugate nuclei heavier than $^{16}$O. Noticeably, very recently, Tohsaki and Itagaki propose the possibility of the existence of long α-chain states with more than sixty α clusters based on the Brink wave function calculations[42]. If their calculations can describe the real nature qualitatively, it would be interesting to see whether these long α-chain states correspond to one-dimensional α condensates in a way similar to those in $^{12}$C and $^{16}$O. One way to study one-dimensional α condensates in heavy self-conjugate nuclei is to extend the THSR analysis to heavier systems, which is possible in principle but might not be easy due to the increasing number of nucleons. Another way is to construct phenomenological models similar to the Yamada-Schuck model in Section 2.2, in which the alpha clusters are regarded as point-like bosons and interact with each other through the effective interactions determined by phenomenological considerations. The corresponding many-body wave function could be approximated by the trial wave function

$$\Psi_{n\alpha}(\{\boldsymbol{r}_i\}, B_\perp, B_\parallel) = \prod_{i=1}^{n} \psi(\boldsymbol{r}_i, B_\perp, B_\parallel), \tag{26}$$

$$\psi(\boldsymbol{r}, B_\perp, B_\parallel) = \left(\frac{2}{\pi B_\perp^2}\right)^{1/2} \left(\frac{2}{\pi B_\parallel^2}\right)^{1/4} \times \exp\left(-\frac{x^2+y^2}{B_\perp^2} - \frac{z^2}{B_\parallel^2}\right), \tag{27}$$

which describes the one-dimensional α condensate is placed along the $z$ direction and is axial symmetric. The parameters $B_\perp$ fm and $B_\parallel$ correspond to the container sizes in the $x$ ($y$) and $z$ direction, respectively, and could be determined by variational calculations with the constraint $B_\perp \approx 1.36$ fm. More discussions on the choice of effective interactions of α clusters and the numerical results for the physical properties of one-dimensional α condensates could be found in an upcoming publication by the authors[41].

## 4 Conclusions

The α condensates are a novel kind of gas-like states of α clusters in self-conjugate nuclei, and could be viewed as the counterpart of the famous Bose-Einstein condensation in nuclear physics. The α condensates in $^8$Be, $^{12}$C, and $^{16}$O have been studied by various authors from both the theoretical and experimental viewpoints. Especially, the α condensate in $^{12}$C, *i.e.*, the Hoyle state, is widely regarded as a flagship of the whole field, and has been studied intensively using various different methods in the past years[3–4]. On the other hand, the physical properties of the α condensates in heavier self-conjugate nuclei are less known, both experimentally and theoretically, and it is important to fill this blank. In this work, we review the basic formalism and some recent progress in the study of α condensates in heavy self-conjugate nuclei, including the THSR wave function as the standard theory for the α condensate, the Yamada-Schuck model as a convenient bosonic model to study α condensates in heavy systems, and the semi-analytic approximation which could simplify the calculations of the Yamada-Schuck model and shed light on the impacts of four-body interactions of α clusters. Moreover, we also comment on some further directions in this field, such as the breakup of α condensates and the one-dimensional α condensate. It is hoped that the present work could be helpful for future experimental and theoretical studies in this direction.

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# 重自共轭原子核中 α 凝聚体物理性质的理论研究

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**摘要：** 原子核多体系统中可以存在一类被称为 α 凝聚体的奇异物理态。该奇异态可以被视为玻色-爱因斯坦凝聚在原子核物理中的推广。一般认为，α 凝聚体不仅可以存在于$^{12}$C中，也可以存在于诸如$^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si等质量更重的自共轭原子核中。重自共轭原子核中的 α 凝聚体的物理性质是核结构理论重要的研究课题，相关理论计算可以为实验研究提供有益参考。主要介绍了该研究方向的基本理论框架，包括Tohsaki-Horiuchi-Schuck-Röpke波函数方法、Yamada-Schuck模型，以及近期提出的半解析近似方法。还讨论了 α 粒子间四体相互作用对 α 凝聚体物理性质的影响，并对 α 凝聚体破裂和一维 α 凝聚体等可能的研究方向做了简要论述。

**关键词：** α 凝聚体；自共轭原子核；α 结团效应

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