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Skyrmion Properties in an Uniform Magnetic Field

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Abstract: The mass and shape of skyrmion in an uniform magnetic field are investigated. Base on the symmetry of the system, an axially symmetric ansatz of the soliton is proposed to perform the study. The baryon number is shown to be always conserved even in a nonzero magnetic background. It is found that with the increase of the strength of magnetic field, the static mass of skyrmion first decreases then increases, as the dominant role shift from the linear term of magnetic field to the quadratic term of magnetic field, while the soliton size first increase then decrease. Finally, in the core part of magnetar, the equation of state have strong dependence of magnetic field, which also modifies the mass limit for magnetar.

Key words: Skyrme model; strong magnetic field; magnetar

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1 Introduction

The hadron properties in the strong magnetic field is one of the hot research topics during the past decades, as the strong magnetic field widely exists in heavy ion collision experiments and magnetar[1−3]. In hadron physics, the strong magnetic field opens a new approach to understand the QCD dynamics, the equation of state of magnetar, *etc*. At present, the mass and shape of meson in the strong magnetic field background is widely studied. For example, results from the Lattice QCD calculation imply that the shape of quark potential is changed in the strong magnetic field[4]. However, the dynamics of baryon in the strong magnetic field background is still unclear.

Following Ref. [5], the mass and shape of baryon in an uniform magnetic field are discussed. In the present study, the Skyrmion is identified as the baryon. In Sec. 2, the way how to construct the Skyrme model for the present propose is described and its typical features relevant to this work are shown. In Sec. 3, the magnetic response on mass and shape of Skyrmion are shown numerically. In the last section a brief conclusion and discussion are given.

2 Model

In the following section, the Skyrme model and the incorporation of magnetic field are briefly reviewed.

2.1 Lagrangian

For the typical Skyrme model[6], its Lagrangian can be described as

$$\varGamma = \int \mathrm{d}^4 x \mathscr{L} + \varGamma_{\mathrm{WZW}} \ , \tag{1}$$

where $\int \mathrm{d}^4 x \mathscr{L}$ is the intrinsic parity even part and $\varGamma_{\mathrm{WZW}}$ is the intrinsic parity odd part.

The intrisic parity even part is expressed as follows

$$\mathscr{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(D_\mu U^\dagger D^\mu U) + \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger - 2) + \frac{1}{32g^2}\mathrm{Tr}([U^\dagger D_\mu U, U^\dagger D_\nu U]^2) \ , \tag{2}$$

where f_π, m_π and g are the pion decay constant, the pion mass and a dimensionless coupling constant, respectively.

The covariant derivative for U is expressed as

$$D_\mu U = \partial_\mu U - \mathrm{i}\mathcal{L}_\mu U + \mathrm{i} U \mathcal{R}_\mu \ , \tag{3}$$

where the external fields $\mathcal{L}$ and $\mathcal{R}$ are expressed as

$$\mathcal{L}_\mu = \mathcal{R}_\mu = eQ_{\mathrm{B}}\mathcal{V}_{B\mu} + eQ_{\mathrm{E}}H_\mu \tag{4}$$

for the present purpose. Here e is the unit electric charge, $Q_{\mathrm{B}} = \frac{1}{3}\mathbb{1}$ is the baryon number charge matrix, $Q_{\mathrm{E}} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3$ is the electric charge matrix, $\mathbb{1}$ is the rank 2 unit matrix, τ_3 is the third pauli matrix, $\mathcal{V}_{B\mu}$ is

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the external gauge field of the $U(1)_V$ baryon number, and H_μ is the magnetic field expressed as

$$H_\mu = -\frac{1}{2}Byg_\mu{}^1 + \frac{1}{2}Bxg_\mu{}^2 , \tag{5}$$

in the symmetric gauge.

The intrinsic parity odd part, *i.e.*, WZW action $\Gamma_{\rm WZW}$, represents the chiral anomaly effects, which is given in Ref. [7].

2.2 Ansatz

In Skyrme model, the ansatz is needed to perform the calculation. Based on the symmetry of the system, an axially symmetric ansatz is introduced for the present purpose.

In the strong magnetic field background, the $SO(3)_{\rm space}$ and $SU(2)_{\rm flavour}$ symmetry of hadron are explicitly broken. To mimic the symmetry patten of the system, the ansatz should satisfies two kinds of formulas, i.e., when magnetic field is small, the ansatz is spherically symmetric hedgehog ansatz with the symmetry of $SO(3)_{\rm spin}\times SU(2)_{\rm isospin}$, which is widely used in the literatures; when magnetic field is strong, the space and isospin symmetry are explicitly broken by the magnetic field, the ansatz only holds the symmetry $SO(2)_{\rm spin}\times U(1)_{\rm isospin}$, which is an axially symmetric ansatz. Additionally, when the strength of the magnetic field is non-zero, the directions of the unbroken symmetries are correlated with each other, therefore, the third direction of iso-spin space, τ_3, is always along with the direction of magnetic field, namely, the z axis.

With respect of the symmetry as stated above, U is decomposed as[8]

$$U = \cos(F(r))\mathbb{1} + \frac{\mathrm{i}\sin(F(r))}{r}\left(\frac{\tau_1}{c_\rho}x + \frac{\tau_2}{c_\rho}y + \frac{\tau_3}{c_z}z\right), \tag{6}$$

where

$$\begin{aligned} x &= c_\rho r\sin(\theta)\cos(\varphi) , \\ y &= c_\rho r\sin(\theta)\sin(\varphi) , \\ z &= c_z r\cos(\theta) . \end{aligned} \tag{7}$$

Here $r = \sqrt{(\frac{x}{c_\rho})^2 + (\frac{y}{c_\rho})^2 + (\frac{z}{c_z})^2}$, c_ρ and c_z are positive dimensionless parameters, and θ and φ are polar angle with $\theta \in [0,\pi]$ and $\varphi \in [0,2\pi]$. The integrate element for a unit volume corresponding to (7) is $\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z = c_\rho^2 c_z r^2 \sin(\theta)\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi$.

2.3 Baryon number current

The baryon number current of the model is obtained by taking a functional derivative of the WZW term with $\mathcal{V}_{\mathrm{B}\mu}$, *i.e.*, $j_B^\mu = \frac{\partial \Gamma_{WZW}}{\partial(e\mathcal{V}_{B\mu})}|_{\mathcal{V}_{B\mu}\to 0}$. The explicit form of the baryon number current is expressed as

$$\begin{aligned} j_{\rm B}^\mu = & \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\Big\{ -\mathrm{i}(\alpha_\nu\alpha_\rho\alpha_\sigma + \beta_\nu\beta_\rho\beta_\sigma) - \\ & 3(\partial_\nu\mathcal{L}_\rho\alpha_\sigma + \partial_\nu\mathcal{R}_\rho\beta_\sigma) + \\ & 3\mathrm{i}(\mathcal{L}_\nu\alpha_\rho\alpha_\sigma - \mathcal{R}_\nu\beta_\rho\beta_\sigma) + \\ & 2(\partial_\nu\mathcal{R}_\rho U^\dagger\mathcal{L}_\sigma U - \partial_\nu\mathcal{R}_\rho\mathcal{R}_\sigma) - \\ & 2(\partial_\nu\mathcal{L}_\rho U\mathcal{R}_\sigma U^\dagger - \partial_\nu\mathcal{L}_\rho\mathcal{L}_\sigma) + \\ & 2\mathrm{i}(U\mathcal{R}_\nu U^\dagger\mathcal{L}_\rho\alpha_\sigma + U^\dagger\mathcal{L}_\nu U\mathcal{R}_\rho\beta_\sigma) + \\ & \mathrm{i}(\mathcal{R}_\nu\mathcal{R}_\rho\mathcal{R}_\sigma - \mathcal{L}_\nu\mathcal{L}_\rho\mathcal{L}_\sigma)\Big\}\Big|_{\mathcal{V}_{B\mu}\to 0} , \end{aligned} \tag{8}$$

where ϵ is the Levi–Civita symbol with $\epsilon^{0123} \equiv -1$, $\alpha_\mu = \frac{1}{\mathrm{i}}(\partial_\mu U)U^\dagger$, and $\beta_\mu = \frac{1}{\mathrm{i}}U^\dagger\partial_\mu U$.

Insert the ansatz Eqs. (6) and (7) into (8), the time component of the baryon number current is obtained as

$$\begin{aligned} j_{\rm B}^0(r,\theta) = & -\frac{1}{c_\rho^2 c_z}\frac{F'\sin^2(F)}{2\pi^2 r^2} + \\ & eB\frac{\left(F'\left(\cos^2(\theta) - \sin^2(\theta)\sin^2(F)\right)\right)}{8\pi^2 c_z} + \\ & eB\frac{\left(\sin^2(\theta)\sin(F)\cos(F)\right)}{8\pi^2 r c_z} . \end{aligned} \tag{9}$$

The baryon number $N_{\rm B}$ is obtained as

$$\begin{aligned} N_{\rm B} = & \int \mathrm{d}V j_{\rm B}^0(r,\theta) \\ = & \left(\frac{\sin(2F) - 2F}{2\pi} + eBc_\rho^2\frac{r^2\sin(2F)}{12\pi}\right)\Bigg|_{F(0)=\pi}^{F(\infty)=0} \\ = & 1 . \end{aligned} \tag{10}$$

Notice in the present calculation the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$ are imposed. Here the baryon number $N_{\rm B}$ is shown to be *always* normalized as one, which is consistent with the fact that the appearance of magnetic field does not break the $U(1)_V$ symmetry.

3 Numerical results

In this section, the soliton mass, the rootmeansquare (RMS) radius of the baryon number density, and also the RMS radius of energy density are shown numerically.

To evaluate the twist effect of skyrmion, the parameters c_ρ and c_z are imported. However, $c_{\rho,z}$ does not only twist the shape of the skyrmion, but also scale the volume. One can easily show that the scale effect of $c_{\rho,z}$ can be absorbed by performing the scale transformation, $r \to r/\lambda$, where λ is an arbitrary nonzero positive real number. With no loose of generality, a further request is imposed as

$$c_\rho \equiv 1/\sqrt{c_z} . \tag{11}$$

Substituting the ansatz Eqs. (6) and (7) into (1), the soliton mass $M_{\rm sol} = \int {\rm d}V \mathscr{M}_{\rm sol}(r,\theta)$ is obtained. Here $\mathscr{M}_{\rm sol}(r,\theta)$ is the energy density of the soliton. In numerically calculating the mass and shape of skyrmion, the standard parameter sets are chosen to reproduce the mass difference of proton and delta[9], *i.e.*, $m_\pi = 138$ MeV, $f_\pi = 108$ MeV and $g = 4.84$.

The parameter c_z is fixed to minimize the soliton mass $M_{\rm sol}$ for a given $|eB|$. The $|eB|$ dependence of $M_{\rm sol}$ and c_z are shown in Fig. 1 and Fig. 2.

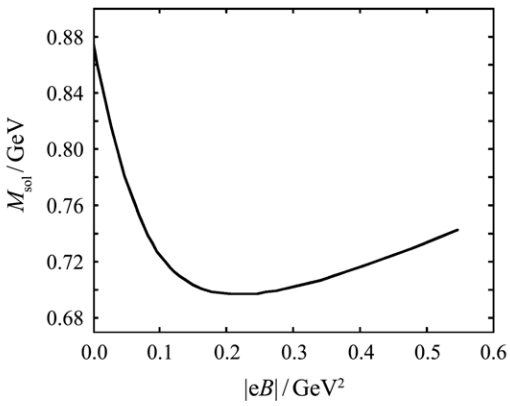

Fig. 1 (color online) $|eB|$ dependence of soliton mass $M_{\rm sol}$.

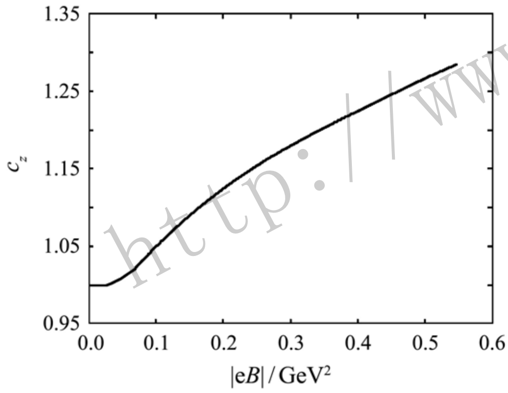

Fig. 2 (color online) $|eB|$ dependence of c_z.

Fig. 1 shows that the soliton mass $M_{\rm sol}$ first decreases and then increases. This is because the $M_{\rm sol}$ contains both (eB) and $(eB)^2$ terms, when the strength of magnetic field increases, the dominant role shift from the term proportional to (eB) to the term proportional to $(eB)^2$, therefore the soliton mass first decreases and then increases.

Fig. 2 shows that the parameter c_z increases with the increase of the magnetic field. Which implies the shape of baryon is stretched along z axis. This is because of that, for a strong $|eB|$, the charged meson $(\pi^{+,-})$ is restricted in the x-y plane, but the neutral meson (π^0) is free to move along z axis, which causes the parameter c_z to increase.

To evaluate the shape deformation of Skyrmion caused by magnetic field numerically, the RMS radius for baryon number density and energy density are introduced. The definition of them are

$$\langle X^2\rangle_{\rm B}^{1/2} = \sqrt{\int_0^\infty {\rm d}V X^2 j_{\rm B}^0}\,, \tag{12}$$

$$\langle X^2\rangle_{\rm E}^{1/2} = \sqrt{\frac{1}{M_{\rm sol}}\int_0^\infty {\rm d}V X^2 \mathscr{M}_{\rm sol}}\,, \tag{13}$$

where X represents R_x, R_z and R. Here $R \equiv \sqrt{x^2+y^2+z^2}$, R_x and R_z represent the projection of R on x and z axis, respectively.

The RMS radius for baryon number density and energy density are shown in Fig. 3 and Fig. 4.

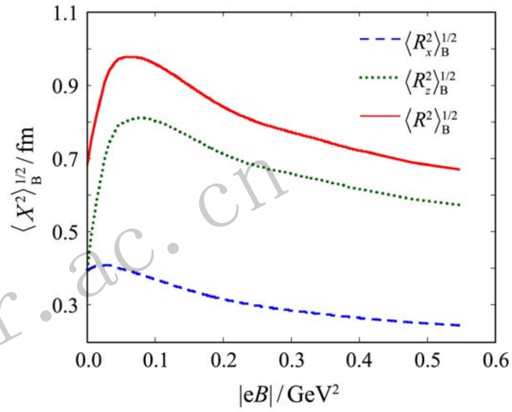

Fig. 3 (color online) $|eB|$ dependence of the RMS radius for the baryon number density.

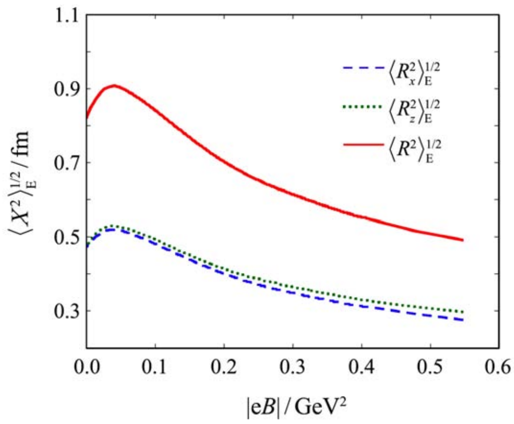

Fig. 4 (color online) $|eB|$ dependence of the RMS radius for the energy density.

Fig. 3 and Fig. 4 show that the RMS radii first increase and then decrease. The reason of this tendency is that: for a weak $|eB|$, the baryon mass decreases, therefore the baryon size increase, *i.e.*, the RMS radii increase; for a strong $|eB|$, the charged meson $(\pi^{+,-})$ is restricted in the x-y plane, which causes the RMS radii to decrease.

Fig. 3 and Fig. 4 also show that when $|eB| \neq 0$, the projection of charge radius R on x axis is always smaller than that on z axis, which is consistent with Fig. 2, *i.e.*, the shape of baryon is stretched along z axis.

4 Conclusions and discussions

By making use of an axially symmetric ansatz, the baryon mass and shape dependence of magnetic field are investigated in a Skyrme model.

It is found that the shape of baryon is stretched along the direction of magnetic field, as the charged pion are restricted in the x-y plane. However, through the baryon number distribution is complicated, the baryon number is shown to be always conserved, as the appearance of magnetic field does not break the $U(1)_V$ symmetry.

The results show that the baryon mass first decrease and then increase when the strength of magnetic field increases, this is because of that the dominant role shift from the term proportional to (eB) to the term proportional to $(eB)^2$. It is also shown that the charge radii of baryon first increase and then decrease with the increase of magnetic field. Since both the mass and charge radii of baryon depend on the strength of magnetic field, the equation of state depend on the strength of magnetic field. The present study shows that the baryon density and the energy density decreases by about 28% and 14%, respectively, compared to that in vacuum, which means the equation of state is significantly modified by the magnetic field. As a result, to get the correct mass-radius relation for magnetars, the effects of magnetic field also should be considered. For example the Skyrmion crystal approach or BPS Skyrmion approach might give a better description of magnetar[10–12].

In the present calculation, the main range of $|eB|$ is less than $m^2_{\rho,\omega}$, and only pion fields are considered. For the ultra strong magnetic field ($|eB| > m^2_{\rho,\omega}$), the inclusion of vector mesons and also scalar mesons becomes necessary[13]. In the present analyse, the leading order of large N_C is discussed, one extension of the model is to include N_C^{-1} effects to separate physical baryon states, *e.g.*, proton and neutron. The magnetic response of physical baryon states is discussed in Ref. [14].

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斯格明子在均匀磁场中的性质

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摘要： 主要研究了斯格明子的质量和电荷半径在均匀磁场中的变化性质。基于系统所拥有的对称性，本研究采用一组拥有轴对称性的拟设方程式以便进行研究工作。本研究证明重子数即使在非零磁场中也是一个守恒量。本研究发现随着磁场强度的增加，斯格明子的质量和电荷半径对磁场的依赖方式不同：由于磁场的主要贡献由线性项过渡到平方项，因此随着磁场的增强，斯格明子的质量先下降然后上升；与此相对应的，随着磁场的增强，斯格明子的电荷半径先增加然后减小。最后，本研究发现磁星内部的状态方程式对磁场强度有强烈的依赖，因此在理论计算磁星的质量上限时应考虑磁场所产生的影响。

关键词： 斯格明模型；强磁场；磁星

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