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Progress in the Study of Multi-quark States

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Abstract: The progress in the study of multi-quark states for the last half century is reviewed schematically and the dibaryon sector is emphasized. By employing the dynamical symmetry, the Gursey-Radicati mass formula, which can give a reasonable description of the masses of multi-quark states, can be reproduced. The dibaryons in bag model and realistic quark model, quark delocalization color screening model, are discussed.

Key words: Multi-quark state; dibaryon

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1 Introduction

Gell-Mann and Zweig proposed the quark model to explain the hadron (including both meson and baryon) structure independently in 1964[1]. Dyson and Xuong extended quark model to 4, 5, 6 valence quark states in the same year[2], they simplified the Gursey-Radicati mass formula[3] for multi-quark states to $SU(2)$ case and fixed the parameters of the spin, isospin dependent terms by fitting the deuteron and the NΔ resonance D_{12} masses, then predicted the masses of D_{03} and D_{30} to be around 2.35 GeV. Kamae *et al.*[4] studied the proton polarization from the γ-deuteron disintegration in 1977—1979 and found a resonance structure around 2.35 GeV which might be related to the D_{03} predicted by Dyson and Xuong. Kamae, Swart, Wong calculated the mass of D_{03} with meson exchange and bag models[5]. Unfortunately these results had not been able to attract the attention of the hadron physics community those years. Jaffe[6] predicted a deep bound dibaryon of $SU(3)$ flavor singlet with strangeness $S=-2$ in 1977, which is called H particle and had inspired tremendous efforts both theoretical calculations and experimental searches for many years. In 1980's and 1990's there were many dibaryons "discovered" and disappeared. These null results depressed the study of multi-quark states[7]. Moscow-Tuebingen-Warsaw-Uppsala collaboration claimed a 2.06 GeV 0^- d′ dibaryon in 1993 which was doubted within the hadron physics community and the same group found it is a spurious signal due to a flaw of their detector[8]. Nakano *et al.*[9] claimed found a $S=1$ penta-quark state Θ^+ around 1540 MeV in 2003. This claim had been "confirmed" by many experimental groups including the high precision Jlab measurement. It is almost the first experimentally confirmed multi-quark state. But the further measurement of the same group at Jlab had not been able to confirm the signal[10].

The real multi-quark era started by the heavy flavor meson spectroscopy. CLEO-c, BaBar, Belle, BESIII, CDF, D0, LHCb, CMS collaborations have observed heavy meson states called X, Y, Z states which can not be accommodated within the simple $Q\bar{Q}$ configuration since 2003. Especially Belle, BESIII, and other collaborations have observed the charged Z_c and Z_b which certainly can not be the pure $c\bar{c}$ and $b\bar{b}$ states since 2008. LHCb observed two J/ψp resonances in 2015 which might be penta-quark states. WASA-at-COSY collaboration observed a resonance structure in the pd → pdππ reaction in 2009 and confirmed it is an $IJ^P=03^+$ dibaryon resonance, which is called d* in a theoretical model prediction[12], through the polarized pd scattering in 2014[11]. Up to now all of the 4, 5, 6 quark states are observed. A new horizon of multi-quark world appeared.

There is already a comprehensive review article of 4,5 quark states[13], we will mainly discuss the dibaryon story in this paper.

2 The discovery of d* dibaryon

As had been mentioned in the introduction, di-

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baryon had been discovered and disappeared many times, the predecessor of WASA-at-COSY collaboration (Moscow-Tuebingen-Warsaw-Uppsala collaboration) had misidentified one dibaryon d′ in 1990's. They continue the dibaryon exploration by a better accelerator and detector with the experience of misidentification of a dibaryon. Thanks to the new high quality experimental facility, the WASA almost 4π detector and COSY with high quality beam and target, they can do an exclusive and kinematical over-complete measurements which made them discover a dibaryon which already rejected by "precise" NN scattering data, in fact still not precise enough and so missed a weak dibaryon signal buried in the few order higher than usual NN scattering processes.

In 2009 they found that their measured pd → pd$\pi^0\pi^0$ reaction cross section can not be explained by the usual t-channel meson exchange ΔΔ excitation process and must include an s-channel resonance in the ΔΔ system. In 2011 with more accurate data they fixed this is an $J^P=3^+$ $di-\Delta$ resonance with mass $M\sim 2.37$ GeV and width $\varGamma\sim 70$ MeV. The subsequent measurements fixed it is an isospin 0 instead of 1 state. In 2014 they measured the analyzing power of polarized **n**p scattering, the resulting data show a resonance structure in the energy dependence-right at the previous observed resonance energy region. Combining their new data and the SAID phase shift data base they co-performed a new partial wave analysis which produces a pole in the NN $^3D_3-^3G_3$ coupled channel at $2\,380\pm10-\mathrm{i}(40\pm5)$ MeV[11]. Confirmed there is really an *inevitable* dibaryon d*[12].

3 Dynamical symmetry in quark model

Before QCD, there was no dynamics for hadron structure. One can only guess the hadron structure with symmetry consideration. Based on $SU(3)$ flavor symmetry, Gell-Mann and Zweig guessed the hadron structure and proposed the quark model[1]. In the study of nuclear spectroscopy, Arima and Ichello developed the dynamical symmetry[14] which is a powerful method for spectroscopy not only for nuclei consist of strong interacting nucleons but also for hadrons consist of strong interacting quarks.

Dynamical symmetry assumed that the Hamiltonian of a strong interacting system is a function of the Casimir operators C or invariants (in our group representation theory it is the class operators C[15]) of a sub-group chain of the dynamic symmetry group G,

$$H=F(C,C_1,C_2,...)\ , \tag{1}$$

here C, C_1, C_2,... are the Casimir or class operators of the dynamic symmetric group chain $G\supset G_1\supset G_2\supset\ldots$. The group G in general does not commute with the Hamiltonian H, but the Casimir or class operators do commute with H, *i.e.*, one has

$$[H,G]\neq0,\quad [H,C]=0,\quad [H,C_i]=0,\quad i=1,2,... \tag{2}$$

The eigen-energy can be directly obtained from the eigenvalues of these operators,

$$E=F(\nu,\ m_1,\ m_2,...)\ , \tag{3}$$

here ν, m_1, m_2,... are the eigenvalues of the Casimir or class operators C, C_1, C_2,….

For u, d, s light quark systems, one has the dynamical symmetry group chain,

$$SU^{cf\sigma x}(18n)\supset SU^c(3)\times\Big(SU^{f\sigma x}(6n)\supset SU^f(3)\times (SU^{\sigma x}(2n)\supset SU^\sigma(2)\times U^x(n))\Big)\ , \tag{4}$$

here σ represents spin, x means orbital, n means how many orbital states occupied by quark. Quark is spin 1/2 fermion, their wave function must be antisymmetric and so must be the totally antisymmetric representation of the permutation group S_f, f is the quark number of the studied quark system. Color confinement restricts any physical states must be the color singlet representation of $SU^c(3)$ group and so any physical state the quark number f must be of module 3 with additional $(\mathrm{q}\bar{\mathrm{q}})^m$, here m is the $\mathrm{q}\bar{\mathrm{q}}$ pair number (we neglect the gluon excitation forming quark-gluon hybrid).

For baryon ground state, q^3 configuration with all quarks occupy the lowest s-wave state is a good approximation (the nucleon spin structure discovered by EMC group[16] calls for $q^3\mathrm{q}\bar{\mathrm{q}}$ components in baryon ground state[17]). So the orbital wave function is the totally symmetric one [3] of $U^x(1)$ and we have the following dynamical group chain for ground state baryons,

$$SU^{cf\sigma x}(18)\supset SU^c(3)\times\Big(SU^{f\sigma x}(6)\supset\Big(SU^f(3)\supset SU^I(2)\times U^Y(1)\Big)\times SU^\sigma(2)\times U^x(1)\Big)\ . \tag{5}$$

The Hamiltonian of baryon states is

$$H=F(C_{SU^f(3)},\ C_{SU^I(2)},\ C_{U^Y(1)},\ C_{SU^\sigma(2)},)\ . \tag{6}$$

The eigenvalues (the masses of ground state baryons) can be parameterized as,

$$M=M_0'+AC_{SU^f(3)}+BJ(J+1)+C'Y+DI(I+1)\ . \tag{7}$$

To take into account the hidden strange hadrons, the $C_{U^Y(1)}$ term should be replaced by the sum of the number of strange quarks and antiquarks. The mass formula reads,

$$M = M_0 + AC_{SU^f(3)} + BJ(J+1) + C\sum|S_i| + DI(I+1)\ . \tag{8}$$

Because the irreducible representation (IR) of color $SU^c(3)$ is fixed by color singlet to be totally antisymmetric $[1^3]$, the orbital $U^x(1)$ IR is fixed to be totally symmetric [3] due to the pure *s*-wave restriction of ground state. Based on the inner product rule of permutation group S_3 the IR of the color-flavor $SU^{f\sigma}(6)$ must be totally symmetric [3], and the IR of spin $SU^\sigma(2)$ and flavor $SU^f(3)$ is restricted to be $[3]\times[3]$ and $[21]\times[21]$. This is the well known ground decuplet and octet baryons. We did an overall fit and the results are shown in Table 1. The fixed parameters are (all of these parameters are in unit MeV),

$$M_0 = 816.92,\quad A = 9.1698,\quad B = 46.683,\quad C = 217.70,\quad D = 34.613\ . \tag{9}$$

Table 1　The mass of ground decuplet and octet baryons (MeV).

	$[f]$	Y	I	J	$\sum\|S_i\|$	$M_{\text{the.}}$	$M_{\text{exp.}}$
N	[21]	1	1/2	1/2	0	933	939
Λ	[21]	0	0	1/2	1	1 116	1 116
Σ	[21]	0	1	1/2	1	1 185	1 193
Ξ	[21]	−1	1/2	1/2	2	1 334	1 318
Δ	[3]	1	3/2	3/2	0	1 232	1 232
Σ^*	[3]	0	1	3/2	1	1 380	1 383
Ξ^*	[3]	−1	1/2	3/2	2	1 529	1 533
Ω	[3]	−2	0	3/2	3	1 677	1 672

The ground state baryon masses fitting is better than any dynamical quark model calculation. More interesting is the same set of parameters (except the parameter M_0, which is readjusted for q^6 system) can be used to predict the dibaryon masses as shown in Table 2. One obtained the almost right $IJ=01$ deuteron, $IJ=10$ di-neutron and $IJ=12$ NΔ threshold resonances (denoted as D_{12} following the Dyson-Xuong notation). The prediction of the D_{03} dibaryon of Dyson and Xuong is also reproduced and it had been discovered by WASA-at-COSY collaboration. The D_{30} state is predicted to be a little higher than the spin-isospin partner D_{03} but still lower than the ΔΔ threshold. Various model calculations support this prediction. However the recent measurement by WASA-at-COSY had not found any resonance in the $\text{pp}\rightarrow\text{pp}\pi^+\pi^+\pi^-\pi^-$ process[18]. One possibility is that the D_{30} production cross section in this process is very small. Other questions should be mentioned here are: (1) The deuteron, di-neutron and $N\Delta$ threshold resonances are all loosely shallow bound hadron molecular states, the six quarks should separate to two baryons

Table 2　The mass of dibaryons (MeV). $M_0 = 1\,672.9$ MeV.

	$[f]$	Y	I	J	$\sum\|S_i\|$	$M_{\text{the.}}$	$M_{\text{exp.}}$
D_{01}	[33]	2	0	1	0	1 876	1 876
D_{10}	[42]	2	1	0	0	1 889	1 878?
D_{03}	[33]	2	0	3	0	2 348	2 357
D_{30}	[6]	2	3	0	0	2 418	?
D_{12}	[42]	2	1	2	0	2 169	2 148
$N\Omega$	[321]	−1	1/2	2	3	2 609	?
$\Omega\Omega$	[6]	−4	0	0	6	2 998	?
H	[222]	0	0	0	2	2 074	?

and occupy two different quark orbital states and so the orbital symmetry is not only totally symmetric one [6] but also mixed symmetry [42]. On the other hand the quark model results of the D_{03}, the d*, is a tightly deep bound six quark state with only totally orbital symmetry [6] almost. This difference has not been taking into account in this dibaryon mass estimate; (2) The mass prediction of the high strangeness dibaryon is different from the dynamical quark model ones (see below), where they are all shallow bound or even unbound ones. On the other hand the mass of single baryon with strangeness is fitted quite well and consistent with dynamical quark model results; (3) The heavy quark baryons have not been discussed because we suppose the flavor symmetry, if extended to $SU(4)$ is badly broken.

We also use the mass formula to do an estimate of the pentaquark masses, the results are shown in Table 3. We assume the pentaquark Θ^+mass is really 1 540 MeV and use it to fix the M_0. For the pentaquark

Table 3　The mass of pentaquarks (MeV). M_b+M_m is the threshold of the corresponding state. $M_0 = 1\,172.25$ MeV.

	$[f]$	Y	I	J	$\sum\|S_i\|$	$M_{\text{the.}}$	M_b+M_m
$N\pi$	[33]	1	1/2	1/2	0	1 348	1 078
$N\pi$	[42]	1	3/2	1/2	0	1 489	1 078
NK	[33]	2	0	1/2	1	1 540	1 434
$N\eta$	[33]	1	1/2	1/2	0	1 348	1 488
$N\eta'$	[33]	1	1/2	1/2	2	1 784	1 897
ΛK	[33]	1	1/2	1/2	2	1 784	1 611
ΣK	[33]	1	1/2	1/2	2	1 784	1 688
ΣK	[42]	1	3/2	1/2	2	1 924	1 688
ΞK	[42]	0	0	1/2	3	2 012	1 813
$N\rho$	[33]	1	1/2	1/2	0	1 348	1 709
$N\rho$	[42]	1	3/2	3/2	0	1 629	1 709
NK^*	[33]	2	0	1/2	1	1 540	1 831
NK^*	[42]	2	1	1/2	1	1 680	1 831
$N\omega$	[33]	1	1/2	1/2	0	1 348	1 722
$N\omega$	[33]	1	1/2	3/2	0	1 488	1 722
$N\phi$	[33]	1	1/2	1/2	2	1 784	1 959
$N\phi$	[33]	1	1/2	3/2	2	1 924	1 959
ΛK^*	[33]	1	1/2	1/2	2	1 784	2 008
ΛK^*	[33]	1	1/2	3/2	2	1 924	2 008
ΣK^*	[33]	1	1/2	1/2	2	1 784	2 085
ΣK^*	[42]	1	3/2	3/2	2	2 064	2 085
ΞK^*	[33]	0	1	1/2	3	2 045	2 210

states with quantum numbers consistent with ground state baryon plus pseudoscalar meson, the theoretical masses are generally higher than the corresponding thresholds ($M_b + M_m$), because the pseudoscalar (the Goldston boson) mass is anomalous small due to chiral symmetry spontaneous breaking. Therefore it is unlikely to have such pentaquark states (Nη might be an exception). For the states consisted of ground state baryon and vector meson, pentaquark resonances are possible, for example, Nϕ, which is consistent with the dynamical quark model calculation. Nϕ might be a resonance a little higher than NK$\bar{\text{K}}$ threshold. The heavy flavor XYZ states and the recent claimed pJ/ψ pentaquark are out of $SU(3)$ flavor light quark system.

4 Bag model for multi-quark system

Bag model is the first dynamical quark model[19]. It has been used to calculate not only single hadron properties but also glueballs, 4, 5, 6 quark states by MIT group. The most influenced both positive and negative one is the flavor singlet called H particle, a uuddss isospin $I=0$ strangeness $S=-2$ $J^p=0^+$ six quark state, it is predicted to have mass $M\sim 2\,150$ MeV and so it is stable against the strong decay[6]. This prediction had caused an enthusiastic attitude in the study of multi-quark states. Theoretically there were a lot of follow up calculations, for example, de Swart *et al.*[20] had used the bag model to calculate the 4-quark, $3n$ quark sates ($n=1$, 2, 3, 4, 5, 6). In 2010—2012, HAL QCD collaboration still use lattice QCD to calculate this H particle and obtained a shallow bound state but with unphysical quark mass[21]. J-PARK still plans to search for the H particle[22]. However, Shanahan *et al.*[23] pointed out that after extrapolating to the physical light quark mass the H particle mass is higher than the $\Lambda\Lambda$ threshold. Since 1977 there were an experimental stampede to search for multi-quark states, about 40 dibaryons had been claimed by different groups. However no one passed the more precise measurements. This caused a pessimistic attitude for the multi-quark search.

Bag model with a confined boundary condition for q$\bar{\text{q}}$ meson and q^3 baryon is physical but this kind confined boundary condition for multi-quark system is unphysical because the multi-quark system can be separated into colorless sub-systems. This unphysical boundary condition introduced unphysical bound multi-quark state. Bag model is also an unrealistic hadron interaction model, for example, it even does not accommodate the deuteron state, say nothing of the vast NN scattering data. A theoretical model for 6-quark system should be able to describe the deuteron properties, the NN and N-hyperon, scattering data well. A model for 4-quark should be able to describe the meson-meson interaction and a model for 5-quark system should be able to describe meson-baryon interaction.

5 Realistic quark model for baryon interaction

To have a convincible prediction of multi-quark state, one must have a model which describes the known hadron interactions well. There are two such kind quark models. One is the chiral quark model, the other is the quark delocalization color screening model (QDCSM).

The chiral quark model includes both one gluon exchange and one boson exchange in addition to the color confinement interaction[24]. The QDCSM introduces quark delocalization (which is similar to the electron delocalization in molecular structure) and color screening (which had been proved to be the effective description of hidden color channel coupling effect) to replace the σ meson exchange and in this way explaining the similarity between molecular force and nuclear force which is a well known fact but never explained by any NN interaction model. These quark models both describe the deuteron properties and the vast NN, N-hyperon scattering data as well as the one boson exchange model and the chiral perturbation model achieved with less than 10 model parameters in comparison with the about 30 parameters of the meson exchange models (see Table 4 and Fig.1).

Table 4 The properties of deuteron.

	ChQM	QDCSM1	QDCSM2
B/MeV	2.0	1.94	2.01
$\sqrt{r^2}$/fm	1.96	1.93	1.94
P_D/%	4.86	5.25	5.25

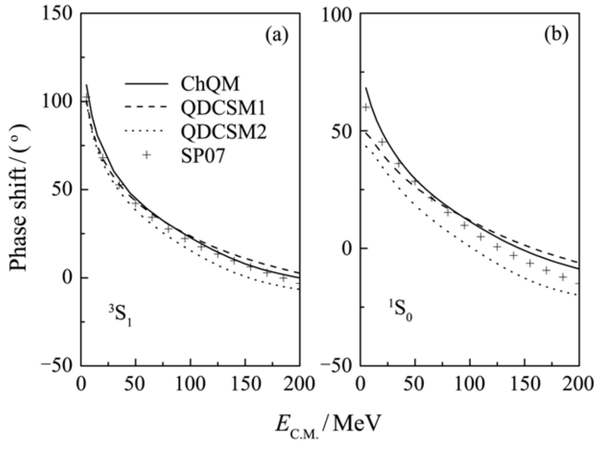

Fig. 1 The phase shifts of NN S-wave scattering.

These quark models confirmed that the d* dibaryon is really an inevitable one. Moreover it predicted that this d* dibaryon can be detected through the Feshbach NN resonance scattering, the d* will couple to NN $^3D_3$ partial wave. All of these predictions had been confirmed by WASA-at-COSY measurements including the predicted resonance scattering width ~ 10 MeV[12]. The total width of d* is ~ 80 MeV, much smaller than the naive estimate of the off shell reduction of the width of Δ. Brodsky *et al.*[25] attributed this small width of d* to the about 80 percent hidden color channel component. In fact there is misunderstanding of hidden color channel in their analysis. The transformation between symmetry bases and cluster bases, the later include both colorless and hidden color channels, are based on the assumption that there are at least two linear independent quark orbital states. Then the 6-quark states have orbital symmetry not only totally symmetry [6] but also [42] and so on. The cluster bases, after quark antisymmetrizaton, are in general not orthogonal if the single quark orbits are not orthogonal. In the limit that all 6 quarks occupy the same one orbital state as assumed by S. Brodsky and others, the d* quantum number $IJ^P=03^+$ channel only have one state. The colorless $\Delta\Delta$ and the hidden color $\Delta\Delta$ channels are exactly the same one. In this case to say there is 80 percent hidden color channel component within d* is meaningless. The real physics might be due to the fact that d* is a very compact (rms radius ~ 1 fm) 6-quark state which is quite different from the $di-\Delta$ structure which is consistent with the partial width measurements of WASA-at-COSY collaboration[11].

These quark models predicted only few promising dibaryon states, such as the d* partner $IJ^P = 30^+$ $di-\Delta$, the threshold resonance $IJ^P = 12^+$ NΔ, the strangeness -3 NΩ with $IJ^P = (1/2)2^+$, the week bound or unbound H particle and $IJ^P = 00^+$ $di-\Omega$. The NΔ mass is predicted to be $\sim 2\,170$ MeV, exactly at the threshold. The NN scattering had found a $^1D_2$ partial wave resonance at (2148-i59) MeV[26]. However it is assumed to be a threshold cusp rather than a dibaryon resonance in the pessimistic period. The $N\Omega$ is predicted to be a bound state with binding energy ~ 10 MeV[27]. The recent lattice QCD calculation obtained a similar NΩ dibaryon state with a binding energy ~ 18.9 MeV[28]. The H particle is still a hot topic both for theoretical calculation and experimental search. The $di-\Omega$ is an interesting one, because the Ω is strong interaction stable baryon and if $di-\Omega$ is bound it will be a strong interaction stable dibaryon similar to H particle. Quark model predict these two are shallow bound ones or unbound.

Quark model might include right physics as evidenced by the predictions on Ω and d*. However quark model predictions are model parameters dependent and even there are vast NN interaction data one still can not fix the model parameters and so leave large uncertainties for the model predictions. If the deuteron were not observed quark model can not answer if deuteron is stable or not. Therefore only if the multi-quark state is bound for a reasonable model parameters set one can say there might be a multi-quark state existed. For strange sector the situation is even worse because the hyperon interaction data are very scarce. Moreover, the multi-quark internal structure might be totally different from the deuteron like molecular one but a genuine compact multi-quark one and the interaction is due to non-perturbative multi-gluon exchange and so the present quark model approach may be irrelevant to the real multi-quark system.

6 Lattice QCD for multi-quark states

Lattice QCD in principle is a nonperturbative approach to calculate hadron properties directly from QCD. It successfully reproduces the ground state hadron masses. There are few lattice QCD collaborations started the ambitious program to study nuclear physics directly from QCD and impressive results obtained. But the state of the art lattice QCD (lattice size $a = 0.085$ fm, lattice volume $V = (8.2\ \text{fm})^4$, (m_π, $m_K = 146$, 525 MeV)) only obtained a qualitative NN interaction and still can not derive the deuteron properties[29]. The calculated multi-quark states, such as H particle, NΩ and $di-\Omega$ are still not reliable[30], it is also not able to pin down the structure of the observed tetra-quark state $Z_c(3900)$ the same as the phenomenological model approach[31].

7 Summary

Multi-quark, a theoretical inspiration since the proposal of quark model in 1964, a new real world of hadron physics now discovered. The journey of the search for this new world is full of pitfalls and disappointments. Many theorists and experimentalists got into this journey due to exciting dawn and got off this journey due to lengthy dark night. Only brave and patient ones insist this scientific long march to the Holy Grail of hadron physics.

Life is always one hard journey after one. There are so many problems remain in multi-quark world. Even in the well known single hadron domain, there are puzzles remain, for example, why the Roper resonance N*(1440) mass is lower than the odd parity N*(1520) and N*(1535), the same puzzle repeats in Δ(1600) and Δ(1620); why the Λ(1405) mass is smaller than the N(1535); how to understand the splitting between $N(1/2)^-$, $N(3/2)^-$; $\Delta(1/2)^-$, $\Delta(3/2)^-$; $\Lambda(1/2)^-$, $\Lambda(3/2)^-$ in a unified manner; are the missing hadron resonance really missing? Are the hadrons really within the $q\bar{q}$ and q^3 pure valence configuration? Dyson-Schwinger Equation approach already shows that the low energy effective quark and gluon masses are around 350 MeV and 450 MeV, why the gluon degree of freedom does not appear in hadron states up to now? Getting into multi-quark domain, the X, Y, Z states are experimentally well established, but are they meson molecule or genuine tetra-quark states, or mixing with hybrid even glueball? The LHCb discovery of pJ/ψ needs an independent experimental check, is it possible to get pJ/ψ correlation data from relativistic heavy ion collision data and in turn to get p-J/ψ interaction information? The WASA-at-COSY had made quite comprehensive measurements of dibaryon d^*, almost all possible decay channels had been measured, but the community still waits for the independent experimental check, the polarized $\gamma d \to d^* \to \mathbf{p}n$ result which in fact had been done in 1977-79 but the results are not conclusive enough[4].

Hidden color components are the radical new degree of freedom of multi-quark systems. It should exist in any multi-quark system even in nuclear structure. However, up to now there is no any experimental observed evidence of this degree of freedom. Theoretically the hidden color component is just an $SU(3)$ color group coupling scheme, it can be completely replaced by the colorless hadron components[32]. A possible flaw is that the wave function of a color singlet multi-quark state is not gauge invariant because we don't have the color gauge link to connect the different color subsystems. In fact even the single hadron wave function is also not gauge invariant because we have not added the color gauge link to connect the individual quark and/or anti-quark yet. There are already papers to use the hidden color components to describe physical effects but these applications should be understood with caution[25, 33].

A journey in the multi-quark world has been started, there might be new pitfalls and disappointments, but the final results will certainly a new exciting.

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多夸克态的研究进展

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摘要: 对多夸克态特别是双重子态的半个世纪的研究进展进行了概述。利用动力学对称性，推导了能够合理地描述多夸克态质量的Gursey-Radicati公式，然后在MIT袋模型和可以很好描述重子-重子相互作用的夸克蜕定域色屏蔽模型中讨论了各种可能的双重子。

关键词: 多夸克态；双重子

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