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Two-nucleon Excitation from *p* to *sd* Shell in ^{12,14}C

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Abstract: The effect of the two-nucleon excitation from p to sd shell is discussed on ^{12}C and ^{14}C in the frame work of the nuclear shell model. The recently suggested shell-model Hamiltonian YSOX provides an suitable tool to investigate the 2 $\hbar\omega$ excitation in psd region. Because the strength of the $<pp|V|sdsd>$ interaction, which represents the interaction between the 0 and 2 $\hbar\omega$ configurations, is considered in the construction of the Hamiltonian YSOX. The level of ^{12}C is almost independent on the $<pp|V|sdsd>$ interaction, but excitation energies of certain states in ^{14}C are strongly affected by it. Further investigation shows that the percentage of 2 $\hbar\omega$ configuration in these states is quite different from that of the ground state. The non-linear effects of the $<pp|V|sdsd>$ interaction on the configurations and transition rates are also discussed.

Key words: nuclear shell model; psd model space; Hamiltonian YSOX; 2 $\hbar\omega$ configuration

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1 Introduction

The new generation of facilities are used to investigate the proton- and neutron-rich nuclei with the radioactive isotope beam. For example, the scientists discovered 45 new neutron-rich nuclei on Radioactive Isotope Beam Factory in 2010[1]. One of the important issue in nuclear physics is to understand the nuclei from the stability line to the drip line. Thanks to the extensive experimental and theoretical study on the properties of proton- and neutron-rich nuclei in recent years, the position of the drip line is known up to oxygen isotopes[2].

In a theoretical view, how to solve the nuclear system is a long standing big challenge. The common theoretical methods are roughly divided into three categories: the *ab initio* methods, the methods based on the mean field approximation, and the nuclear shell model[3–4]. The nuclear shell model solves the many-body Schrodinger equation in a truncated model space. The modern nuclear shell model includes the configuration mixing and the residue interactions. The binding energies and the wave functions of both the ground and excited states can be given simultaneously after the diagonalization process. The observed binding energies, levels, electromagnetic properties, β decays, and many other properties can be well described through shell model approach in light and medium mass region[3–4].

For light nuclei in psd region, some well determined shell-model Hamiltonians are constructed by fitting to the observed binding energies and levels, such as MK[5], WBT and WBP[6]. The fitting procedure of these Hamiltonians is limited in the $0\sim1$ $\hbar\omega$ model space and considers the strength of $<pp|V|pp>$, $<sdsd|V|sdsd>$ and $<psd|V|psd>$ parts of the interaction. The two-nucleon excitation from p to sd shell and the corresponding $<pp|V|sdsd>$ interaction are not included in the construction of the Hamiltonians. Recently, a new Hamiltonian for psd shell, YSOX, is introduced[7]. YSOX well describes the binding energies, levels, electromagnetic properties, and Gamow-Teller transitions of boron, carbon, nitrogen, and oxygen isotopes[7]. The construction of the YSOX includes the consideration of the $<pp|V|sdsd>$ interaction, which allows an investigation on the two-nucleon excitation from the p to sd shell.

The percentage of the 2 $\hbar\omega$ configuration dramatically decreases from ^{16}O to ^{24}O, which indicates its

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importance in the description of nuclei from the stability line to the drip line[7]. It is easy to understand such changes on the isospin degree of freedom. The main configuration of ^{16}O fully occupies the p shell in the independent particle model (IPM). After the inclusion of the residue interaction, the ground state of ^{16}O shows strong mixing between the 0 and 2 $\hbar\omega$ configurations. Actually, the higher $\hbar\omega$ configuration is also important for the exactly description of ^{16}O[3]. From ^{16}O to ^{24}O, the valence neutrons in sd shell increase. The mixing between the 0 and 2 $\hbar\omega$ configurations contribute less in the total energies.

The two-nucleon excitation shows its effect on the degree of freedom of excitation energy. In ^{10}B and ^{17}C, levels of some excited states are strongly affected by the strength of the $<pp|V|sdsd>$ interaction, but others are almost independent on that interaction. The present work investigates how and why the 2 $\hbar\omega$ configuration influences the levels, configurations, and transition rates in ^{12}C and ^{14}C in the frame work of shell model.

2 Shell model

Modern shell model includes the residue two-body interactions and treats the states of nuclei as the mixing of all possible configurations. In principle, the energies and wave functions can be obtained through solving the Schrodinger equation. But for the majority of nuclei with many protons and neutrons, the model space is too huge. One can limit the cost of the calculations by selecting a core, normally a doubly magic nuclei. The model space is then reduced to several valence proton and neutron orbits. The energies and wave functions can be obtained by solving the Schrodinger equation in the truncated model space.

Because the model space is reduced to the effective model space, a corresponding effective Hamiltonian is needed to be used when solving the Schrodinger equation. Effective Hamiltonian is normally derived from the nucleon-nucleon potential in two ways, one is phenomenological by fitting the nucleon-nucleon potential to the experimental binding energies and energy levels, the other is realistic by using the nucleon-nucleon potential derived from the pion-nucleon scattering and the nucleon-nucleon scattering data.

In this paper, the effective model space is psd space, and corresponding effective Hamiltonians are MK[5], WBT[6], WBP[6], and recently suggested YSOX[7]. These phenomenological Hamiltonians fit their two-body matrix elements (TBME) to the nuclear structure data, especially the binding energies and energy levels. Shell-model calculations are forwarded through the programme OXBASH[8].

The new Hamiltonian for psd region YSOX[7] is developed from V_{MU}[11], SFO[9] and SDPF-M[10]. The $\langle pp|V|pp\rangle$ and $\langle sdsd|V|sdsd\rangle$ parts of TBME are from SFO and SDPF-M, respectively. The $\langle psd|V|psd\rangle$ ($\langle pp|V|sdsd\rangle$) TBME are calculated through V_{MU}[11] plus M3Y[12] spin-orbit force as follows,

$$V = 0.85(0.55)V_{\mathrm{central}} + V_{\mathrm{tensor}}(\pi+\rho) + V_{\mathrm{spin-orbit}}(M3Y)\ . \quad (1)$$

The $V_{\mathrm{central}}+V_{\mathrm{tensor}}(\pi+\rho)$ is the original V_{MU}. In the present study, we reduce the central force in TBME in $\langle psd|V|psd\rangle$ and $\langle pp|V|sdsd\rangle$ by factors 0.85 and 0.55 from the original V_{MU}, respectively. More details can be found in Ref. [7].

3 Results

The percentage of 2 $\hbar\omega$ configuration changes in each isotope when neutron number increasing, such as, from 25% in ^{16}O to 10% in ^{24}O[7]. It may also change from ground to excited states. Ref. [7] discussed the effect of the $<pp|V|sdsd>$ interaction on the energy levels of ^{10}B and ^{17}C. The $<pp|V|sdsd>$ interaction is the interaction between 0 and 2 $\hbar\omega$ configurations. If the excited state has the similar 2 $\hbar\omega$ configuration to the ground state, the strength of the $<pp|V|sdsd>$ interaction rarely affects the excitation energy, and vice versa. But both in ^{10}B and ^{17}C, the excitation energies of some states change a lot when the strength of the $<pp|V|sdsd>$ interaction changes. Such results indicate the percentage of 2 $\hbar\omega$ configuration in these excited states are different from those of the ground states. It is interesting to see that some low lying states in ^{10}B have relatively large 2 $\hbar\omega$ configuration. Although the valence nucleons of ^{10}B are mainly active in the p shell, the 1_1^+ state has 16% 2 $\hbar\omega$ configuration, around 10% larger than that of the ground state. The previous investigations show that the mixing of 0 and 2 $\hbar\omega$ configurations is important in the description of both the ground and excited states in both the stable and exotic nuclei.

The present work aims to extend the knowledge on the effect of the two-nucleon excitation from p shell to sd shell. Figs. 1 and 2 present the levels of ^{12}C and ^{14}C as the function of the strength of the $<pp|V|sdsd>$ interaction. As discussed before, if the excited states have similar 2 $\hbar\omega$ configuration to the ground state, the excitation energies do not change much when the $<pp|V|sdsd>$ interaction changes. In Fig. 1, it is seen that the levels of ^{12}C change little as the $<pp|V|sdsd>$ interaction increasing or decreasing. But for ^{14}C, the excitation energies of 0_2^+, 2_2^+, and

4_1^+ states are dramatically dependent on the strength of the $<pp|V|sdsd>$ interaction. The relationships between these three energies and the strength of the $<pp|V|sdsd>$ interaction are quite linear, which indicates a sudden change on energy is not expected.

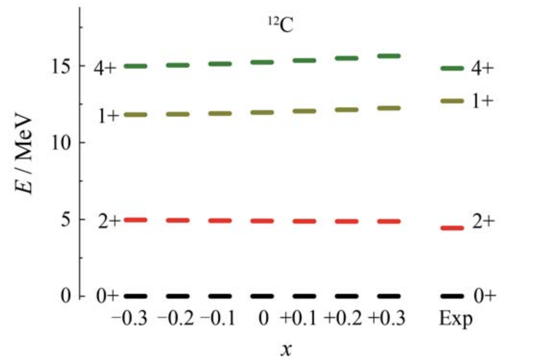

Fig. 1 (color online) The levels of ^{12}C as the function of x, which specifies the strength of the $<pp|V|sdsd>$(central)$=(0.55+x)V_{\rm MU}$(central). Experimental data is from NNDC[13].

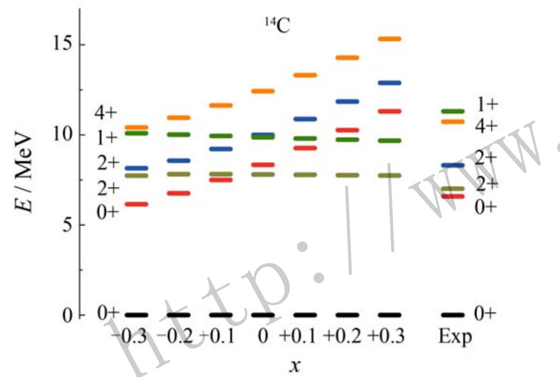

Fig. 2 (color online) The same as Fig. 1 but for ^{14}C.

To further investigate the 2 $\hbar\omega$ configuration in different states of ^{12}C and ^{14}C, the percentages of both 0 and 2 $\hbar\omega$ configurations presented in Fig. 3 as the function of the strength of the $<pp|V|sdsd>$ interac-

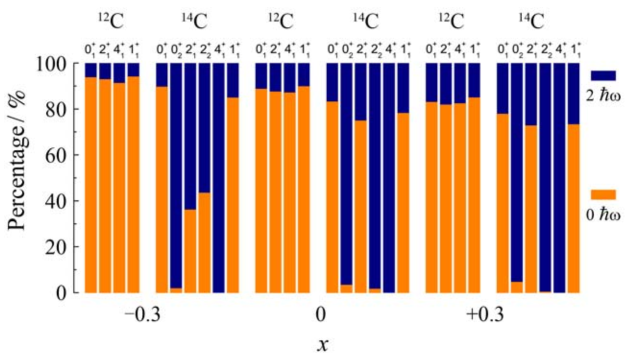

Fig. 3 (color online) The same as Fig. 1 but for the 0 and 2 $\hbar\omega$ configurations in ground and excited states of ^{12}C and ^{14}C.

tion. It is clearly seen that the 2 $\hbar\omega$ configuration is almost the same in each state of ^{12}C, even when the strength of the $<pp|V|sdsd>$ interaction changes a lot. Such configurations can be used to explain the little dependance between the excitation energies of ^{12}C and the strength of the $<pp|V|sdsd>$ interaction. The valence nucleons of ^{12}C are mostly excited inside p shell. The two-nucleon excitation from p to sd shell is around 10% in each state and does not change much the level of ^{12}C.

But both the 0 and 2 $\hbar\omega$ configurations are very different in each state of ^{14}C. The 0_1^+ and 1_1^+ states are dominated by the 0 $\hbar\omega$ configuration, while the 0_2^+ and 4_1^+ states are dominated by the 2 $\hbar\omega$ configuration. The configurations of these four states are little dependent on the strength of the $<pp|V|sdsd>$ interaction. The configurations of $2_{1,2}^+$ states are dominated by 0 and 2 $\hbar\omega$ configurations, respectively, and almost not changed by the shift of the strength of central part of the $<pp|V|sdsd>$ interaction from 0.85 to 0.55 of its original value. But when the strength is small, 0.25 of its original value, the configurations of $2_{1,2}^+$ states are not dominated by single configuration but strongly mixed between 0 and 2 $\hbar\omega$ configurations, which indicates a small gap between p and sd shell. The sudden changes of the configurations show the non-linear effect contributed by the $<pp|V|sdsd>$ interaction, while a linear effect is presented on the excitation energies.

In ^{14}C, the most important configuration for ground state is $\pi(p_{3/2})^4\nu(p_{3/2})^4(p_{1/2})^2$. If the original YSOX is used, the 2_1^+ state is mainly excited by one proton moving to $p_{1/2}$ orbit, which does not increase or decrease the 2 $\hbar\omega$ configuration compared with that of ground state. The 0_2^+ and 2_2^+ states are mainly excited by moving two neutrons to sd shell, rather than two protons to $p_{1/2}$ orbit. The 2 $\hbar\omega$ configuration becomes the most important configuration ($>95\%$). The 4_1^+ state is all contributed by the 2 $\hbar\omega$ configuration. Because the two proton holes in p shell can not couple to the angular momentum 4.

As a results of the 2 $\hbar\omega$ configuration in each state, the levels are affected by the $<pp|V|sdsd>$ interaction. When the strength of the $<pp|V|sdsd>$ interaction is increasing, the ground state, 2_1^+, and 1_1^+ states are more binding because the 2 $\hbar\omega$ configuration is around 20%. The 0_2^+, 2_2^+, and 4_1^+ states are (almost) purely 2 $\hbar\omega$ configuration. Therefore their binding energies are rarely dependent on the strength of the $<pp|V|sdsd>$ interaction. As a result, their levels change relative to the ground state. But when the strength of the $<pp|V|sdsd>$ interaction is decreasing, complicated configuration mixing contributes to the $2_{1,2}^+$ states, which gives similar trends on the energies but quite different trends on the configurations. It is expected that the sudden changes of the configurations affect the transition rates.

The $B(\text{E2};2^+_1 \rightarrow \text{g.s.})$ value of ^{12}C and the $B(\text{E2};2^+_{1,2} \rightarrow \text{g.s.})$ values of ^{14}C are presented in Fig. 4, which shows the strong non-linear effect of the strength of the $< pp|V|sdsd >$ interaction on the transition rates in ^{14}C. The $B(\text{E2};2^+_{1,2} \rightarrow \text{g.s.})$ values of ^{14}C are almost very similar when the strength of the central part of the $< pp|V|sdsd >$ interaction shifts from 0.85 to 0.35 of its original value. But when the strength is 0.25 of its original value, the $B(\text{E2};2^+_1 \rightarrow \text{g.s.})$ value of ^{14}C dramatically decreases and agrees with the observed data. The $B(\text{E2};2^+_2 \rightarrow \text{g.s.})$ value of ^{14}C also increases significantly. Such effects can be explained by the strong mixing between 0 and 2 $\hbar\omega$ configurations. It should be concentrated that such effects can also be contributed by the decrement of the gap between the p and sd shell. So it is difficult to obtain an exact strength of the $< pp|V|sdsd >$ interaction purely by properties of ^{14}C. The Hamiltonian YSOX is deduced from global properties of all nuclei in boron, carbon, nitrogen, and oxygen isotopes.

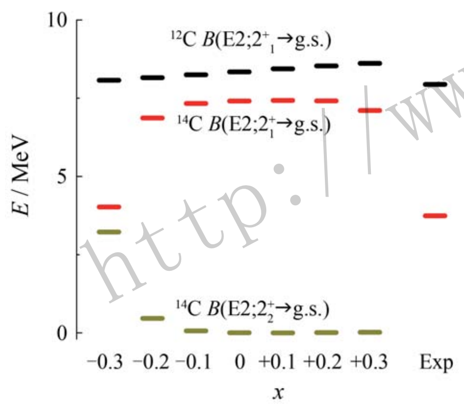

Fig. 4 (color online) The same as Fig. 1 but for the $B(\text{E2};2^+_1 \rightarrow \text{g.s.})$ of ^{12}C and $B(\text{E2};2^+_{1,2} \rightarrow \text{g.s.})$ of ^{14}C.

4 Summary

The present work discusses the two-nucleon excitation from p to sd shell in light nuclei based on the framework of the nuclear shell model. The 2 $\hbar\omega$ configuration is important in the investigation of binding energy in a long isotope chain and in the different states in one nucleus. The relationship between the strength of the $< pp|V|sdsd >$ interaction and the levels of ^{12}C and ^{14}C are researched through Hamiltonian YSOX. It is shown that the strength of the $< pp|V|sdsd >$ interaction is very influential to the levels of the states with different 2 $\hbar\omega$ configurations from that of the ground state. The non-linear effect of such interaction on the configurations and transition rates in ^{14}C is also discussed. The strength of such interaction linked the 0 and 2 $\hbar\omega$ configurations is rarely investigated because its effect is not obvious in the levels of many nuclei. It is interesting to further find the nuclei of which the level is sensitive to the strength of the $< pp|V|sdsd >$ interaction, which is helpful for the understanding how the nuclear force drives the nuclear structure.

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