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Time-dependent Calculations for Two-proton Decay Width with Schematic Density-dependent Contact Pairing Interaction

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Abstract: We calculate the two-proton decay width of the ^{6}Be nucleus employing the schematic density-dependent contact potential for the proton-proton pairing interaction. The decay width is calculated with a time-dependent method, in which the two-proton emission is described as a time-evolution of a three-body meta-stable state. Model-dependence of the two-proton decay width has been shown by comparing the results obtained with the two different pairing models, schematic density-dependent contact and Minnesota interactions, which have zero and finite ranges, respectively.

Key words: two-proton radioactivity; nuclear pairing interaction; time-dependent method
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1 Introduction

Description of the pairing correlation of atomic nuclei has been a major subject in recent nuclear physics[1–2]. After the establishment of the traditional mean-field theory for atomic nuclei, there have been enormous theoretical and computational developments in this field[3–5]. These developments have led to a deeper insight beyond the pure mean-field picture[4–6].

There have been several types of models for nuclear pairing interactions used within a framework of Hartree-Fock-Bogoliubov (HFB) or nuclear energy density functional (EDF) theory[7–9]. These theoretical approaches have provided the successful results in an excellent agreement with empirical binding energies or pairing gaps for stable nuclei. Even with the various efforts toward the nuclear mean-field theory with superfluidity, however, the detailed characters of nuclear pairing correlations have not been determined. Namely, for example, whether the phenomenological pairing interaction should be a volume or surface type[10–11], or whether it should have a finite or zero range[12–13], are still open questions.

Recently, it is expected that the two-proton (2p) radioactivity may provide a new way to investigate the nuclear pairing correlations[14–17]. In true 2p-decays[14–15], a pair of protons is emitted simultaneously from the parent nucleus, whereas the single proton decay is forbidden because of the pairing correlation energy. The important advantage of the 2p-radioactivity is that it provides a new kind of observable quantities which cannot be found for the stable nuclei; the 2p-decay width or lifetime. The proton-proton pairing correlation may play an essential role to determine not only the released Q-value, which corresponds to the pairing gap, but also the decay width.

In this article, we study whether the different characters of pairing models can be reflected on the 2p-decay width or not, connected to a specific interest in improving the pairing models. For this purpose, we focus on the 2p-emission from the ground state of ^{6}Be, which is the simplest 2p-emitter. We basically employ the same model to that used in our previous work[18]; time-dependent three-body model. The formalism of our model is given in Sec. 2. In Sec. 3, we present the results of our calculations. Finally, Sec. 4 is devoted to summarize the article.

2 Formalism

2.1 Three-body model Hamiltonian

Within our model, the 2p-decay of ^{6}Be is described as an emission of the valence two protons from the spherical mean-field generated by the α core. The

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total three-body Hamiltonian is

$$H_{3\mathrm{b}} = h_1 + h_2 + \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{A_{\mathrm{c}} m} + v_{\mathrm{pp}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \ , \tag{1}$$

$$h_i = \frac{\boldsymbol{p}_i^2}{2\mu} + V_{\mathrm{cp}}(r_i) \quad (i = 1, 2) \ , \tag{2}$$

where h_i is the single particle (s.p.) Hamiltonian between the core and the i-th proton. $\mu \equiv m A_{\mathrm{c}}/(A_{\mathrm{c}}+1)$ is the reduced mass where m and $A_{\mathrm{c}} = 4$ are the nucleon mass and the mass number of the core nucleus, respectively.

The mean-field potential between the α core and a valence proton reads

$$V_{\mathrm{cp}}(r) = V_{\mathrm{WS}}(r) + V_{\mathrm{Coul}}(r), \tag{3}$$

where the Woods-Saxon part is given as

$$V_{\mathrm{WS}}(r) = V_0 f(r) + U_{ls} (\boldsymbol{l} \cdot \boldsymbol{s}) \frac{1}{r} \frac{\mathrm{d}f(r)}{\mathrm{d}r}, \tag{4}$$

with

$$f(r) = \frac{1}{1 + \mathrm{e}^{(r - R_0)/a_0}}. \tag{5}$$

For V_{cp} we use the same parameters to those used in our previous work[18], with which one can reproduce the resonance energy and the width of the $(p_{3/2})$-channel for α-p scattering, given as $E_r(p_{3/2}) = 1.96$ MeV and $\Gamma_r(p_{3/2}) = 1.56$ MeV, respectively[22].

Employing the Coulomb potential for point charges, the proton-proton pairing interaction, v_{pp}, is given as

$$v_{\mathrm{pp}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_{\mathrm{pp}}^{(N)}(r_{12}) + \frac{e^2}{r_{12}}, \tag{6}$$

where $r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$. In Ref. [18], for $v_{\mathrm{pp}}^{(N)}$, we employed the finite-ranged Minnesota potential[23]. We modified one parameter from its original value, so as to reproduce the empirical Q-value, $Q_{2\mathrm{p}} = 1.37$ MeV for the emitted two protons from ^{6}Be[22].

In this article, we try to use a pairing interaction with zero range: we employ a schematic density-dependent contact (DDC) potential. That is,

$$v_{\mathrm{pp}}^{(N)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_{\mathrm{d}} \left[1 - c_{\mathrm{d}} f\left(\left| \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2} \right| \right) \right] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \tag{7}$$

where $f(r)$ is the same function to Eq.(5). Note that a more general form of DDC interaction, which has been used in a lot of EDF calculations, is given as $v_{\mathrm{pp}}^{(N)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_{\mathrm{d}} \left[1 - c_{\mathrm{d}} \{ \rho(\left| \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2} \right|) / \rho_0 \}^{\alpha} \right] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$, whereas the density term is schematically replaced for the three-body model calculations[19–21]. The bare strength parameter, v_d, can be determined so as to reproduce the empirical scattering length of neutrons in vacuum[19–20]: $v_{\mathrm{d}} = -767.398$ MeV consistently to the energy-cutoff, 40 MeV, in this article. In order to reproduce the empirical Q-value, $Q_{2\mathrm{p}} = 1.37$ MeV[22], we also need the phenomenological coefficient, $c_{\mathrm{d}} = 1.04$. Thus, we consider an equivalent condition for both the schematic DDC and Minnesota interactions, with the same value of $Q_{2\mathrm{p}}$ corresponding to the same pairing gap.

2.2 Wave function

The eigen-states of the three-body Hamiltonian, $H_{3\mathrm{b}}$, can be obtained by expanding the wave function on the uncorrelated basis:

$$\left| E_{\mathrm{N}} \right\rangle = \sum_{\mathrm{M}} U_{\mathrm{NM}} \left| \Phi_{\mathrm{M}} \right\rangle \ . \tag{8}$$

Here the uncorrelated basis functions after the anti-symmetrization can be obtained as

$$\Phi_{\mathrm{M}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{\sqrt{2(1 + \delta_{n_{\mathrm{a}}, n_{\mathrm{b}}})}} \sum_m C(j, m; j, -m \,|\, 0, 0) \times \left[\phi_{n_{\mathrm{a}} l j m}(\boldsymbol{r}_1) \phi_{n_{\mathrm{b}} l j -m}(\boldsymbol{r}_2) + \phi_{n_{\mathrm{a}} l j m}(\boldsymbol{r}_2) \phi_{n_{\mathrm{b}} l j -m}(\boldsymbol{r}_1) \right] \ , \tag{9}$$

with the simplified notation, $M = (n_{\mathrm{a}}, n_{\mathrm{b}}, l, j)$. Each s.p. state satisfies $h_i \phi_{nljm}(\boldsymbol{r}_i) = \epsilon_{nlj} \phi_{nljm}(\boldsymbol{r}_i)$. We assumed the 0^+ configuration for both the uncorrelated basis and the α core, consistently to that the total spin-parity is also 0^+ for the ground state of ^{6}Be. The expansion coefficients, U_{NM}, are determined by diagonalizing the Hamiltonian matrix for $H_{3\mathrm{b}}$. The N-th state then satisfies $H_{3\mathrm{b}} \left| E_{\mathrm{N}} \right\rangle = E_{\mathrm{N}} \left| E_{\mathrm{N}} \right\rangle$.

We notify that all the calculations are performed in the truncated space defined by the energy-cutoff, $\epsilon_{n_{alj}} + \epsilon_{n_{blj}} \leqslant E_{\mathrm{cut}} = 40$ MeV. The continuum s.p. states are discretized within the radial box of $R_{\mathrm{box}} = 80$ fm. Thus, the eigen-states, $\left| E_{\mathrm{N}} \right\rangle$, are also discretized. For the angular momentum channels, we include up to the $h_{11/2}$ channel. In order to take into account the effect of the Pauli principle, we exclude the first $s_{1/2}$ state, that is occupied by the protons in the core nucleus.

2.3 Time-dependent method

In order to fix the initial state for time-evolutions, we employ the same confining potential as that used in our previous work[18]. This kind of method with confining potentials has been used for a good approximation for quantum resonant states[24–27]. The initial state, $\left| \Psi(0) \right\rangle$, can be expanded on the eigen-state basis of the total Hamiltonian. That is,

$$\left| \Psi(0) = \sum_{\mathrm{N}} F_{\mathrm{N}}(0) \left| E_{\mathrm{N}} \right\rangle \right. \ . \tag{10}$$

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After the time-evolution with H_{3b}, the state can be represented as

$$\left|\Psi(t)=\exp\left[-\mathrm{i}t\frac{H_{3b}}{\hbar}\right]\right|\Psi(0)=\sum_{N}F_{N}(t)\left|E_{N}\right\rangle\ , \quad (11)$$

where $F_{N}(t)=\mathrm{e}^{-\mathrm{i}tE_{N}/\hbar}F_{N}(0)$. Note that the discrete energy-distribution can be given as $\mathrm{d}(E_{N})=|F_{N}(0)|^{2}=|F_{N}(t)|^{2}$. If we consider the continuous energy limit, $\mathrm{d}(E)$ corresponds to the Breit-Wigner spectrum.

3 Results

In Fig. 1, we plotted the density-distribution for the initial state, $\rho(r_1,r_2,\theta_{12})=|\Psi(t=0;r_1,r_2,\theta_{12})|^{2}$, as a function of $r_{p-p}=(r_1^2+r_2^2-2r_1r_2\cos\theta_{12})^{1/2}$ and $r_{c-pp}=(r_1^2+r_2^2+2r_1r_2\cos\theta_{12})^{1/2}/2$, where θ_{12} is the opening angle between two protons. The initial wave function is well confined inside the potential barrier, showing a similar structure like as the initial state obtained with the Minnesota potential, as well as the two-neutron bound state in ^{6}He, which is the mirror nucleus to ^{6}Be (see Fig. 4 and Fig. 6 in Ref. [18]). The higher peak at $r_{p-p}\cong 2.0$ fm and $r_{c-pp}\cong 2.5$ fm indicates a strong localization of two protons. We also confirmed that this localization is attributable to the spin-singlet configuration, whose ratio is 83% in this initial state. Thus, it suggests a diproton correlation by the schematic DDC pairing interaction[28].

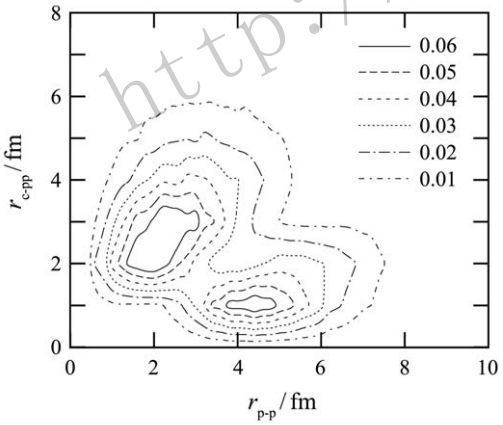

Fig. 1　The density-distribution for the initial 2p-state as a function of r_{p-p} and r_{c-pp}.

From the decay probability defined as $N_{d}(t)=1-\left|\langle\Psi(0)\left|\Psi(t)\right\rangle\right|^{2}$, the 2p-decay width can be calculated as,

$$\Gamma(t)\equiv-\hbar\frac{\mathrm{d}}{\mathrm{d}t}\ln\left[1-N_{d}(t)\right]=\frac{\hbar}{1-N_{d}(t)}\frac{\mathrm{d}}{\mathrm{d}t}N_{d}(t)\ . \quad (12)$$

It is worthwhile to mention that if the time-evolution follows the exponential decay law, $\Gamma(t)$ should be constant and related to the lifetime of this meta-stable state: $\Gamma=\hbar/\tau$. In Fig. 2, we present the calculated decay width. With both types of the pairing interaction, we have obtained finely converged results after the sufficient time-evolution. Values of the decay width at $ct=1\,000$ fm are 34.7 and 88.2 keV with the schematic DDC and Minnesota interactions, respectively. From this result, we find that the schematic DDC interaction significantly underestimates the width, whereas the modified Minnesota interaction shows a good agreement with experimental data. Remember that both interactions are fitted to reproduce the same empirical Q-value. Thus, it is shown that the calculated 2p-decay width has a remarkable dependence to the pairing models, even though we consider an equivalent condition with respect to the pairing gap. In order to find the origin of this dependence, it is worth investigating the effective potential surface for two protons, which is, however, a complicated product of all the interactions between particles and at this moment we have several technical problems to achieve it.

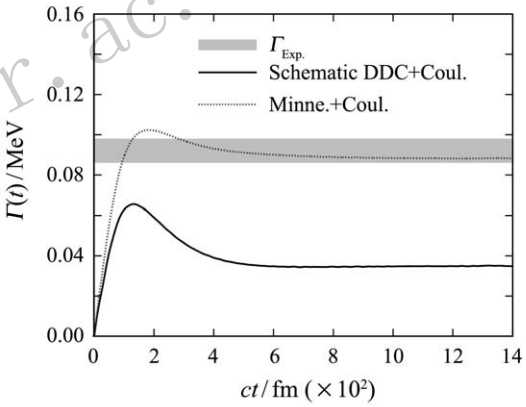

Fig. 2　The 2p-decay width for ^{6}Be obtained with the time-dependent three-body model. Results are obtained with the schematic DDC and modified Minnesota pairing interactions. The experimental value is also shown by the shaded area.

4 Summary

In this article, we reported the new result by a time-dependent three-body model, in order to discuss the dependence of 2p-decay width on the theoretical models of pairing interactions. We compared the two different models, the schematic DDC and Minnesota with zero and finite ranges, respectively. The calculated 2p-decay width is remarkably suppressed by the schematic DDC interaction. This result can be instructive for further systematic studies based on, *e.g.*, HFB or nuclear EDF theories[29]. Further investigation about this suppressing effect, which should be connected to the essential characters of pairing models, *e.g.* whether the interaction has finite or zero range, is an important future work.

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Another possibility of progress is to study the dynamical property with its sensitivity to the pairing models. For this purpose, our time-dependent model is an advantageous tool that we can monitor the time-dependent decay probabilities[18]. Work toward this direction is also in progress now.

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