第 31 卷第 3 期 原 子 核 物 理 评 论 Vol. 31, No. 3
2014 年 9 月 Nuclear Physics Review Sep., 2014

Article ID: 1007-4627(2014) 03-0333-06

Symmetry Energy with the Non-nucleonic Constituents in Nuclear Matter

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Abstract: While the nuclear symmetry energy is usually studied in finite nuclei and nucleonic matter, we study the symmetry energy in relativistic mean-field models with the inclusion of the hyperon and quark degrees of freedom at high densities. Apparent softening of the symmetry energy with the inclusion of hyperon and quark degrees of freedom is found and demonstrated in the relativistic mean-field model. This softening would have associations with the symmetry energy extraction which needs detailed discriminations in dense matter with the admixture of new degrees of freedom created by heavy-ion collisions.

Key words: symmetry energy; hyperon; quark degrees of freedom; relativistic mean-field model

CLC number: O572.21 **Document code:** A **DOI:** 10.11804/NuclPhysRev.31.03.333

1 Introduction

The nuclear symmetry energy of isospin asymmetric nuclear matter is important for understanding the structure of neutron- or proton-rich nuclei, the reaction dynamics of heavy-ion collisions, see, *e.g.*, Refs. [1–3], and a number of important issues in astrophysics, see, *e.g.*, Refs. [4–6]. Though data have been accumulated to constrain quite well the symmetry energy at normal density and subsaturation densities, the accurate extraction of the symmetry energy is restrained by the uncertainty of the many-body theory. It is well-known that there exists a Coster band for the nuclear saturation which reflects the deviations of various famous nuclear potentials. Due to the uncertainty induced by the Coster band, the symmetry energy at saturation density should also be allowed in a reasonable region, depending on nuclear models and extraction approaches. Of course, this is a home truth in the nuclear community. However, the striking point is that the density dependence of the symmetry energy is still poorly known at supra-normal densities[3, 7–9], although appreciable progresses have been achieved on constraining the symmetry energy at saturation and subsaturation densities either through the extraction by virtue of astrophysical observations or terrestrial experiments[10–15].

It is not strange that theoretical models predict diverse density dependencies of the symmetry energy at high densities, because nuclear models are just calibrated by properties of finite nuclei and nuclear matter at saturation. Nevertheless, it is worth noticing that completely diverse density dependencies of the symmetry energy can be extracted from analyzing the FOPI/GSI data on the π^-/π^+ ratio in relativistic heavy-ion collisions with various transport models[7–9]. Indeed, the diversity of experimental extraction of high-density symmetry energy is also associated with the in-medium properties of the strong interaction. The theoretical uncertainty of high-density symmetry energy is usually regarded to be associated with the tensor force that originates from the exchange terms[16−17]. On the other hand, the mean-field approximation is regarded as a much better approximation at high densities. In this work,

Received date: 28 Sep. 2013; **Revised date:** 18 Oct. 2013

Foundation item: National Natural Science Foundation of China(10975033, 11275048)

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we thus do not carry on the tensor force that appears beyond the Hartree approximation but consider the non-nucleonic degrees of freedom in the Hartree approximation.

The new degrees of freedom considered here are hyperons and quarks that would appear in dense matter roughly around the density $2\sim4\rho_0$, depending on the parametrization of models[18–22]. Nuclear matter at this density domain can be produced via heavy-ion collisions, and it usually includes the admixture of non-nucleonic degrees of freedom. In the past, the effects of new constituents on the symmetry energy are seldom investigated. It is the aim of this work to reveal the variation of the symmetry energy in phases featuring these new constituents. The paper is organized in the following. In Sec. 2, we present brief formalism for the symmetry energy in pure hadron and mixed phases within the relativistic mean-field (RMF) framework. In Sec. 3, the numerical results and discussion are given. At last, we give a brief summary.

2 Brief formalism

In the parabolic approximation, the energy per nucleon in isospin asymmetric nuclear matter can be written as

$$\frac{\mathcal{E}}{\rho_{\mathrm{N}}}=\frac{E}{A}=e_0(\rho_{\mathrm{N}})+E_{\mathrm{sym}}(\rho_{\mathrm{N}})\delta^2\ , \tag{1}$$

where $e_0(\rho_{\mathrm{N}})$ is the energy per nucleon in symmetric nuclear matter with $\rho_{\mathrm{N}}=\rho_{\mathrm{n}}+\rho_{\mathrm{p}}$ being the nucleonic number density, the $E_{\mathrm{sym}}(\rho_{\mathrm{N}})$ is the symmetry energy, and $\delta=(\rho_{\mathrm{n}}-\rho_{\mathrm{p}})/\rho_{\mathrm{N}}$ is the isospin asymmetry. The symmetry energy in the RMF models can be given as

$$E_{\mathrm{sym}}=\frac{1}{2}C_\rho^2\rho_{\mathrm{N}}+\frac{k_{\mathrm{F}}^2}{6E_{\mathrm{F}}^*}\ , \tag{2}$$

where $C_\rho=g_{\rho\mathrm{N}}^*/m_\rho^*$ and $E_{\mathrm{F}}^*=\sqrt{k_{\mathrm{F}}^2+m_{\mathrm{N}}^{*\,2}}$ with m_{N}^* the effective mass of nucleon, k_{F} the nucleon Fermi momentum, and m_ρ^* the in-medium ρ-meson mass.

For non-nucleonic baryons in this work, we just consider the Λ hyperon which is an isoscalar. In principle, we can include the isovector components $(\Sigma^\pm,\Sigma^0)$ and (Ξ^0,Ξ^-) and generally introduce in the energy density the new symmetry energy terms for these isovector components as in Eq. (1). It would be interesting to analyze the properties of the hyperon symmetry energies and the effects on the nuclear symmetry energy. However, here we just focus on the effect of the isoscalar Λ hyperon by ignoring the complication of the isovector hyperon components, because the Λ fraction is usually dominant in nuclear matter. In this case, Eq. (1) still holds for hyperonized matter. The nuclear symmetry energy now reads

$$E_{\mathrm{sym}}=\frac{1}{2}C_\rho^2\frac{\rho_{\mathrm{N}}^2}{\rho_{\mathrm{B}}}+\frac{k_{\mathrm{F}}^2}{6E_{\mathrm{F}}^*}\frac{\rho_{\mathrm{N}}}{\rho_{\mathrm{B}}}\ , \tag{3}$$

where ρ_{N} is the number density of nucleons, and $\rho_{\mathrm{B}}=\rho_{\mathrm{N}}+\rho_\Lambda$. The symmetry energy is now suppressed due to the factor $\rho_{\mathrm{N}}/\rho_{\mathrm{B}}$. On the other hand, as one source term of meson fields, the Λ hyperon has led to a moderate decrease to the nucleon effective mass and E_{F}^* in the kinetic term. Together with the suppressed nucleon Fermi momentum in E_{F}^*, the suppression of the kinetic term can be partially compensated. Nevertheless, the symmetry energy eventually turns out to be suppressed in either the baryon-density or nucleon-density profile. While the formula (3) applies to the case of the given ratio $\rho_\Lambda/\rho_{\mathrm{B}}$, in chemically equilibrated and charge neutral matter where the particle fractions are obtained from solving coupled equations, we may calculate the nuclear symmetry energy using the following relation

$$E_{\mathrm{sym}}=\frac{1}{4\delta}(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})\frac{\rho_{\mathrm{N}}}{\rho_{\mathrm{B}}}\ , \tag{4}$$

where μ_{n} and μ_{p} are the neutron and proton chemical potentials, respectively. With Eq. (4), one can obtain the symmetry energy using the difference of nucleon chemical potentials in asymmetric matter. As the isospin asymmetry parameter δ runs to vanishing, the formula (4) reduces to Eq. (3).

After the hadron-quark phase transition occurs, hadrons and quarks coexist in a mixed phase. The construction of the mixed phase is based on the mechanical and chemical equilibriums, namely, the Gibbs conditions which are given as[19]

$$\begin{aligned} &p^{\mathrm{H}}=p^{\mathrm{Q}},\ \mu_{\mathrm{u}}=\frac{\mu_{\mathrm{p}}}{3}-\frac{\mu_{\mathrm{e}}}{3}\ ,\\ &\mu_{\mathrm{d}}=\mu_{\mathrm{s}}=\frac{\mu_{\mathrm{n}}}{3}+\frac{\mu_{\mathrm{e}}}{3}\ . \end{aligned} \tag{5}$$

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Here, we invoke the MIT bag model to describe the hadron-quark admixture and the quark phase[23]. In terms of the quark phase proportion Y, the total baryon density can be expressed as

$$\rho_{\rm B}=\frac{Y}{3}\rho_{\rm Q}+(1-Y)\rho_{\rm H}\ , \tag{6}$$

where $\rho_{\rm H}=\rho_{\rm N}+\rho_\Lambda$ is the baryon density on the hadronic level and $\rho_{\rm Q}$ is the quark density. Using Gibbs conditions, one can obtain the quark phase proportion Y. The total energy density and isospin asymmetry parameters are written as

$$\mathcal{E}=(1-Y)\mathcal{E}_{\rm H}+Y\mathcal{E}_{\rm Q},\ \alpha=(1-Y)\delta_{\rm H}+Y\delta_{\rm Q}, \tag{7}$$

with $\delta_{\rm H}=(\rho_{\rm n}-\rho_{\rm p})/\rho_{\rm N}$ and $\delta_{\rm Q}=(\rho_{\rm u}-\rho_{\rm d})/(\rho_{\rm u}+\rho_{\rm d})$. The energy density in the mixed phase thus depends on the Y. In the parabolic approximation, the energy density can be expressed as

$$\frac{\mathcal{E}}{\rho_{\rm B}}=e_0(\rho_{\rm B},Y)+E^{\rm H}_{\rm sym}(\rho_{\rm B},Y)\delta^2_{\rm H}+E^{\rm Q}_{\rm sym}(\rho_{\rm B},Y)\delta^2_{\rm Q}. \tag{8}$$

Because the quark phase proportion depends on the isospin asymmetry, we limit the derivation of the symmetry energy $E^{\rm H}_{\rm sym}$ in symmetric matter, namely $\alpha=0$. In this way, the nuclear symmetry energy is defined as $E^{\rm H}_{\rm sym}=(1/2)\partial^2(\mathcal{E}/\rho_{\rm B})/\partial\delta^2_{\rm H}$ at $\delta_{\rm H}=0$, and the definition of the quark symmetry energy is similarly given as $E^{\rm Q}_{\rm sym}=(1/2)\partial^2(\mathcal{E}/\rho_{\rm B})/\partial\delta^2_{\rm Q}$ at $\delta_{\rm Q}=0$. The symmetry energy is eventually dependent on the quark phase proportion and the bag constant.

3 Numerical results

The symmetry energy for various Λ fractions is calculated in symmetric matter at $\delta=0$. The effect of Λ hyperons on the nuclear symmetry energy with the RMF model NL3[24] is illustrated in Fig. 1. It is shown in Fig. 1 that the symmetry energy is softened clearly with the increase of the Λ fraction. Compared with Eqs. (2) and (3), we see that the softening is dominated by the suppression factor $\rho_{\rm N}/\rho_{\rm B}$. However, even without this suppression factor, the symmetry energy in hyperonized matter is still modified by the isoscalar Λ hyperons provided there exists the isoscalar-isovector coupling $\Lambda_{\rm v}$ (for model details, see Refs. [2, 21]). This is clearly seen in the inset of the lower panel of Fig. 1 where the potential part of the symmetry energy is displayed. Similarly, if the charged hyperons are included, the potential part of the symmetry energy can be modified even without the isoscalar-isovector coupling. This can be verified numerically, while it is beyond the scope of the present work. Nevertheless, we may infer that the symmetry energy may be modified significantly by taking into account the hyperons.

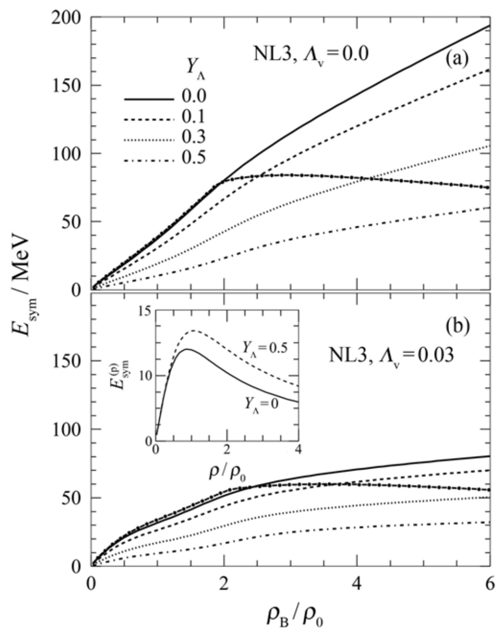

Fig. 1 The symmetry energy as a function of density in the presence of the Λ hyperons.
The curves depicted are for various hyperon fractions. The upper panel presents the NL3 results without the isoscalar-isovector coupling, while the lower panel include such a coupling that softens the symmetry energy. The potential part of the symmetry energy in the inset of the lower panel is drawn for two cases with and without hyperons. The solid dotted curve is obtained in chemically equilibrated and charge neutral matter.

With the increase of density, the hadron-quark phase transition may occur. In this work, quark matter, regarded as the free fermion gas without interactions, is described with the MIT bag model[23]. In quark matter, we include the up, down and strange quarks. For the hadron phase, we choose the density-dependent RMF models SLC and SLCd[25−26] and a few nonlinear RMF models in the calculation: NL3[24], TM1[27] and NL3w3[28]. The TM1 param-

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eter set has a much softer vector potential than the NL3 and NL3w3, while the SLC and SLCd have additional rearrangement term. Because of these distinctions, these models produce rather different nucleon chemical potentials and quite different critical densities according to Gibbs conditions. Moreover, these models have differences in the symmetry energy. The RMF model NL3w3 has a softer symmetry energy than the NL3. The SLCd and SLC also have different density profile of the symmetry energy, as pointed out above. These specific model factors can affect the occurrence of the phase transition. The mixed phase consists of high-density quark matter and low-density nuclear matter with the quark phase proportion Y being determined according to Gibbs conditions. We do not reiterate here the detailed solutions which can be found in the literature[19].

In Fig. 2, it shows the critical density as a function of isospin asymmetry for various bag constants and RMF models. Considering that neither the chemical equilibrium nor the charge neutral condition can generally match the given isospin asymmetries in asymmetric matter, we take a simple treatment by neglecting the charge chemical potential $\mu_{\rm e}$ in Eq. (5) in determining the quark chemical potentials. Our treatment for the quark chemical potentials is different from that in Refs. [29–31] where such a charge chemical potential is actually included considering the baryon and isospin conservations. We see in Fig. 2 that the critical density with the nonlinear RMF models (TM1, NL3, and NL3w3) decreases generally with the increase of the isospin asymmetry, consistent with that observed in Ref. [29], though the change rate is smaller in the present calculation. Note that the critical density with the TM1 and NL3 at $\alpha=0$ is consistent with that in Ref. [31] as long as we rule out the strange quarks. The big difference appears in the density-dependent RMF models SLC and SLCd. We see that the critical density with the SLC and SLcd is much higher than other models, and it first decreases and then increases with the rise of the isospin asymmetry, especially with a smaller bag constant. This feature is originated mainly from the density-dependent properties induced by the parameter scaling in SLC and SLCd. Compared to other models in the present work, the SLC and SLCd feature the rearrangement term and are characteristic of a much smaller nucleon effective mass[25]. Thus, the resulting nucleon chemical potentials with the SLC and SLCd are clearly smaller than those with other models. Together with the different density-dependent property of the chemical potentials, the properties of the critical density with the SLC and SLCd, as shown in Fig. 2, can be well understood.

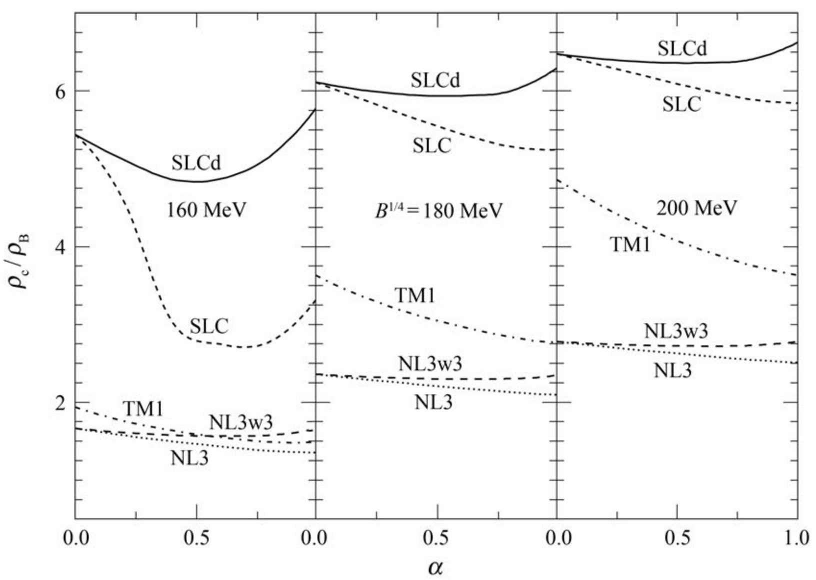

Fig. 2 The critical density for the hadron-quark phase transition as a function of isospin asymmetry with various RMF models and bag constants as marked.

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Moreover, the nonlinear RMF model TM1 has a larger critical density than the NL3 and NL3w3. This is also attributed to the smaller nucleon chemical potential due to the softening of the vector potential in TM1[27]. As seen in Fig. 2, another two factors, the bag constant and symmetry energy, can affect the transition density. The rise of the bag constant reduces the pressure of quark matter and thus results in larger critical densities. The softening of the symmetry energy can modify the nucleon chemical potentials in asymmetric matter and reduces moderately the critical density, as observed in Fig. 2. We stress that the results interpreted here is of qualitative significance, while more specific results depend on the specific treatments[29].

Besides the critical density, the quark phase proportion Y also depends on the isospin asymmetry. In this way, the symmetry energy in the mixed phase obtained in symmetric matter can not simply be used to predict the properties of asymmetric matter because the quark phase proportion changes with the isospin asymmetry in asymmetric matter. Nevertheless, the symmetry energy obtained in symmetric matter is instructive to exhibit its variation trend in the mixed phase. Shown in Fig. 3 is the nuclear symmetry energy as a function of baryon density for various Λ-hyperon fractions with the bag constant

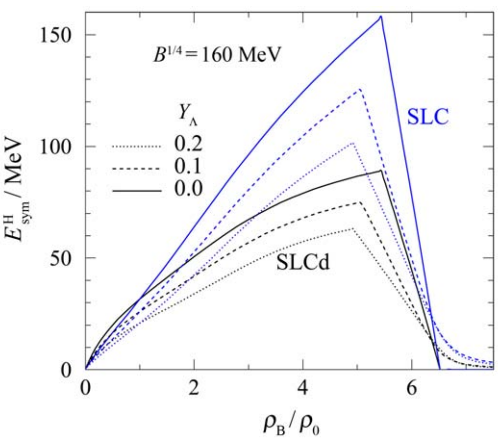

Fig. 3 (color online) The symmetry energy as a function of density for different Λ-hyperon fractions in hyeronized matter with the hadron-quark phase transition.
The RMF models SLC and SLCd are adopted here and the bag constant is (160 MeV)^4. Taken from Ref. [32].

$B = (160\ \mathrm{MeV})^4$. Apparent decrease of the symmetry energy can be observed after the hadron-quark phase transition occurs. With the increase of density, the nucleon phase proportion decreases, which causes a straightforward reduction of the nuclear symmetry energy. As the nucleon phase proportion reduces to zero, the nuclear symmetry energy vanishes. The inclusion of Λ hyperons suppresses the symmetry energy in the hadronic phase, consistent with those shown in Fig. 1. For other bag constants and RMF models, the conclusion is qualitatively similar. To save space, these numerical results are thus not displayed.

4 Summary

We have studied the effect of Λ hyperons and quarks on the nuclear symmetry energy at high densities with relativistic models. The softening of the nuclear symmetry energy is observed either in chemically equilibrated matter or matter with an given Λ fraction. With the occurrence of the hadron-quark phase transition, we find that the nuclear symmetry energy obtained in the mixed phase reduces quickly with the rise of quark phase proportion. Though the specific softening depends on the parametrizations of models, we conclude that the effect of phase transitions is important on the symmetry energy. Because high-density matter is created by the heavy-ion collisions, one can extract the symmetry energy until the effect of the phase transition on the symmetry energy is appropriately figured out.

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非核子自由度物质的对称能

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摘要： 通常人们在有限核与核子物质中研究对称能，而本工作利用相对论平均场模型研究包含超子和夸克自由度物质的对称能。发现了含超子和夸克自由度物质中对称能的表观软化，并用相对论模型对此做了阐述。该软化现象提示由重离子碰撞产生的含非核子自由度致密物质的对称能提取将有待细致的甄别。

关键词： 对称能；超子；夸克自由度；相对论平均场理论

收稿日期： 2013-09-28； **修改日期：** 2013-10-18

基金项目： 国家自然科学基金资助项目(10975033, 11275048)

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