Article ID: 1007-4627(2011)04-0433-06

Winger Function for Spin Half Non-commutative Landau Problem*

WANG Ya-hui, YAN Jiang-feng, YUAN Yi

(*Department of Physics, Shaanxi University of Technology, Hanzhong* 723001, *Shaanxi, China*)

Abstract: With great significance in describing the state of quantum system, the Wigner function of the spin half non-commutative Landau problem is studied in this paper. On the basis of the review of the Wigner function in the commutative space, which is subject to the *-eigenvalue equation, Hamiltonian of the spin half Landau problem in the non-commutative phase space is given. Then, energy levels and Wigner functions in the form of a matrix of the spin half Landau problem in the non-commutative phase space are obtained by means of the *-eigenvalue equation (or Moyal equation).

Key words: spin half Landau problem; Wigner function; non-commutative phase space; uniform magnetic field

CLC number: O413.1 **Document code**: A

1 Introduction

In recent years the Wigner function has enjoyed a wide popularity in virtually all areas of physics. As a quasi-probability distribution function in phase-space and a highly semi-classical approximation[1], the Wigner function has been useful in describing quantum transport in quantum optics, nuclear physics, decoherence (e.g. quantum computing), quantum chaos, signal processing, etc.. Nevertheless, a remarkable aspect of the Wigner function was not pioneered until 1975 by Moyal according to the internal logic of Quantum Mechanics[2]. In fact, with the Moyal *-eigenvalue equation as its general form, the Wigner function is not only as valuable as other formulations, such as Schrödinger, Heisenberg regularization operator, Feynman path integral quantization, etc., but also of great significance in modern quantum measurement. For example, the Wigner function of an ensemble of helium atoms was discussed in Ref. [3].

Furthermore, the emergence of the noncommutative geometry with a neutral way in string theory/(M-Theory) in a definite limit not only provides an effective analysis of the duality, BPS state and D-brane dynamics, but also causes a revolution in the whole physical theory[4−5]. In the ultra-micro field the space-time coordinates which are never commutative can satisfy the uncertain-relation of space-time. With this, the space-time point loses its original sense and the geometry which describes the original physical phenomenona is not consistent with the new physics in this space-time area. Therefore, it is necessary to have a new space-time geometry-noncommutative space-time geometry to describe the gravitation[6]. Especially,

* **Received date**: 10 Jan. 2011; **Revised date**: 17 Feb. 2011

* **Foundation item**: Open Topic of State Key Laboratory for Superlattices and Microstructures(CHJG200902); Scientific Research Project in Shaanxi Province(2009K01-54); Foundation of Shanxi University of Technology(SLGKY10-02)

Biography: Wang Ya-hui(1978—), male, Baoji, Shaanxi, Master, Lecturer, Working on Quantum Field Theory; E-mail: angyahui8312469@163.com

Yang-Mills theory (NCYM) in the non-commutative flat space, the relationship between non-commutative geometry and the D-membrane kinetic need paying great attention. Recent studies show that the $*$-eigenvalue equation which the Wigner function obeys in the phase space is the $*$-eigenvalue in the Moyal equation, and it is the same as Moyal-Weyl product in ultra-spin theories. The causes of such sameness are worth pondering. Thus, in recent years there have been a lot of delightful achievements in the noncommutative field theory, the topological phase and the correction of noncommutative energy levels, etc.[7-20]. For example, Refs. [21 - 26] have studied the Wigner function in noncommutative space. With the introduction of the Wigner function in commutative space and its $*$-eigenvalue equation, the Wigner functions and energy levels of spin half Landau problem in the non-commutative phase space are analyzed and obtained by means of the $*$-eigenvalue equation in this article.

2 Wigner Function and Its $*$-eigenvalue Equation

This part intends to make a review of the Wigner function and its $*$-eigenvalue equation. It is known that the Wigner function is of great significance in the physical measurements and theoretical studies. In the phase space with the degree of freedom n the general form of the Wigner function is

$$W(\boldsymbol{x},\boldsymbol{p},t)=\frac{1}{(2\pi\hbar)^n}\int_{-\infty}^{\infty}\mathrm{d}\boldsymbol{y}\mathrm{e}^{-\mathrm{i}\boldsymbol{y}\boldsymbol{p}}\langle\boldsymbol{x}-\frac{\boldsymbol{y}}{2}|\hat{\rho}|\boldsymbol{x}+\frac{\boldsymbol{y}}{2}\rangle\,. \tag{1}$$

This is a special representation of the density matrix. Alternatively, it may be regarded as the auto-correlation function of the wave function $\psi(\boldsymbol{x})$ in the quantitative subsystem.

According to Eq. (1), we can prove that the time-dependent Wigner function has the following dynamic evolution equation

$$\frac{\partial W}{\partial t}=-\frac{\boldsymbol{p}}{m}\frac{\partial W}{\partial\boldsymbol{x}}+\frac{\partial V}{\partial\boldsymbol{x}}\frac{\partial W}{\partial\boldsymbol{p}}\,. \tag{2}$$

Of course, we can also solve the Moyal equation instead of the Schrödinger equation to get the following equation with $*_\hbar$-eigenvalue[19],

$$\frac{\partial W}{\partial t}=\frac{H*_\hbar W-W*_\hbar H}{\mathrm{i}\hbar}\,, \tag{3}$$

where the $*_\hbar$-product is

$$*_\hbar=\exp\left[\frac{\mathrm{i}\hbar}{2}(\overleftarrow{\partial}_x\overrightarrow{\partial}_p-\overleftarrow{\partial}_p\overrightarrow{\partial}_x)\right]. \tag{4}$$

Since the $*_\hbar$-product involves exponential operators, much difficulty exists in the real calculation. In fact, $\hbar$ is a very small volume, so $*_\hbar$-product, as a series expansion, can be expressed as[19]

$$f(\boldsymbol{x},\boldsymbol{p})*_\hbar g(\boldsymbol{x},\boldsymbol{p})=f\left(\boldsymbol{x}+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ \boldsymbol{p}-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right)g(\boldsymbol{x},\boldsymbol{p}) \tag{5}$$

or

$$f(\boldsymbol{x},\boldsymbol{p})*_\hbar g(\boldsymbol{x},\boldsymbol{p})=f(\boldsymbol{x},\boldsymbol{p})g\left(\boldsymbol{x}-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ \boldsymbol{p}+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right). \tag{6}$$

In this way, the Wigner function meets the binding $*_\hbar$-eigenvalue equations[19]

$$H(\boldsymbol{x},\boldsymbol{p})*_\hbar W(\boldsymbol{x},\boldsymbol{p})=H\left(\boldsymbol{x}+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ \boldsymbol{p}-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right)\times W(\boldsymbol{x},\boldsymbol{p})=EW(\boldsymbol{x},\boldsymbol{p}) \tag{7}$$

and

$$W(\boldsymbol{x},\boldsymbol{p})*_\hbar H(\boldsymbol{x},\boldsymbol{p})=W(\boldsymbol{x},\boldsymbol{p})H\left(\boldsymbol{x}-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ \boldsymbol{p}+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right)=EW(\boldsymbol{x},\boldsymbol{p})\,. \tag{8}$$

Here E is the energy eigenvalue of $H\psi=E\psi$. Although Eqs. (7) and (8) completely describe the nature of the Wigner function, what is still worth mentioning is that the method of transformation from $*$-product in the non-commutative phase space to the general multiplication is the same as that of conversion between Moyal $*$-eigenvalue Eqs. (7) and (8). Therefore, the study on the Wigner function in non-commutative phase space is of great significance.

3 Hamiltonian of Electrons in a Magnetic Field in the Non-commutative Phase Space

This section aims at getting Hamiltonian for electrons in a magnetic field in the non-commutative phase space. It is known that in the non-commutative phase space, the steady state Schrödinger equation is usually written as[22]

$$H(\boldsymbol{x},\ \boldsymbol{p}) * \psi(\boldsymbol{x}) = \hat{E}\psi(\boldsymbol{x})\ . \tag{9}$$

Here, Moyal-Weyl product or the $*$-product is defined as

$$*= \exp[\ \frac{i\theta_{ij}}{2\alpha^2}\overleftarrow{\partial}_{x_i}\overrightarrow{\partial}_{x_j} + \frac{i\bar{\theta}_{ij}}{2\alpha^2}\overleftarrow{\partial}_{p_i}\overrightarrow{\partial}_{p_j}]. \tag{10}$$

Thus, as a series expansion, $*$-product can be expressed as

$$\begin{aligned}(f * g)(\boldsymbol{x},\ \boldsymbol{p}) = f(\boldsymbol{x},\ \boldsymbol{p})g(\boldsymbol{x},\ \boldsymbol{p}) + \\ \frac{i}{2\alpha^2}\theta_{ij}\ \partial_i^x f\partial_j^x g\,|_{x_i=x_j} + \\ \frac{i}{2\alpha^2}\bar{\theta}_{ij}\ \partial_i^p f\partial_j^p\ g\,|_{p_i=p_j} + O(\theta^2)\ ,\end{aligned} \tag{11}$$

where $f(x)$ and $g(x)$ are two arbitrary functions. Now, we begin to discuss the energy level of electrons in the magnetic field in the non-commutative phase space. According to Ref. [22], in two dimensions we can rewrite Eq. (11) as

$$\hat{x}_1 = \alpha x_1 - \frac{1}{2\alpha\hbar}\theta p_2,\ \hat{x}_2 = \alpha x_2 + \frac{1}{2\alpha\hbar}\theta p_1,$$

$$\hat{p}_1 = \alpha p_1 + \frac{1}{2\alpha\hbar}\bar{\theta}x_2,\ \hat{p}_2 = \alpha p_2 - \frac{1}{2\alpha\hbar}\bar{\theta}x_1, \tag{12}$$

here, θ and $\bar{\theta}$ are both small non-commutative parameters. From Ref. [22] we know $\bar{\theta}=4\alpha^2\hbar^2(1-\alpha^2)/\theta$, $\theta \leqslant (10^4\ \text{GeV})^{-2}$, where α is another non-commutative parameter.

Now, let's discuss the movement of the electron with the quantity of electric charge e in the uniform magnetic field $\boldsymbol{B}(0,\ 0,\ B)$. In terms of the electronic mass μ and spin 1/2, when the electron moves in the o-xy plane, its Hamiltonian in the phase space is

$$\begin{aligned}H = \frac{1}{2\mu}\Big[\Big(p_1 + \frac{eB}{2c}x_2\Big)^2 + \Big(p_2 - \frac{eB}{2c}x_1\Big)^2\Big] + \\ \frac{e\hbar\ B}{2\mu\ c}\sigma_Z.\end{aligned} \tag{13}$$

According to the Moyal-Weyl product, we can change the $*$-product to the ordinary multiplication only by means of a Bopp change $H(x,\ p)\rightarrow \hat{H}(\hat{x},\ \hat{p})$. As a result, the Hamiltonian of the electron in the magnetic field in the non-commutative phase space is

$$\begin{aligned}\hat{H} = \frac{1}{2\mu}\Big[\Big(\hat{p}_1 + \frac{eB}{2c}\hat{x}_2\Big)^2 + \\ \Big(\hat{p}_2 - \frac{eB}{2c}\hat{x}_1\Big)^2\Big] + \frac{e\hbar B}{2\mu c}\hat{\sigma}_Z.\end{aligned} \tag{14}$$

Inserting Eq. (12) into Eq. (14), we get the Hamiltonian in non-commutative phase space

$$\begin{aligned}\hat{H} = \frac{1}{2\mu}\Big[\Big(\alpha + \frac{eB}{4\hbar\alpha c}\theta\Big)p_1 + \Big(\frac{eB}{2c}\alpha + \frac{\bar{\theta}}{2\hbar\alpha}\Big)x_2\Big]^2 + \\ \frac{1}{2\mu}\Big[\Big(\alpha + \frac{eB}{4\hbar\alpha c}\theta\Big)p_2 - \Big(\frac{eB}{2c}\alpha + \frac{\bar{\theta}}{2\hbar\alpha}\Big)x_1\Big]^2 + \frac{e\hbar B}{2\mu c}\hat{\sigma}_Z.\end{aligned} \tag{15}$$

Ordering

$$\tilde{\mu} = \frac{\mu}{(\alpha + \frac{eB}{4\hbar\alpha c}\theta)^2},\ \tilde{\omega}_L = \frac{\frac{eB\alpha}{c} + \frac{\bar{\theta}}{\hbar\alpha}}{2\tilde{\mu}\Big(\alpha + \frac{eB}{4\hbar\alpha c}\theta\Big)}, \tag{16}$$

we obtain

$$\begin{aligned}\hat{H} = \frac{1}{2\tilde{\mu}}(p_1^2 + p_2^2) + \frac{1}{2}\tilde{\mu}\tilde{\omega}_L^2(x_1^2 + x_2^2) + \\ \tilde{\omega}_L(x_2 p_1 - x_1 p_2) + \frac{e\hbar B}{2\mu c}\hat{\sigma}_Z.\end{aligned} \tag{17}$$

Here $\tilde{\mu}$ and $\tilde{\omega}_L$ are the electron's equivalent mass and equivalent frequency respectively in the magnetic field in the non-commutative phase space.

4 The Wigner Function and Energy Level of the Spin Half Landau Problem in NC Phase Space

This section provides the Wigner function and energy level of the spin half Landau problem in NC phase space. From the third dsection we know the Hamiltonian of spin half Landau problem in the non-commutative phase space

$$\hat{H}=\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)+\frac{e\hbar B}{2\mu c}\hat{\sigma}_Z. \tag{18}$$

Taking into account the electron spin as well as a Pauli matrix

$$\hat{\sigma}_Z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

we know that in the representation of $\hat{\sigma}_Z$ the Wigner function has the following form

$$\hat{W}=\begin{pmatrix}\hat{W}_1 & 0\\ 0 & \hat{W}_2\end{pmatrix}. \tag{19}$$

Thus, in the phase space the spin half Landau problem is described by the following equation

$$\left[\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2({x_1}^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)+\frac{e\hbar B}{2\mu c}\hat{\sigma}_Z\right]*\begin{pmatrix}\hat{W}_1 & 0\\ 0 & \hat{W}_2\end{pmatrix}=\hat{E}\begin{pmatrix}\hat{W}_1 & 0\\ 0 & \hat{W}_2\end{pmatrix}. \tag{20}$$

After calculation, we get the Wigner function of the spin half Landau problem in the non-commutative phase space

$$\left[\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)+\frac{e\hbar B}{2\mu c}\right]*\hat{W}_1=\hat{E}_1\hat{W}_1. \tag{21}$$

and

$$\left[\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)-\frac{e\hbar B}{2\mu c}\right]*\hat{W}_2=\hat{E}_2\hat{W}_2. \tag{22}$$

In terms of

$$\hat{\varepsilon}_1=\hat{E}_1-\frac{e\hbar B}{2\mu c},\quad \hat{\varepsilon}_2=\hat{E}_2+\frac{e\hbar B}{2\mu c}, \tag{23}$$

Eqs. (21) and (22) change to

$$\left[\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)\right]*\hat{W}_1=\hat{\varepsilon}_1\hat{W}_1, \tag{24}$$

and

$$\left[\frac{1}{2\tilde{\mu}}(p_1^2+p_2^2)+\frac{1}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)+\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)\right]*\hat{W}_2=\hat{\varepsilon}_2\hat{W}_2. \tag{25}$$

Inserting the Moyal eigenvalue equation into Eqs. (24)and (25) respectively, we have

$$\frac{1}{2\tilde{\mu}}\Big[(p_1^2+p_2^2)+\tilde{\mu}^2\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)-\frac{\hbar^2}{4}\tilde{\mu}^2\tilde{\omega}_{\mathrm{L}}^2(\partial_{p_1}^2+\partial_{p_2}^2)-\frac{\hbar^2}{4}(\partial_{x_1}^2+\partial_{x_2}^2)+2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)+\frac{\hbar^2}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}\times(\partial_{x_1}\partial_{p_2}-\partial_{x_2}\partial_{p_1})\Big]\hat{W}_1=\hat{\varepsilon}_1\hat{W}_1, \tag{26}$$

and

$$\frac{1}{2\tilde{\mu}}\Big[(p_1^2+p_2^2)+\tilde{\mu}^2\tilde{\omega}_{\mathrm{L}}^2(x_1^2+x_2^2)-\frac{\hbar^2}{4}\tilde{\mu}^2\tilde{\omega}_{\mathrm{L}}^2(\partial_{p_1}^2+\partial_{p_2}^2)-\frac{\hbar^2}{4}(\partial_{x_1}^2+\partial_{x_2}^2)+2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}(x_2p_1-x_1p_2)+\frac{\hbar^2}{2}\tilde{\mu}\tilde{\omega}_{\mathrm{L}}(\partial_{x_1}\partial_{p_2}-\partial_{x_2}\partial_{p_1})\Big]\hat{W}_2=\hat{\varepsilon}_2\hat{W}_2. \tag{27}$$

This equation is similar to the Landau problem. Now let's introduce four new variables X_i ($i=1, 2, 3, 4$) for Eq. (26),

$$X_1=\left(\sqrt{\frac{1}{2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}}p_2+\sqrt{\frac{\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}{2}}x_1\right),$$

$$X_2=\left(\sqrt{\frac{1}{2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}}p_1+\sqrt{\frac{\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}{2}}x_2\right),$$

$$X_3=\left(\sqrt{\frac{1}{2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}}p_1-\sqrt{\frac{\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}{2}}x_2\right),$$

$$X_4=\left(\sqrt{\frac{1}{2\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}}p_2-\sqrt{\frac{\tilde{\mu}\tilde{\omega}_{\mathrm{L}}}{2}}x_1\right). \tag{28}$$

Straightforward calculation leads to

$$\frac{1}{2\tilde{\mu}}\Big[3\tilde{\mu}\tilde{\omega}_L(X_2^2+X_4^2)-\tilde{\mu}\tilde{\omega}_L(X_1^2+X_3^2)-\frac{3\hbar^2}{8}\tilde{\mu}\tilde{\omega}_L(\partial_{X_2}^2+\partial_{X_4}^2)+\frac{\hbar^2}{8}\tilde{\mu}\tilde{\omega}_L(\partial_{X_1}^2+\partial_{X_3}^2)\Big]\times \hat{W}_1=\hat{\varepsilon}_1\hat{W}_1. \tag{29}$$

With two more new variables ξ and η,

$$\xi:=\frac{2}{\hbar}(X_1^2+X_3^2)\ ,$$

$$\eta:=\frac{2}{\hbar}(X_2^2+X_4^2)\ . \tag{30}$$

Eq. (29) may be rewritten as follows,

$$\frac{\tilde{\omega}_L}{2}\left[6\left(\frac{\eta}{4}-\eta\partial_\eta^2-\partial_\eta\right)-2\left(\frac{\xi}{4}-\xi\partial_\xi^2-\partial_\xi\right)\right]\times \hat{W}_1=\hat{\varepsilon}_1\hat{W}_1. \tag{31}$$

With the separation of variables, $\hat{W}_1(\xi,\eta)=\hat{W}_1(\xi)^1\hat{W}_1(\eta)^2$, $\hat{\varepsilon}_1=6\hat{\varepsilon}^2-2\hat{\varepsilon}^1$, we have

$$\frac{\tilde{\omega}_L}{2}\left[\frac{\xi}{4}-\xi\partial_\xi^2-\partial_\xi-\hat{\varepsilon}^1\right]\hat{W}_1(\xi)^1=0\ , \tag{32}$$

and

$$\frac{\tilde{\omega}_L}{2}\left[\frac{\eta}{4}-\eta\partial_\eta^2-\partial_\eta-\hat{\varepsilon}^2\right]\hat{W}_1(\eta)^2=0\ . \tag{33}$$

Finally, we can find the solutions for Eqs. (32) and (33)

$$\hat{W}_1(\xi)_m^1=\frac{(-1)^m}{\pi\hbar}\ e^{-\xi/2}\ L_m(\xi)\ ,$$

$$\hat{\varepsilon}^1=\left(m+\frac{1}{2}\right)\frac{\tilde{\omega}_L}{2}\hbar,\quad m=0,1,\cdots \tag{34}$$

and

$$\hat{W}_1(\eta)_n^2=\frac{(-1)^n}{\pi\hbar}\ e^{-\eta/2}\ L_n(\eta)\ ,$$

$$\hat{\varepsilon}^2=\left(n+\frac{1}{2}\right)\frac{\tilde{\omega}_L}{2}\hbar,\quad n=0,1,\cdots \tag{35}$$

Thus, we have

$$\hat{W}_1(\xi,\eta)_{mn}=\frac{(-1)^{m+n}}{(\pi\hbar)^2}\ e^{-(\xi+\eta)/2}\ L_m(\xi)L_n(\eta)\ . \tag{36}$$

$$\hat{E}_1=3\left(m+\frac{1}{2}\right)\hbar\tilde{\omega}_L-\left(n+\frac{1}{2}\right)\hbar\tilde{\omega}_L+\frac{e\hbar B}{2\mu c}\ . \tag{37}$$

Symmetrically, we get

$$\hat{W}_2(\mu,\nu)_{mn}=\frac{(-1)^{m+n}}{(\pi\hbar)^2}\ e^{-(\mu+\nu)/2}\ L_m(\mu)L_n(\nu)\ . \tag{38}$$

$$\hat{E}_2=3\left(m+\frac{1}{2}\right)\hbar\tilde{\omega}_L-\left(n+\frac{1}{2}\right)\hbar\tilde{\omega}_L-\frac{e\hbar B}{2\mu c}\ . \tag{39}$$

These are the very Wigner function and energy level of the spin half Landau problem in non-commutative phase space where the additional items of both non-commutative parameters θ, $\bar{\theta}$ and α are included. When both $\bar{\theta}$ and θ are close to 0, α is close to 1, and the results are back to be that in the commutative space.

5 Conclusion

There is widespread charged particles moving in a magnetic field in nature, so discussing the charged particles is of great theoretical significance and application value in the magnetic field, such as cyclotron mass spectrometry, magnetic focusing, Hall effect, Zeeman effect and the Landau problem and so on. As a quasi-probability distribution function on phase space, a special representation of the density matrix, and a good semi-classical approximation, the Wigner function is very important in quantum measurement. In this article, on the basis of the revision of the nature of Wigner function in the commutative space the Wigner function of the spin half Landau problem in non-commutative phase space is obtained by the *-eigenvalue equation (Moyal equation). The result is of great significance in many practical problems because it not only shows that the Moyal method in the phase space and other quantization methods are equivalent, but also supports that it is compatible with classical mechanics. This is the very importance of the Wigner function.

References:

[1] Wigner E. Phys Rev, 1932, **40**: 749.

[2] Lee Hai-Woong. Phys Repts, 1995, **259**(3): 147.

[3] Kurtsiefer Ch, Pfau T, Mlynek J. Nature, 1997, **386**: 150.

[4] Seiberg N, Witten E. JHEP, 1999, **9909**: 032.

[5] Chaichian M, Sheikh J abbari M M, Tureanu A. Phys Rev Lett, 2001, **86**: 2716.

[6] Wei Gaofeng, Long Chaoyun, Long Zhengweng, *et al*. Chin Phys C, 2008, **32**(4): 247.

[7] Zachos Cosmas. Int J Mod Phys, 2002, **A17**: 297.

[8] Kim Y, Wigner E. Am J Phys, 1990, **58**: 439.

[9] Hillery M, O'Connell R, Scully M, *et al*. Phys Repts, 1984, **106**(3): 121.

[10] Li Kang, Dulat Sayipjamal. Eur Phys J, 2006, **C46**: 825.

[11] Mirza B, Zarei M. Eur Phys J, 2004, **32**: 583.

[12] Li Kang, Wang Jianhua. Eur Phys J, 2007, **C50**: 1007.

[13] Yuan Yi, Li Kang, Wang Jianhua, *et al*. Chin Phys C, 2010, **34**(5): 543.

[14] Wang Jianhua, Li Kang. J Phys, 2007, **A40**: 2197.

[15] Wang Jianhua, Li Kang. Chin Phys Lett, 2007, **24**: 5.

[16] Dulat Sayipjamal, Li Kang. Chin Phys, 2008, **C32**: 92.

[17] Dulat Sayipjamal, Li Kang. Eur Phys J, 2008, **C54**: 333.

[18] Li Kang, Wang Jianhua, Chen Chiyi. Mod Phys Lett, 2005, **A20**(28): 2165.

[19] Wang Yahui, Wang Jianhua, Ren Yajie, *et al*. Nuclear Physcis Review, 2008, **25**(3): 218(in Chinese).
(王亚辉，王剑华，任亚杰，等. 原子核物理评论，2008，**25**(3)：218.)

[20] Wang Yahui, Wang Jianhua, Ren Yajie, *et al*. Nuclear Physcis Review, 2009, **26**(1): 19(in Chinese).
(王亚辉，王剑华，任亚杰，等. 原子核物理评论，2009，**26**(1)：19.)

[21] Wang Yahui, Ren Yajie, Wang Jianhua, *et al*. Nuclear Physcis Review, 2009, **26**(4): 304(in Chinese).
(王亚辉，任亚杰，王剑华. 原子核物理评论，2009，**26**(4)：304.)

[22] Chen Desheng, Wang Yahui, Wang Yao, *et al*. Nuclear Physcis Review, 2010, **27**(2): 155(in Chinese).
(陈德胜，王亚辉，王垚，等. 原子核物理评论，2010，**27**(2)：155.)

[23] Yuan Yi, Li Kang, Wang Jianhua, *et al*. Int J Theor Phys, 2010, **49**: 1993.

[24] Heng T H, Li P, Jing S C. Chin Phys Lett, 2007, **24**: 592.

[25] Heng T H, Lin B S, Jing S C. Chin Phys Lett, 2008, **25**: 3535.

[26] Li Kang, Wang Jianhua, Dulat Sayipjamal, *et al*. Int J Theor Phys, 2010, **49**: 134.

自旋 1/2 非对易朗道问题的 Wigner 函数*

王亚辉[1)]，刘江峰，袁 毅

(陕西理工学院物理系，陕西 汉中 723001)

摘　要：Wigner 函数在对量子体系状态的描述方面具有重要的意义。讨论了自旋 1/2 非对易朗道问题的 Wigner 函数。首先回顾了对易空间中 Wigner 函数所服从的星本征方程，然后给出了非对易相空间中自旋 1/2 朗道问题的 Hamiltonian，最后利用星本征方程(Moyal 方程)计算了非对易相空间中自旋 1/2 朗道问题具有矩阵表示形式的 Wigner 函数及其能级。

关 键 词：自旋 1/2 朗道问题；Wigner 函数；非对易相空间；均匀磁场

* **收稿日期：** 2011-01-10；**修改日期：** 2011-02-17

* **基金项目：** 半导体超晶格国家重点实验室开放研究课题(CHJG200902)；陕西省科学技术研究发展计划项目(2009K01-54)；陕西理工学院科研项目(SLGKY10-02)

1) E-mail：wangyahui8312469@163.com