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Effects of Tensor Force on Properties of Finite Nuclei*

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Abstract：The impact of the tensor force on the properties of finite nuclei is discussed by analyzing the spin-orbit splittings and the multipole giant resonances in nuclei. It is found that the tensor force do plays an important role in nuclear structure. The experimental isospin dependence of the spin-orbit splitting is very well depicted when the tensor force is included. The tensor force has a larger effect on the spin flip magnetic dipole states than on the natural parity isoscalar quadrupole (2^+) states. By analyzing the modifications to the Hartree-Fock mean field induced by the tensor terms，and the specific features of the residual particle-hole tensor interaction，we find that the tensor force gives an attractive contribution to the particle-hole matrix elements.

Key words：tensor force；spin-orbit splitting；multipole giant resonance

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1　Introduction

The known presence of a strong tensor force in the realistic nucleon-nucleon interaction demands that the effective nucleon-nucleon interaction shall also contain the tensor components[1]. More than 30 a ago，several authors have pointed out the important effect of the tensor part of the nucleon-nucleon effective interaction on the spin-orbit splitting of the Hartree-Fock (HF) single-particle spectrum[2-6]. In self-consistent mean field theory，such as the Skyrme HF mean field approach proposed by Skyrme in 1950's[7]，the effective zero-range nonlocal interaction already contained a zero-range tensor force，but the first applications of Skyrme interaction in self-consistent mean-field models neglected the tensor force. Stancu and Brink have studied the effects of tensor force in the splitting of spin-orbit of nuclei by adding the tensor force on a perturbatively way in 1970's[8]. Later most of the Skyrme forces are fitted without considering the contributions of tensor part except the work done by Liu et al.[9]. Recently，the authors of Refs.[10-11] have claimed that the tensor force plays crucial role for the evolution of shell structure in exotic nuclei based on shell model and self-consistent mean field calculations. This has illumined the study of the role of tensor force in various self-consistent mean field approaches. Tensor terms were added perturbatively into the existing standard parametrizations SIII[12] and SLy5[13] in Refs.[14-16]，respectively. And the inclusion of tensor terms in the Skyrem HF calculations achieved considerable success in explaining some features of the evolution of single-particle states in medium mass nuclei. Various new parameter sets have been fitted by considering the tensor terms on the same footing as other typical Skyrme parameters and have been applied

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to investigated the properties of finite nuclei not only in spherical but also in deformed nuclei[17−18]. There are also other papers to stress the important of tensor force[19−21]. Then, very recently, the effects of tensor force on the multipole giant resonances have been analyzed based on fully self-consistent random phase approximation[22−23]. Both the charge-exchange and non-charge-exchange strength distributions have been studied.

In this paper we will review the progress of the tensor force in recently years. This article is organized as follows: section 2 we present the contribution of central exchange and tensor parts to the single-particle evolution in $Z=50$ isotopes and $N=82$ isotones; the effects of tensor force on the multipole giant resonances are given in section 3; a summary is given in Section 4.

2　Evolution of Single-particle Energy in $Z=50$ Isotopes and $N=82$ Isotones

In this section we will discuss the contribution of the tensor force to the spin-orbit splitting of single-particle states in the Skyrme HF+BCS framework. Meanwhile, the central exchange term is also included because it plays the same role as the tensor term. The spherical approximation for the nucleus is assumed in the calculations.

The tensor force have triplet-even and triplet-odd zero-range terms, which read as

$$\begin{aligned} v_{\mathrm{T}} = \frac{T}{2}\Big\{ & \big[(\sigma_1\ k')(\sigma_2\ k') - \\ & \frac{1}{3}(\sigma_1\ \sigma_2)\ k'^2\big]\delta(r_1-r_2) + \delta(r_1-r_2)\times \\ & \big[(\sigma_1\ k)(\sigma_2\ k) - \frac{1}{3}(\sigma_1\ \sigma_2)\ k^2\big]\Big\} + \\ & U\ \Big\{(\sigma_1\ k')\ \delta(r_1-r_2)(\sigma_2\ k) - \\ & \frac{1}{3}(\sigma_1\ \sigma_2)\big[k'\,\delta(r_2-r_2)k\big]\Big\}\ , \end{aligned} \tag{1}$$

the operator $k=(\nabla_1-\nabla_2)/2i$ acts on the right and $k'=-(\nabla_1-\nabla_2)/2i$ acts on the left. The coupling constants T and U denote the strength of the triplet-even and triplet-odd tensor interactions, respectively. The tensor force and the central exchange term lead to the contribution to the energy density as

$$\begin{aligned} \Delta H(\boldsymbol{r}) = & \frac{1}{2}\alpha(J_{\mathrm{n}}^2(\boldsymbol{r}) + J_{\mathrm{p}}^2(\boldsymbol{r})) + \\ & \beta J_{\mathrm{n}}(\boldsymbol{r})\ J_{\mathrm{p}}(\boldsymbol{r})\ , \end{aligned} \tag{2}$$

where $J_{\mathrm{q}}(\boldsymbol{r})$ is the proton or neutron spin-orbit densities defined as

$$\begin{aligned} J_q(r) = & \frac{1}{4\pi r^3}\sum_i v_i{}^2(2j_i+1)\times \\ & \left[j_i(j_i+1) - l_i(l_i+1) - \frac{3}{4}\right]R_i{}^2(r)\ . \end{aligned} \tag{3}$$

In this expression $q=0(1)$ labels neutrons (protons), where $i=n,\ l,\ j$ runs over all states having the given q. The $v_i{}^2$ is the BCS occupation probability of each orbital and $R_i(r)$ is the radial part of the wavefunction.

In Eq. (2) $\alpha=\alpha_{\mathrm{C}}+\alpha_{\mathrm{T}}$ and $\beta=\beta_{\mathrm{C}}+\beta_{\mathrm{T}}$ are the parameters related to the central exchange and the tensor terms. The central exchange contributions are written in terms of the usual Skyrme parameters,

$$\begin{aligned} \alpha_{\mathrm{C}} &= \frac{1}{8}(t_1-t_2) - \frac{1}{8}(t_1x_1+t_2x_2)\ , \\ \beta_{\mathrm{C}} &= -\frac{1}{8}(t_1x_1+t_2x_2)\ . \end{aligned} \tag{4}$$

Basic definitions of all quantities derived from the Skyrme parameters can be found in Refs. [4, 7, 13]. The tensor contributions are expressed as

$$\alpha_{\mathrm{T}} = \frac{5}{12}U\ ,\qquad \beta_{\mathrm{T}} = \frac{5}{24}(T+U)\ . \tag{5}$$

The modified spin-orbit potential with tensor and central exhcnage term is given by

$$U_{\mathrm{s.o.}}^{(q)} = \frac{W_0}{2r}\left(2\frac{\mathrm{d}\rho_q}{\mathrm{d}r} + \frac{\mathrm{d}\rho_{q'}}{\mathrm{d}r}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right)\ , \tag{6}$$

where the first term on the r. h. s comes from the Skyrme spin-orbit interaction, the second term includes both the central exchange and the tensor contributions.

For the purpose of this paper we use the parameter set T41[17] which is fitted using the Saclay-Lyon fit protocol, the tensor parameters T and U are fitted on the same footing with the other typical Skyrme parameters. For T41, the parameters $\alpha_T = -180.649$ MeV fm^5 and $\beta_T = 94.037$ MeV fm^5, we should mention that for the force T41, $\alpha_C = 20.649$ MeV fm^5 and $\beta_C = 25.963$ MeV fm^5. For the pairing force, we simply use a constant pairing strength G, for $Z=50$ isotopes ($N=82$ isotones), the neutron (proton) pairing strength is fitted by reproducing the pairing gap in ^{120}Sn(^{146}Gd). Before present the results, there is one thing we want to stress, the sign of the J_q depends on the quantum numbers of the orbital: the orbital with $j_> = l + 1/2$ ($j_< = l - 1/2$) gives a positive(negative) contribution to J_q.

Fig. 1(a) shows the HF+BCS results for proton single particle energy difference between $h_{11/2}$ and $g_{7/2}$ in Sn isotopes with and without tensor force. In the case of T41 without tensor, the energy differences are much larger than the empirical data, it fails to reproduce the experimental trend. Whereas, we can see an substantial improvement due to the inclusion of the tensor force. The results given by HF+BCS with tensor can be qualitatively understood by simple arguments according to Eq. (6). For the protons, only the proton $g_{9/2}$ orbital dominates the proton spin density which gives a positive contribution, consequently, with a negative value of α_T, the spin-orbit potential is enlarged, so the proton spin-orbit splittings are increased, and the energy difference between $h_{11/2}$ and $g_{7/2}$ is reduced with respect to HF+BCS without tensor. This reduction is seen better around $N-Z=20$, here ^{120}Sn is spin-saturated in neutron case so that one gets no contribution from J_n. It is obvious that for a fixed proton number the term in α does not give any isospin dependence to the spin-orbit potential, but only the term in β can be responsible for the isospin dependence. In a pure HF description, from $N-Z=6$ to 14, the $g_{7/2}$ neutron orbit is gradually filled and J_n from this orbital is negative. Then, with the positive value of β_T, it enlarges in absolute value of the spin-orbit potential and increases the spin-orbit splitting, so that the energy difference between $h_{11/2}$ and $g_{7/2}$ becomes smaller. Moreover, from $N-Z=14$ to 20, the $s_{1/2}$ and $d_{3/2}$ neutron orbits are occupied and in this region the spin density is not so much changed since the $s_{1/2}$ orbital does not provide any contribution. For $N-Z=20$ to 32, the $h_{11/2}$ orbital is gradually filled. This gives a positive contribution to the spin-orbit potential and the spin-orbit splitting becomes smaller. $\varepsilon(h_{11/2}) - \varepsilon(g_{7/2})$ consequently increases, and this effect is well pronounced in our theoretical results.

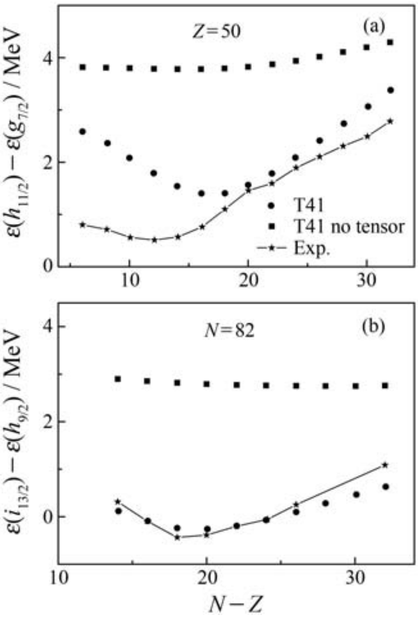

Fig. 1 Energy differences (a) between the $1h_{11/2}$ and $1g_{7/2}$ single-proton states in Sn isotopes and (b) between the $1i_{13/2}$ and $1h_{9/2}$ single-neutron states in $N=82$ isotones.

In Fig. 1(b), the neutron single particle energy difference between $i_{13/2}$ and $h_{9/2}$ in $N=82$ isotones is plotted as a function of the neutron excess. Certainly, the same arguments already made in the previous paragraph can be applied in order to understand the results. The proton $1g_{7/2}$ and $2d_{5/2}$ orbitals are gradually filled from right to left. These two proton orbitals have opposite effect on the spin orbit potential. Because of its larger value of j, the $1g_{7/2}$ orbital plays a more important role on the spin-orbit potential when the tensor interaction is included. Accordingly, with positive β_T the neutron spin-orbit potential is enlarged in absolute value: the spin-orbit splitting is made larger for these isotones. These changes make the energy gap between $i_{13/2}$ and $h_{9/2}$ smaller for the nuclei from $N-Z=32$ (^{132}Sn) to $N-Z=24$ (^{140}Ce). Then, the occupation of the $2d_{5/2}$ orbital reverses the trend around $N-Z=22$ (^{142}Nd). The theoretical trend remains the same until $N-Z=14$, since the effect of the $2d_{3/2}$ occupation is counterbalanced by the occupation of the $1h_{11/2}$ which is not much higher and enters the active BCS space.

3 Tensor Effects on the Multipole Giant Resonances

It is well known that the RPA theory is a very useful tool in describing the properties of giant resonances in finite nuclei. Inspired by existing studies on giant resonances, we try to investigate the effects of tensor force on the multipole giant resonance within self-consistent RPA approach. For the details of the calculations we refer the readers to the Ref. [23]. Here we take the non-spin flip isoscalar quadrupole and spin flip magnetic dipole (M1) giant resonances as an example to show the effects of tensor force. We shall use isoscalar quadrupole operator, that is,

$$\hat{F}_L = \sum_{i=1}^{A} \sqrt{2L+1}\, r_i^L\, Y_{\mathrm{LM}}(\hat{r}_i) \qquad (7)$$

with $L=2$. For spin-flip M1 states the following operators are used,

$$\hat{F}_J(\mathrm{IS}) = \sum_{i=1}^{A} \frac{g^{\mathrm{IS}} e\hbar}{2mc} \sqrt{J(2J+1)}\, \sigma_i{}^{\mu} Y_{00}(\hat{r}_i)\ , \quad (8)$$

in the isoscalar case, and

$$\hat{F}_J(\mathrm{IV}) = \sum_{i=1}^{A} \frac{g^{\mathrm{IV}} e\hbar}{2mc} \sqrt{J(2J+1)}\, \sigma_i{}^{\mu} Y_{00}(\hat{r}_i)\, \tau_i{}^{z} \qquad (9)$$

in the isovector case. In the above equations the nuclear magneton $\mu_{\mathrm{N}} = \frac{e\hbar}{2mc}$ appears together with the quantities $g^{\mathrm{IS}} = \frac{1}{4}(g_s^n + g_s^p - 1) = 0.19$, and $g^{\mathrm{IV}} = \frac{1}{4}(g_s^n - g_s^p + 1) = -2.10$.

First let us look at the effect of tensor force on the non-spin flip giant quadrupole resonance. For all the figures in the following paragraphs, full (or without tensor) means the calculations done by including (excluding) the tenor terms both in ground states and excited states, without tenor in RPA means we include tensor for ground states but drop it in the particle-hole residual interactions of RPA calculations. Fig. 2 displays in the upper (lower) panel the Hartree (IS RPA) response functions of giant quadrupole resonance in the nucleus ^{40}Ca. For the $\vec{l}\,\vec{s}$-closed nucleus and a natural-parity excitation spectrum, we do not expect that the tensor interaction plays any significant role. In fact, the microscopic calculations show that the results are exactly the same in the case of with and without the tensor force.

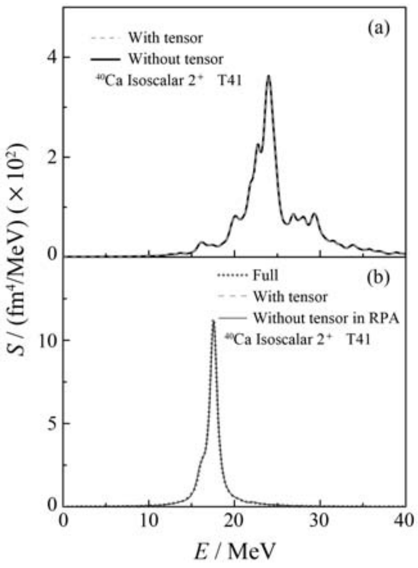

Fig. 2 Quadrupole unperturbed (a) and RPA (b) strength functions for ^{40}Ca given by T41. The discrete RPA peaks have been smeared out by using Lorentzian functions with $\Gamma=1$ MeV.

For a comparison, we have calculated the same IS response in the nucleus ^{208}Pb and show the results in Fig. 3. The response in the giant resonance region is not affected strongly by the inclusion of the tensor force, the main impact of the tensor terms is visible in the states below the isoscalar giant quadrupole resonance.

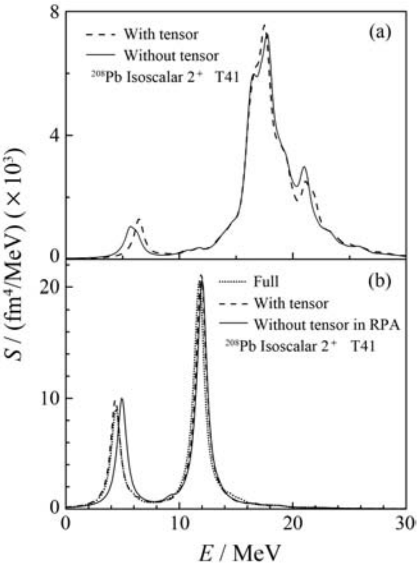

Fig. 3 The same as in Fig. 2 but for ^{208}Pb.

We have analyzed the changes of these properties induced by the tensor force. With the interaction T41, the spin-orbit splittings are increased compared to the results without tensor force. In general, at least for well-bound states, the spin-orbit splittings should increase more for larger values of l. Consequently, the low-lying 2^+ state in ^{208}Pb, which is mainly due to the neutron $i_{13/2} \rightarrow g_{9/2}$ and proton $h_{11/2} \rightarrow f_{7/2}$ as well as $h_{11/2} \rightarrow h_{9/2}$ transitions, is pushed upward in the unperturbed response by the tensor force, which can be seen clearly in the figure. We denote this shift by ΔE_{HF}. The effect of the particle-hole residual interaction produced by tensor force V_{tensor} included in RPA can be estimated by means of

$$\Delta E_{\mathrm{RPA}} \approx \Delta E_{\mathrm{HF}} + \langle V_{\mathrm{tensor}} \rangle , \tag{10}$$

where ΔE_{RPA} indicates the difference between the RPA result with and without the tensor force, and $\langle \ \rangle$ means that we extract here an average value of the residual force. With T41 parameter set, the value of ΔE_{HF} is 0.81 MeV and from the shift of the RPA peak ($\Delta E_{\mathrm{RPA}} = 0.05$ MeV) we extract $\langle V_{\mathrm{tensor}} \rangle = -0.76$ MeV. This means that the particle-hole residual interaction of tensor force is attractive, albeit not large, which can be understood using a separable approximation for the tensor interaction (see the appendix B in Ref. [23]). For the tensor-even and tensor-odd terms, the diagonal particle-hole matrix elements are proportion to $-T$ and U, respectively, for T41 parameter set, we get $T = 884.93$ MeV fm^5 and $U = -433.56$ MeV fm^5. It is shown that both the tensor-even and tensor-odd terms produce indeed attractive interaction in the present case.

Because the tensor force is spin-dependent, it can be expected that the effects of the tensor force are larger for spin (or for spin-isospin) states, both at the mean field level since unperturbed configurations are sensitive to the spin-orbit splittings, and also as far as RPA correlations are concerned. Many earlier works in the literature have emphasized the role of the tensor force in the spin and spin-isospin channel, but not in a self-consistent framework like in the present case or in the case of Ref. [22]. The self-consistent calculation of the uniform matter response performed using the Skyrme force T44 in Ref. [25], confirms that larger and non-trivial effects from the tensor terms may be expected in the spin channel. With this background, we have made calculations for the magnetic dipole response in ^{208}Pb. We make the analysis of the results for ^{208}Pb which are reported in Fig. 4 with the parameter set T41. In the unperturbed spectrum, the main contribution comes from the proton configuration $h_{11/2} \rightarrow h_{9/2}$ and neutron configuration $i_{13/2} \rightarrow i_{11/2}$. They are clearly visible in Fig. 4(a). In the case of the T41 interaction, the proton configuration lies at 5.71 MeV while the neutron configuration is at 8.46 MeV. As we know, in the case of the T41, the spin-orbit splittings are reduced when the tensor terms are turned off, which leads the energies is shifted downward, becomes 4.88 and 6.62 MeV, respectively, for the proton and neutron configurations we have mentioned.

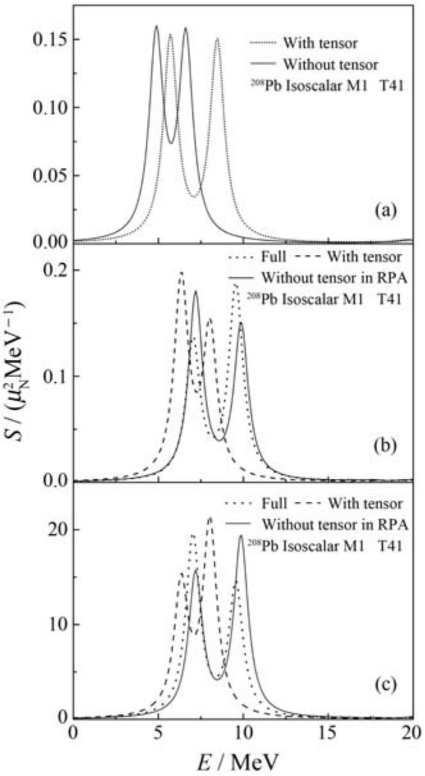

Fig. 4 M1 unperturbed(a) and RPA strength functions in ^{208}Pb associated with the isoscalar(b) and isovector(c) given by T41. The discrete RPA peaks have been smeared out by using Lorentzian functions with $\Gamma=1$ MeV.

For the RPA without the tensor terms in Fig. 4(b) and(c), the lowest and highest peaks are located at 6.35 and 8.04 MeV, respectively. Within RPA with tensor, we have found two peaks at 7.04 and 9.57 MeV. The lowest peak is mainly composed by the proton $h_{11/2}\rightarrow h_{9/2}$ configuration with an small admixture of the neutron $i_{13/2}\rightarrow i_{11/2}$ configuration having different sign in its amplitude; the highest peak is mainly based on the neutron $i_{13/2}\rightarrow i_{11/2}$ configuration, with some admixture of the proton $h_{11/2}\rightarrow h_{9/2}$ configuration, having the same sign in its amplitude. In other words, the lowest(highest) state has more IV(IS) character in present calculation. This isospin character is not strongly pronounced because, with the values of T and U that have been employed, the non-diagonal matrix element which mixes the proton and neutron configurations is small. Since experimentally one finds that the lowest(highest) state has more IS(IV) character, there is a doublet inversion which is probably related to the values of the Landau parameters. If we neglect the small mixing between the proton and neutron configurations, we can extract the values of the matrix elements of the residual tensor force from Eq. (10) separately for the proton and neutron states. We find, respectively, −0.14 and −0.32 MeV. Again the tensor force also gives an attractive contribution to the matrix elements in RPA.

4 Summary

In this review, we briefly discuss the necessity to include the tensor component in the Skyrme mean field theory. The first attempts in this direction were focusing on the effect of the tensor force on the isospin dependence of single-particle energies. Our results show that the introduction of the tensor force can fairly well explain the isospin dependence of energy differences between single-particle proton states in Sn isotopes, and neutron states in $N=82$ isotones within the parameter set we used. We have also done the analysis to disentangle the effects due to the modifications of the HF mean field, and those due to the particle-hole residual interaction. To do that, we calculate the isoscalar quadrupole and magnetic dipole giant resonance within self-consistent HF plus RPA approach. The dominant character(attractive or repulsive) of the residual interaction in the matrix elements has been extracted from the numerical calculations. The magnetic dipole states are more affected by the inclusion of the tensor terms at the mean field level, since unperturbed configurations correspond exactly to energy jumps between spin-orbit partners. Our work is exploratory and we have shown that the tensor force plays a role in RPA since the matrix elements are in general not negligible.

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张量力对原子核性质的影响*

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摘　要：最近一段时期，对张量力性质研究成为原子核结构研究的热点之一。基于自洽的 Skyrme 平均场理论讨论了张量力研究的最新进展。同时，讨论了张量力和 Skyrme 能量密度中的中心交换项对原子核单粒子态的演化以及多极巨共振的贡献。发现考虑张量力贡献后，利用 Skyrme 平均场计算可以基本描述 $Z=50$ 和 $N=82$ 原子核单粒子态演化的实验结果。而张量力对于原子核电多极巨共振基本没有影响，只对其低能集体态有一定的影响。张量力的引入使得原子核磁单极巨共振的能量和强度发生显著的改变。通过对数值结果的分析，发现张量力会产生吸引的粒子-空穴剩余相互作用。

关 键 词：张量力；自旋-轨道劈裂；多极巨共振

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