第 27 卷　第 3 期
2010 年 9 月

原子核物理评论
Nuclear Physics Review

Vol. 27, No. 3
Sep., 2010

Article ID: 1007－4627(2010)02－0274－06

Transverse Mass and Rapidity Distributions of Λ and $\bar{\Lambda}$ Hyperons Produced in Central Pb-Pb Collisions at High Energies*

XIE Wen-jie

(*Department of Physics*, *Yuncheng University*, *Yuncheng* 044000, *Shanxi*, *China*)

Abstract: The transverse mass and rapidity distributions of Lambda and Antilambda hyperons produced in central Pb-Pb collisions at 40, 80, and 158 *A*GeV are described by the multisource ideal gas model and the three-fireball model. The results calculated by the models are compared and found to be in agreement with the experimental data of the NA49 Collaboration.

Key words: transverse mass distribution; rapidity distribution; Pb-Pb collision; multisource ideal gas model; three-fireball model

CLC number: O571.6　　**Document code**: A

1 Introduction

Relativistic heavy-ion collisions are important subjects in theoretical and experimental nuclear physics because these collisions provide a unique approach to explore the new matter state. The quark matter predicted by the quantum chromodynamics (QCD) has been studied in high-energy heavy-ion interactions and some properties of nuclear reactions have been explained by the knowledge of current physics.

In relativistic heavy-ion collisions[1—3], many particles are produced and some of them are strange particles[4]. The strange particle production has been an important observable in high-energy heavy-ion reactions and its increase was one of the first suggested signatures for quark-gluon plasma (QGP).

In recent years, a lot of experimental data of heavy-ion collisions have been reported[5—10] and many theoretical models are suggested to explain these data[11—13]. Explanation of transverse mass spectra of particles produced in heavy-ion collisions turns out to be one of most difficult tasks[14].

In this paper we concentrate on the study of transverse mass and rapidity distributions of Lambda and Antilambda hyperons produced in Pb-Pb collisions at 40, 80, and 158 *A*GeV based on the multisource ideal gas model[15—18] and the three-fireball model[19—22]. We use the Monte Carlo method to calculate the transverse mass and rapidity distributions and compare our calculated results with the experimental data of the NA49 Collaboration[23].

2 Theoretical Models

In the investigation of transverse mass distributions, we will use the multisource ideal gas model. The multisource ideal gas model can be

---

* **Received date**: 8 Feb. 2010;　**Revised date**: 31 Mar. 2010

* **Foundation item**: National Natural Science Foundation of China(10975095); Natural Science Foundation of Shanxi Province (2007011005)

Biography: Xie Wen-jie(1980—), male(Han Nationality), Yuncheng, Shanxi, China, Master, working on the field of high energy physics; E-mail: wenjiexie@yeah.net

found in Refs. [15—18]. To give a whole presentation of the current work, we introduce shortly the model in the following.

Let the heavy-ion beam direction be oz axis and the reaction plane be xoz plane. A lot of emission sources of particles are supposed to be formed and we assume that the particles are isotropically emitted in the rest frame of emission source. According to the multisource ideal gas model[16—18], the three components p_x^*, p_y^* and p_z^* of particle momentum obey a Gaussian distribution and have the same standard deviation σ in the source rest system. Considering the expansions of the emission sources and the movement of the emission sources center, the final particle momentum components can be obtained by revising p_x^*, p_y^* and p_z^*. The relations between p_x and p_x^*, p_y and p_y^* and p_z and p_z^* are supposed to be linear. We have

$$p_x = a_x p_x^* + b_x \sigma , \tag{1}$$

$$p_y = a_y p_y^* + b_y \sigma , \tag{2}$$

and

$$p_z = a_z p_z^* + b_z \sigma , \tag{3}$$

where a_x, a_y, a_z, b_x, b_y and b_z are free parameters which can describe an expansion of the emission source and a movement of the emission source center in the ox, oy and oz directions respectively, and σ is the coefficient that stands for the width of momentum distribution in the source rest system. In the transverse space, the transverse momentum and transverse mass are defined by

$$p_{\mathrm{T}} = \sqrt{p_x^2 + p_y^2} \tag{4}$$

and

$$m_{\mathrm{T}} = \sqrt{p_{\mathrm{T}}^2 + m_0^2} \tag{5}$$

respectively, where m_0 is the rest mass of a particle.

Let R_1, R_2, R_3, R_4, R_5, and R_6 denote even random variables distributed in [0,1]. We have

$$p_x^* = \sqrt{-2\ln R_1}\ \cos(2\pi R_2)\sigma , \tag{6}$$

$$p_y^* = \sqrt{-2\ln R_3}\ \cos(2\pi R_4)\sigma , \tag{7}$$

and

$$p_z^* = \sqrt{-2\ln R_5}\ \cos(2\pi R_6)\sigma . \tag{8}$$

Considering Eqs. (1), (2) and (4) to (7), the transverse mass can be written as

$$m_{\mathrm{T}} = \sigma \sqrt{[a_x A + b_x]^2 + [a_y B + b_y]^2 + m_0^2} , \tag{9}$$

$$A = \sqrt{-2\ln R_1}\ \cos(2\pi R_2),$$

$$B = \sqrt{-2\ln R_3}\ \cos(2\pi R_4).$$

According to the three-fireball model[19—22], a projectile fireball (P^*), a target fireball (T^*) and a central fireball (C^*) are formed in high-energy nuclear-nuclear collisions. Each fireball can be regarded as a emission of particles, and particles in each fireball are assumed to emit isotropically. Three momentum components p_x^*, p_y^* and p_z^* of particles obey Gaussian distribution with the same standard deviation σ in the source rest system. In the Monte Carlo calculations, the p_x^*, p_y^* and p_z^* can be written as the Eqs. (6)—(8).

In the source rest frame, the rapidity of particle is defined by

$$y^* = \frac{1}{2}\left(\frac{\varepsilon + p_z}{\varepsilon - p_z}\right) , \tag{10}$$

where $\varepsilon = \sqrt{p_x^{*2} + p_y^{*2} + p_z^{*2} + m_0^2}$ is the energy of a particle.

In the laboratory system or the the center-of-mass frame system, the rapidity variable y is given by

$$y = y^* + y_x \quad , \tag{11}$$

where y_x indicates the rapidity of the projectile fireball, target fireball, or central fireball in the concerned reference frame. Let δy denote the rapidity shift of the projectile fireball and the target fireball and let Δy denote the rapidity shift of the leading projectile particle and leading target particle. The rapidity distribution is finally contributed by the projectile fireballs, the target fireballs, the central fireballs, leading projectile particles and leading target particles. In current work, we assume that the contributions of three fireballs (P^*,

T^*, and C^*) are equal to each other, and the contributions of leading projectile particles and leading target particles are equal to each other, too. Let k denote the contributions of leading projectile particles or leading target particles, then, the contribution of each fireball is $(1-2k)/3$. We can fit the experimental data to obtain the value of k. Considering the expressions (6) to (8), (10) and (11), we can give the value of rapidity.

In the above discussions, we can use Eqs. (9) and (11) to calculate the transverse mass and rapidity distributions, respectively. For transverse mass distributions of Lambda and Antilambda, we used multisource emission picture. For rapidity distributions of Lambda and Antilambda, we used three-source emission picture. In the multisource ideal gas model, we can use the parameters $a_{x,y}$ and $b_{x,y}$ to denote the transverse expansion of emission sources and transverse movement of center of emission sources, respectively. Therefore, it is convenient for us to calculate transverse distributions of particles produced in high-energy nucleus-nucleus collisions. But for the rapidity distribution, we do not need to consider the transverse expansion of emission sources. So, we can use the three-fireball model to describe the rapidity distributions of particles produced in nucleus-nucleus at high energies. The transverse mass distributions, $(1/m_T)\,d^2N/dm_Tdy$, and the rapidity distributions, dN/dy, are finally obtained by the statistic method, where N and y denote particle number and rapidity respectively.

We would like to point out that the Eqs. (1) to (3) are valid only in the non-relativistic limit. But in the emission source rest frame the relativistic effect is small. As a first approximation, Eqs. (1)—(3) are used in present work.

3 Comparison with Experimental Results

The transverse mass distributions, $(1/m_T)\,d^2N/dm_Tdy$, of Lambda and Antilambda hyperons produced in the central Pb-Pb collisions at 158 AGeV are shown in Fig. 1. The full circles denote the experimental data of NA49 Collaboration[23] and the curves indicate our calculated results by using Monte Carlo method. The parameter values obtained by fitting the experimental data are displayed in Table 1. In the selection of the parameter values, the χ^2-testing method is used and we can see that our calculated results are in agreement with the NA49 experimental data[23].

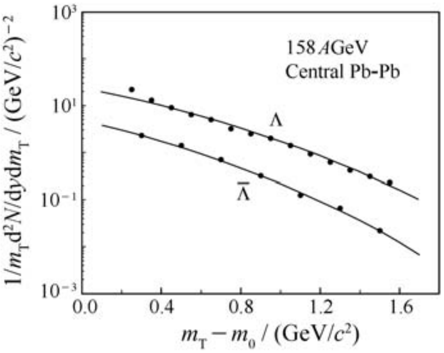

Fig. 1 The transverse mass distributions of Lambda and Antilambda hyperons produced in central Pb-Pb collisions. The circles represent the experimental data of NA49 Collaboration[23] and the curves denote our calculated results.

Table 1 Parameter values for curves in Figs. 1—3

Particle	E/AGeV	a_x	b_x	a_y	b_y	σ/(GeV/c)
Λ	40	1.30	−0.05	1.40	−0.05	0.51
	80	1.35	−0.05	1.40	−0.05	0.52
	158	1.40	−0.1	1.35	−0.1	0.57
$\bar{\Lambda}$	40	1.32	−0.05	1.40	−0.05	0.56
	80	1.32	−0.05	1.40	−0.05	0.56
	158	1.32	−0.05	1.40	−0.05	0.53

Fig. 2 presents the transverse mass distributions of Lambda and Antilambda hyperons produced in central Pb-Pb collisions at 80 AGeV. The circles are the experimental data of NA49 Collaboration[23] and the curves are the Monte Carlo calculated results. The parameter values obtained by fitting the experimental data are shown in Table 1 and the χ^2-testing method is used in the selection of parameter values. One can see that the multisource ideal gas model can describe the experimental data in high-energy heavy-ion collisions.

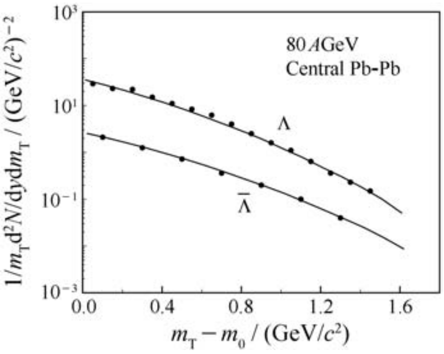

Fig. 2　As for Fig. 1, but displaying the results.

In Fig. 3, the transverse mass distributions, $(1/m_T)\,d^2N/dm_Tdy$, of Lambda and Antilambda hyperons produced in 40 *A*GeV central Pb-Pb collisions are given. The circles and curves represent the experimental data of NA49 Collaboration[23] and the calculated results by using Monte Carlo method based on the multisource ideal gas model[15—18], respectively. In our calculations, The parameter values obtained by fitting experimental data are shown in Table 1. The same conclusion from Fig. 3 as from Figs. 1 and 2 can be obtained. The calculated results from Eq. (9) are in agreement with the experimental data in high-energy Pb-Pb collisions.

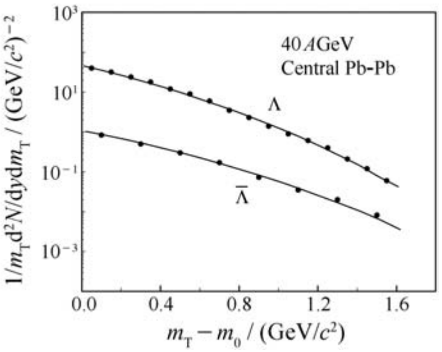

Fig. 3　As for Fig. 1, but showing the distributions.

The rapidity distributions of Λ and $\bar{\Lambda}$ hyperons produced in the central Pb-Pb collisions at 40(a), 80(b), 158 *A*GeV(c) are displayed in Figs. 4 and 5.

The circles denote the NA49 experimental data[23] and the curves represent our calculated results by the Monte Carlo method based on the three-fireball model[19—22]. In our calculations, we regard σ, k, Δy and δy as the free parameters and their values, obtained by fitting the experimental data, are showed in Table 2. In the selection of parameter values, the χ^2-testing method is used. To see the quality of our fits, we have calculated the values of χ^2/degree of freedom(dof) for all fits. In Fig. 4, the values of χ^2/dof are 0.025(a), 0.012(b), and 0.059(c) respectively. In Fig. 5, the values of χ^2/dof are 0.016(a), 0.076(b), and 0.063(c) respectively. One can see that our calculated results are in agreement with the experimental data in central Pb-Pb collisions at high energies.

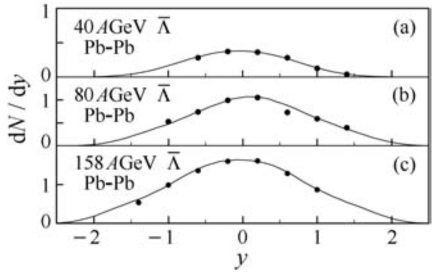

Fig. 4　The rapidity distributions of $\bar{\Lambda}$ hyperon produced in central Pb-Pb collisions at different energies. The circles denote the experimental data of NA49 Collaboration[23] and the curves indicate our calculated results.

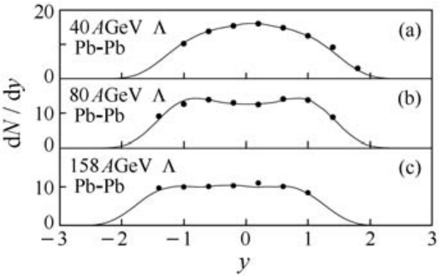

Fig. 5　As for Fig. 4, but showing the results of Λ hyperon.

Table 2　Parameter values for curves in Figs. 4—5

Particle	E/AGeV	k	Δy	δ_y	σ/(GeV/c)
Λ	40	0.20	1.05	0.48	0.52
	80	0.25	1.12	0.49	0.53
	158	0.26	1.13	0.50	0.58
$\bar{\Lambda}$	40	0.07	1.05	0.50	0.57
	80	0.12	1.30	0.52	0.58
	158	0.13	1.35	0.57	0.60

4 Conclusions and Discussions

In the multisource ideal gas model, there are three kinds of parameters: the expansion coefficient a_x, a_y, and a_z, the movement coefficient b_x, b_y and b_z and the excitation degree σ of the source. The parameters a_x, a_y, b_x and b_y reflect the transverse structure of the emission source in the momentum space. The condition $a_x(a_y)>1$ means that the emission source has expansions in $ox(oy)$ direction. The conditions $b_x(b_y)>0$ and $b_x(b_y)<0$ mean that the emission source has movements in the positive $ox(oy)$ and negative $ox(oy)$ directions, respectively. From Table 1, we can see that the emission sources formed in high-energy heavy-ion collisions have a relative strong expansion ($a_x>1$ and $a_y>1$) in the ox and oy axes, and the center of emission sources have a movement ($b_x<0$ and $b_y<0$) along the negative ox and negative oy directions. In our calculations, the allowed ranges of parameters a_x and a_y, b_x and b_y are estimated to be ±0.02 and ±0.03, respectively.

From Table 2, we can see that the values of k increase with the incident energy, that is, the higher the incident energy is, the larger the contributions of leading particles are. The rapidity shift (δy) of projectile fireball and the target fireball and the rapidity shift (Δy) of leading particles increase with the incident energy, too. We can imagine, when the incident energy is lower, the three fireballs are close, and when the incident energy is higher, the projectile fireball and the target fireball is far from the center fireball. When the incident energy is very high, the particles of projectile and target often stay in fragmentation region. This leads to the larger leading particle effect.

In fact, we can regard the multisource ideal gas model as the expansion of the three-fireball model. The multisource ideal gas model are successful in describing the transverse momentum[24, 25] and the angular distributions[26—29], and the three-fireball model are successful in describing the (pseudo) rapidity and transverse momentum distributions[22]. The present work shows that the multisource ideal gas model describes the transverse mass distributions of the particles produced in high-energy heavy-ion collisions.

The multisource ideal gas model is a reasonable picture in high-energy heavy-ion collisions. A recent work[30] shows that in the previous work of Liu et al., however, the multisource ideal gas model has not considered the relativistic and/or quantum effects in many cases. We hope to improve the model by considering the relativistic and quantum effects in our future work.

Acknowledgment The author would like to thank Prof. Dr. Liu Fuhu for useful discussions.

References:

[1] Bertsch G, Amsden A A. Phys Rev, 1978, **C18**: 1293.

[2] Quack E, Henning P A. Phys Rev, 1996, **D54**: 3125.

[3] Back B B, Baker M D, Barton D S, *et al*. Phys Rev Lett, 2002, **89**: 222301.

[4] Rafelski J, Müller B. Phys Rev Lett, 1982, **48**: 1066; Koch P, Müller B, Rafelski J. Phys Rep, 1986, **142**: 167.

[5] Partlan M D, Albergo S, Bieser F, *et al*. (EOS Collaboration). Phys Rev Lett, 1995, **75**: 2100.

[6] Hong B, Herrmann N, Ritman J L, *et al*. (FOPI Collaboration). Phys Rev, 1998, **C57**: 244.

[7] Barrette J, Bellwied R, Bennett S, *et al*. (E877 Collaboration). Phys Rev, 2000, **C63**: 014902.

[8] Klay J L, Ajitanand N N, Alexander J M, *et al*. (E895 Collaboration). Phys Rev, 2003, **C68**: 054905.

[9] Anticic T, Baatar B, Bama D, *et al*. (NA49 Collaboration). Phys Rev, 2009, **C79**: 044904.

[10] Aggarwal M M, Ahammed Z, Angelis A L S, *et al*. (WA98 Collaboration). Phys Rev Lett, 2004, **93**: 022301.

[11] Gustafson G. Nucl Phys, 1994, **A566**: 233.

[12] Werner K. Nucl Phys, 1994, **A566**: 477.

[13] Wang X N. Phys Rev, 1991, **D43**: 104.

[14] Wagner M, Larionov A B, Mosel U. Phys Rev, 2005, **C71**: 034910.

[15] Liu Fuhu. Nuclear Physics Review, 1998, **15**(1): 21(in Chinese).

(刘福虎. 原子核物理评论, 1998, **15**(1): 21.)

[16] Liu F H. Europhys Lett, 2003, **63**: 193.

[17] Liu F H, Abd Allah N N, Singh B K. Phys Rev, 2004, **C69**:

057601.

[18] Liu F H，Li J S，Duan M Y. Phys Rev，2007，**C75**：054613.

[19] Liu L S，Meng T C. Phys Rev，1983，**D27**：2640.

[20] Chou K C，Liu L S，Meng T C. Phys Rev，1983，**D28**：1080.

[21] Liu Fuhu，Sun Hancheng. HEP & NP，1994，**18**：1073(in Chinese).
(刘福虎,孙汉城. 高能物理与核物理，1994，**18**：1073.)

[22] Liu F H. Can J Phys，2002，**80**：525.

[23] Anticic T，Baatar B，Barna D，*et al*.(NA49 Collaboration). Phys Rev Lett，2004，**93**：022302.

[24] Liu F H. Chin J Phys，2001，**39**：163.

[25] Liu F H. Chin J Phys，2003，**41**：357.

[26] Liu F H，Abd Allah N N，Singh B K，*et al*. Acta Physica Polonica，2006，**B37**：2695.

[27] Liu F H，Abd Allah N N，Zhang D H，*et al*. Chin J Phys，2004，**42**：152.

[28] Liu F H. Chin J Phys，2001，**39**：248.

[29] Liu F H，Ma Y Q，Duan M Y. Chin Phys，2005，**14**：2458.

[30] Meng C R. Chin Phys Lett，2009，**26**：102501.

高能 Pb-Pb 中心碰撞中产生的 Λ 和 $\bar{\Lambda}$ 强子的横质量分布和快度分布*

谢文杰[1]

（运城学院物理与电子工程系，山西 运城 044000）

摘　要：用多源理想气体模型和三火球模型分析并计算了能量在 40，80，158 *A*GeV 下中心 Pb-Pb 碰撞中所产生的 Λ 和 $\bar{\Lambda}$ 强子的横质量分布和快度分布，发现模型计算的结果与 NA49 合作组的实验结果相一致。

关 键 词：横质量；快度；Pb-Pb 碰撞；多源理想气体模型；三火球模型

---

* **收稿日期**：2010 - 02 - 08；**修改日期**：2010 - 03 - 31

* **基金项目**：国家自然科学基金资助项目(10975095)；山西省自然科学基金资助项目(2007011005)

1) E-mail：wenjiexie@yeah.net