第 26 卷 增刊 原 子 核 物 理 评 论 Vol. 26, Suppl.
2009 年 7 月 Nuclear Physics Review Jul., 2009

Article ID: 1007-4627(2009)Suppl.-0129-04

QCD Sum Rule Study of $\pi_1 s$ States*

CHEN Hua-xing[1,2], A. Hosaka[2], ZHU Shi-lin[1]

(1 *Department of Physics*, *Peking University*, *Beijing* 100871, *China*;
2 *Research Center for Nuclear Physics*, *Osaka University*, *Ibaraki* 567-0047, *Japan*)

Abstract: We study the states $\pi_1(1600)$ and $\pi_1(2015)$ in the QCD sum rule. We classify the tetraquark currents of the quantum numbers $I^G J^{PC}=1^- 1^{-+}$, and find that the flavor structure $(\bar{3}\otimes\bar{6})\oplus(6\otimes 3)$ is preferred when using a diquark-antidiquark construction. By using both the SVZ and finite energy sum rules, we obtain a mass around 1.6 and 2.0 GeV, for the states with the quark contents $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$, respectively. We also study their decay patterns.

Key words: exotic mesons; tetraquark; QCD sum rule

CLC number: O572.33+9 **Document code**: A

1 Introduction

Since 1976, Jaffe has suggested to use the four-quark system which contains two strongly correlated diquarks to explain the low-lying scalar mesons ($\sigma(600)$, $\kappa(800)$, $a_0(980)$ and $f_0(980)$)[1]. Besides these states, the meson states $\pi_1 s$ with exotic quantum numbers $I^G J^{PC}=1^- 1^{-+}$ are also tetraquark candidates[2]. There are altogether three $\pi_1 s$ experimentally. They are $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$. Their masses and widths are (1376 ± 17, 300 ± 40) MeV, (1653^{18}_{-15}, 225^{+45}_{-28}) MeV, and ($2014\pm20\pm16$, $230\pm21\pm73$) MeV, respectively[3,4]. They are interpreted as hybrid states in many papers[5]. Tetraquark states can also carry these exotic quantum numbers. In this proceeding, we will use the tetraquark structure to explain $\pi_1(1600)$ and $\pi_1(2015)$ by using the method of QCD sum rule, which has proven to be a very powerful and successful non-perturbative method[6]. The detailed analysis can be found in Ref. [7].

In order to classify the flavor structure of four-quark system with quantum numbers $J^{PC}=1^{-+}$, we start with the consideration of the charge-conjugation symmetry. The charge-conjugation transformation changes diquarks into antidiquarks, while it maintains their flavor structures. Therefore, the internal diquark (qq) and antidiquark ($\bar{q}\bar{q}$) can have the symmetric flavor structure $6_f\otimes\bar{6}_f$ (S), the antisymmetric flavor structure $\bar{3}_f\otimes 3_f$ (A), and the combination of $\bar{3}_f\otimes\bar{6}_f$ and $6_f\otimes 3_f$:

$$
\begin{aligned}
&qq\bar{q}\bar{q}\,(S),\ qs\bar{q}\bar{s}(S)\sim 6_f\otimes\bar{6}_f\,(S),\\
&qs\bar{q}\bar{s}(A)\sim\bar{3}_f\otimes 3_f\,(A),\\
&qq\bar{q}\bar{q}(M),\ qs\bar{q}\bar{s}\,(M),\\
&\sim(\bar{3}_f\otimes\bar{6}_f)\oplus(6_f\otimes 3_f)\,(M),
\end{aligned}\tag{1}
$$

where q represents an up or down quark, and s represents a strange quark. The flavor structures are shown in Fig. 1 in terms of $SU(3)$ weight diagrams. The ideal mixing scheme is used since it is expected to work well for hadrons except for the pseudoscalar mesons.

* **Received date**: 29 Aug. 2008; **Revised date**: 2 Sep. 2008
* **Foundation item**: National Natural Science Foundation of China(10625521, 10721063); Scientific Research((C) 19540297) from the Ministry of Education, Culture, Science and Technology, Japan
Biography: Chen Hua-xing(1982-), male(Han Nationality), Anshun, Guizhou, China, PhD Student, working on the field of high energy physics; E-mail: hxchen@rcnp.osaka-u.ac.jp

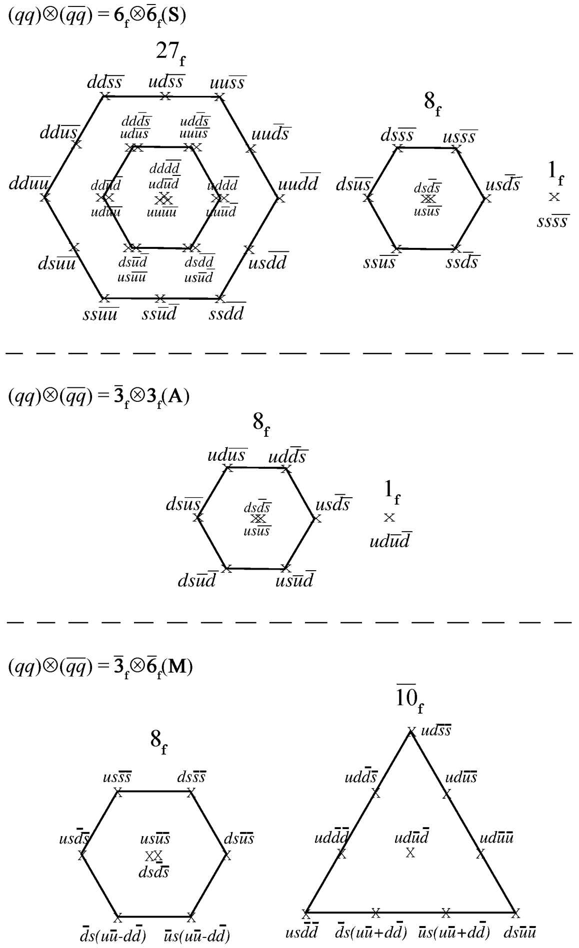

Fig. 1 Weight diagrams for $6_f\otimes\bar{6}_f$ (S) (top panel), $\bar{3}_f\otimes 3_f$ (A) (middle panel), $\bar{3}_f\otimes\bar{6}_f$ (M) (bottom panel). Weight diagram for $6_f\otimes 3_f$ (M) is the charge-conjugation transformation of the bottom one.

For each state, there are several independent currents. We list them in the following. For the two isospin triplets of $6_f\otimes\bar{6}_f$ (S):

$$
\begin{aligned}
\eta^S_{1\mu} \sim\ & u_a^T C\gamma_5 d_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma_\mu\gamma_5 C\bar{d}_a^T) + \\
& u_a^T C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma_5 C\bar{d}_a^T),\\
\eta^S_{2\mu} \sim\ & u_a^T C\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu} C\bar{d}_b^T - \bar{u}_b\sigma_{\mu\nu} C\bar{d}_a^T) + \\
& u_a^T C\sigma_{\mu\nu} d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T - \bar{u}_b\gamma^\nu C\bar{d}_a^T),\\
\eta^S_{3\mu} \sim\ & u_a^T C\gamma_5 s_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}_b^T + \bar{u}_b\gamma_\mu\gamma_5 C\bar{s}_a^T) + \\
& u_a^T C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma_5 C\bar{s}_b^T + \bar{u}_b\gamma_5 C\bar{s}_a^T),\\
\eta^S_{4\mu} \sim\ & u_a^T C\gamma^\nu s_b(\bar{u}_a\sigma_{\mu\nu} C\bar{s}_b^T - \bar{u}_b\sigma_{\mu\nu} C\bar{s}_a^T) + \\
& u_a^T C\sigma_{\mu\nu} s_b(\bar{u}_a\gamma^\nu C\bar{s}_b^T - \bar{u}_b\gamma^\nu C\bar{s}_a^T).
\end{aligned}
\qquad (2)
$$

where the sum over repeated indices (μ, ν, $\cdots$ for Dirac spinor indices, and a, b, $\cdots$ for color indices) is taken. C is the charge-conjugation matrix, T represents the matrix transposition. $\eta^S_{1\mu}$ and $\eta^S_{2\mu}$ are the two independent currents containing only light flavors, and $\eta^S_{3\mu}$ and $\eta^S_{4\mu}$ are the two independent ones containing one $s\bar{s}$ pair. We use "～" to make clear that the quark contents here are not exactly correct. However, the QCD sum rule results are the same and so correct.

For the isospin triplet of $\bar{3}_f\otimes 3_f$ (A):

$$
\begin{aligned}
\eta^A_{1\mu} \sim\ & u_a^T C\gamma_5 s_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}_b^T - \bar{u}_b\gamma_\mu\gamma_5 C\bar{s}_a^T) + \\
& u_a^T C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma_5 C\bar{s}_b^T - \bar{u}_b\gamma_5 C\bar{s}_a^T),\\
\eta^A_{2\mu} \sim\ & u_a^T C\gamma^\nu s_b(\bar{u}_a\sigma_{\mu\nu} C\bar{s}_b^T + \bar{u}_b\sigma_{\mu\nu} C\bar{s}_a^T) + \\
& u_a^T C\sigma_{\mu\nu} s_b(\bar{u}_a\gamma^\nu C\bar{s}_b^T + \bar{u}_b\gamma^\nu C\bar{s}_a^T),
\end{aligned}
\qquad (3)
$$

where $\eta^A_{1\mu}$ and $\eta^A_{2\mu}$ are the two independent currents.

For the two isospin triplets of $(\bar{3}_f\otimes\bar{6}_f)\oplus(6_f\otimes 3_f)$(M):

$$
\begin{aligned}
\eta^M_{1\mu} \sim\ & u_a^T C\gamma_\mu d_b(\bar{u}_a C\bar{d}_b^T + \bar{u}_b C\bar{d}_a^T) + \\
& u_a^T C d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T + \bar{u}_b\gamma_\mu C\bar{d}_a^T),\\
\eta^M_{2\mu} \sim\ & u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar{u}_a\gamma^\nu\gamma_5 C\bar{d}_b^T + \\
& \bar{u}_b\gamma^\nu\gamma_5 C\bar{d}_a^T) + \\
& u_a^T C\gamma^\nu\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{d}_b^T + \\
& \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{d}_a^T),\\
\eta^M_{3\mu} \sim\ & u_a^T C d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T - \bar{u}_b\gamma_\mu C\bar{d}_a^T) + \\
& u_a^T C\gamma_\mu d_b(\bar{u}_a C\bar{d}_b^T - \bar{u}_b C\bar{d}_a^T),\\
\eta^M_{4\mu} \sim\ & u_a^T C\gamma^\nu\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{d}_b^T - \\
& \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{d}_a^T) + \\
& u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar{u}_a\gamma^\nu\gamma_5 C\bar{d}_b^T - \\
& \bar{u}_b\gamma^\nu\gamma_5 C\bar{d}_a^T),\\
\eta^M_{5\mu} \sim\ & u_a^T C\gamma_\mu s_b(\bar{u}_a C\bar{s}_b^T + \bar{u}_b C\bar{s}_a^T) + \\
& u_a^T C s_b(\bar{u}_a\gamma_\mu C\bar{s}_b^T + \bar{u}_b\gamma_\mu C\bar{s}_a^T),\\
\eta^M_{6\mu} \sim\ & u_a^T C\sigma_{\mu\nu}\gamma_5 s_b(\bar{u}_a\gamma^\nu\gamma_5 C\bar{s}_b^T + \\
& \bar{u}_b\gamma^\nu\gamma_5 C\bar{s}_a^T) + \\
& u_a^T C\gamma^\nu\gamma_5 s_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{s}_b^T + \\
& \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{s}_a^T),\\
\eta^M_{7\mu} \sim\ & u_a^T C s_b(\bar{u}_a\gamma_\mu C\bar{s}_b^T - \bar{u}_b\gamma_\mu C\bar{s}_a^T) + \\
& u_a^T C\gamma_\mu s_b(\bar{u}_a C\bar{s}_b^T - \bar{u}_b C\bar{s}_a^T),\\
\eta^M_{8\mu} \sim\ & u_a^T C\gamma^\nu\gamma_5 s_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{s}_b^T - \\
& \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{s}_a^T) +
\end{aligned}
$$

$$u_a^T C \sigma_{\mu\nu} \gamma_5 s_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma^\nu \gamma_5 C \bar{s}_a^T) , \quad (4)$$

where $\eta^{M}_{1,2,3,4}$ are the four independent currents containing only light flavors, and $\eta^{M}_{1,2,3,4}$ are the four independent ones containing one $s\bar{s}$ pair.

We have performed the OPE calculation up to dimension 12. The full OPE expressions are too lengthy and are omitted here. In our numerical analysis, we use the following values for various condensates and m_s at 1 GeV and α_s at 1.7 GeV[3,8]:

$$\begin{aligned}
&\langle \bar{q}q \rangle = -(0.240\ \text{GeV})^3 ,\\
&\langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.240\ \text{GeV})^3 ,\\
&\langle g_s^2 GG \rangle = (0.48 \pm 0.14)\ \text{GeV}^4 ,\\
&\langle g_s \bar{q}\sigma Gq \rangle = -M_0^2 \times \langle \bar{q}q \rangle ,\\
&M_0^2 = (0.8 \pm 0.2)\ \text{GeV}^2 ,\\
&m_s(1\ \text{GeV}) = 125 \pm 20\ \text{MeV} ,\\
&\alpha_s(1.7\ \text{GeV}) = 0.328 \pm 0.03 \pm 0.025 .
\end{aligned} \quad (5)$$

There is a minus sign in the definition of the mixed condensate $\langle g_s \bar{q}\sigma Gq \rangle$, which is different from that used in some other QCD sum rule studies. This difference just comes from the definition of coupling constant g_s[9].

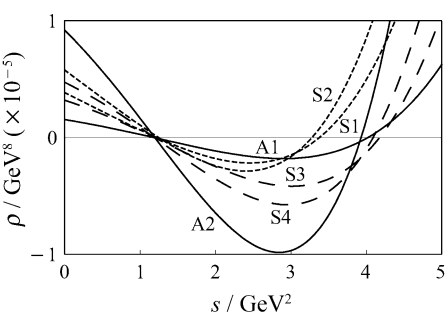

Fig. 2 Spectral densities for the current $\eta^{A}_{1\mu}$, $\eta^{A}_{2\mu}$ (—), $\eta^{S}_{1\mu}$, $\eta^{S}_{2\mu}$ (---), $\eta^{S}_{3\mu}$ and $\eta^{S}_{4\mu}$ (— — —). The labels besides the lines indicate the flavor symmetry (S or A) and suffix i of the current $\eta^{S,A}_{i\mu}$ (i=1,2,3,4).

There are altogether five possibilities: $qq\bar{q}\bar{q}$ (S), $qs\bar{q}\bar{s}$ (S), $qs\bar{q}\bar{s}$ (A), $qq\bar{q}\bar{q}$ (M), and $qs\bar{q}\bar{s}$ (M). However, we find that the spectral densities are negative in the energy region 1—2 GeV for the currents belonging to the flavor representations $6_f \otimes \bar{6}_f$ (S) and $\bar{3}_f \otimes 3_f$ (A). They are shown in Fig. 2. Therefore, the flavor structures $6_f \otimes \bar{6}_f$ and $\bar{3}_f \otimes 3_f$ are not preferred. Then there are two possibilities left: $qq\bar{q}\bar{q}$(M), and $qs\bar{q}\bar{s}$(M), which belong to the mixed flavor structure $(\bar{3}_f \otimes \bar{6}_f) \oplus (6_f \otimes 3_f)$. Their spectral densities are positive as shown in Fig. 3. The convergence of the OPE is very good in the region of 2 GeV^2 < M_B^2 < 5 GeV^2. Therefore, we will use these currents to perform the QCD sum rule analysis.

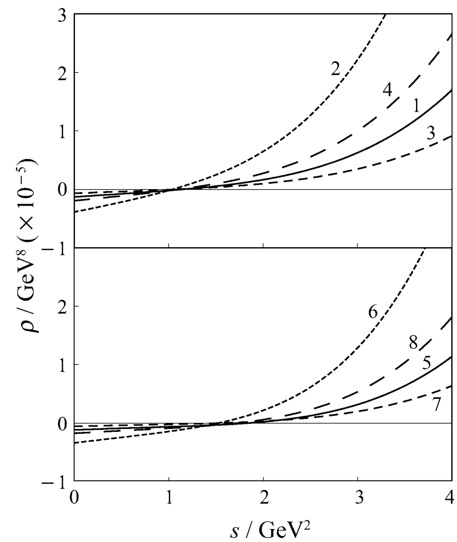

Fig. 3 Spectral densities for the current $\eta^{M}_{i\mu}$. The spectral densities for the currents with the quark contents $qq\bar{q}\bar{q}$ are shown on the top panel, and those with the quark contents $qs\bar{q}\bar{s}$ are shown on the bottom panel. The labels besides the lines indicate the suffix i of the current $\eta^{M}_{i\mu}$ (i=1,…,8).

The mass is obtained as functions of Borel mass M_B and threshold value s_0. As an example, we show the mass calculated from currents $\eta^{M}_{2\mu}$ and $\eta^{M}_{6\mu}$ in Figs. 4 and 5, respectively. The first one has quark contents $qq\bar{q}\bar{q}$; while the last one has quark contents $qs\bar{q}\bar{s}$. From figures on the top panel, we find that the dependence on Borel mass is weak. In figures on the bottom panel, the mass is shown as functions of s_0, we find that there is a mass minimum, which is around 1.6 and 2.0 GeV for $\eta^{M}_{2\mu}$ and $\eta^{M}_{6\mu}$, respectively. Although there are four independent currents, we find that they give a similar result. Therefore, our QCD sum rule analysis leads to a mass around 1.6 and 2.0 GeV for $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$, respectively. For comparison, we have also used the method of finite energy sum rule, and arrive at the same results.

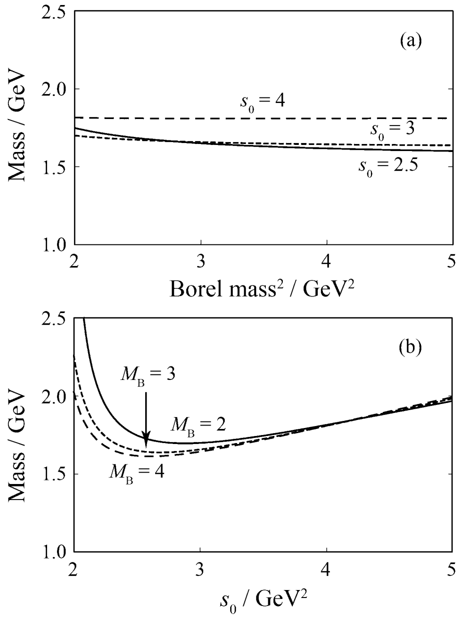

Fig. 4 The mass of the state $qq\bar{q}\bar{q}$ calculated by using the current $\eta^{M}_{2\mu}$, as functions of M_B^2 (a) and s_0 (b) in units of GeV.

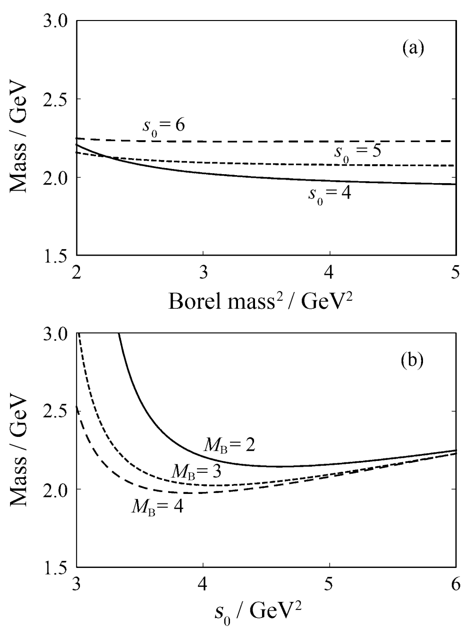

Fig. 5 The mass of the state $qs\bar{q}\bar{s}$ calculated by using the current $\eta^{M}_{6\mu}$, as functions of $M_B{}^2$ (a) and s_0 (b) in units of GeV.

We have also studied their decay patterns and found that the possible final states are S-wave b_1 (1235)π as well as P-wave modes $\pi\eta$ and $\pi\eta'$, etc.

References:

[1] Jaffe R L. Phys Rev, 1977, D15: 267; Jaffe R L. Phys Rev, 1977, D15: 281.

[2] Klempt E, Zaitsev A. Phys Rept, 2007, 454: 1.

[3] Yao W M, Amsler C, Asner D, *et al*. J Phys, 2006, G33: 1.

[4] Adams G S, Adams T, Bar-Yam Z, *et al*. Phys Lett, 2007, B657: 27; Abele A, Adomeit J, Amsler C, *et al*. Phys Lett, 1999, B446: 349; Thompson D R, Adams G S, Adams T, *et al*. Phys Rev Lett, 1997, 79: 1 630; Ivanov E I, Stienike D L, Ryabchikov D I, *et al*. Phys Rev Lett, 2001, 86: 3 977; Lu M, Adams G S, Adams T, *et al*. Phys Rev Lett, 2005, 94: 032 002; Nozar M, Salgado C, Weygand D P, *et al*. arXiv: hep-ex/08054438.

[5] Hedditch J N, Kamleh W, Lasscock B G, *et al*. Phys Rev, 2005, D72: 114 507; Bernard C, Burch T, Gregory E B, *et al*. Phys Rev, 2003, D68: 074 505; Iddir F, Semlala L. arXiv: hep-ph/07105352; Chetyrkin K G, Narison S. Phys Lett, 2000, B485: 145; Yang K C. Phys Rev, 2007, D76: 094 001.

[6] Shifman M A, Vainshtein A I, Zakharov V I. Nucl Phys, 1979, B147: 385; Reinders L J, Rubinstein H, Yazaki S. Phys Rept, 1985, 127: 1.

[7] Chen H X, Hosaka A, Zhu S L. arXiv: hep-ph/08061998.

[8] Narison S. Camb Monogr Part Phys Nucl Phys Cosmol, 2002, 17: 1; Gimenez V, Lubicz V, Mescia F, *et al*. Eur Phys J, 2005, C41: 535; Jamin M. Phys Lett, 2002, B538: 71; Ioffe B L, Zyablyuk K N. Eur Phys J, 2003, C27: 229; Ovchinnikov A A, Pivovarov A A. Sov J Nucl Phys, 1988, 48: 721; [Yad Fig2, 1988, 48: 1135].

[9] Yang K C, Hwang W Y P, Henley E M, *et al*. Phys Rev, 1993, D47: 3001; Hwang W Y P, Yang K C. Phys Rev, 1994, D49: 460.