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介子束缚态相对论本征方程的非微扰重整化*

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摘　要： 介子结合态本征方程中 δ 相互作用可用 T 矩阵进行非微扰重整化，深入理解重整化的一些基本问题：物理结果与重正化点的选取无关，T 矩阵非微扰重整化的物理实质.

关 键 词： 介子结合态本征方程；T 矩阵非微扰重整化；本征波函数

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基于光锥 QCD 和迭代的预解式投射算子方法，得到无味混合的介子质量谱的等效本征方程：

$$\begin{aligned}&[M^2+(E_1(k)+E_2(k))^2]\varphi_{s_1s_2}(\boldsymbol{k})\\&=\sum_{s_1s_2}\int\mathrm{d}^3\boldsymbol{k}'U_{s_1s_2,s_1's_2'}(\boldsymbol{k},\boldsymbol{k}')\varphi_{s'_1s'_2}(\boldsymbol{k}')\ ,\\&E_i(k)=\sqrt{m_i+\boldsymbol{k}^2}\ .\end{aligned}\tag{1}$$

等效势 $U_{s_1s_2,s_1's_2'}(\boldsymbol{k},\boldsymbol{k}')$ 包含需要重整化的发散项. (1)式的紧奏形式可写为

$$\hat{H}\Psi_n=M_n^2\Psi_n\ ,\tag{2a}$$

$$\hat{H}=\hat{H}_0\hat{V}_\delta\ .\tag{2b}$$

$\hat{V}_\delta$为 δ 型相互作用：$\hat{V}_\delta(\boldsymbol{r})=\hat{V}_0\delta(\boldsymbol{r})$.
Green 函数 G 与 T 矩阵的关系：$\hat{H}$ 的 Green 函数 G 可用 $\hat{H}_0$ 的 Green 函数 G_0 表示：

$$G=\frac{1}{M^2-\hat{H}}=G_0+G_0\hat{V}_\delta G\ ,\tag{3a}$$

$$\hat{V}_\delta G=TG_0\ .\tag{3b}$$

T 矩阵满足，

$$T=\hat{V}_\delta+\hat{V}_\delta G_0T\ ,\tag{4a}$$

G 与 T 矩阵的关系

$$T=\hat{V}_\delta GG_0^{-1}.\tag{4b}$$

$\hat{H}_0$的本征解为

$$\hat{H}_0\mid n\rangle=M_{0n}^2\mid n\rangle\ ,\tag{5}$$

$\hat{H}$ 的本征解 Ψ_n 可展开为

$$\Psi_n=\sum_n C_{mn}\mid m\rangle\ ,\tag{6}$$

算得

$$G=\sum_n\frac{\mid\Psi_n\rangle\langle\Psi_n\mid}{M^2-M_n^2}\ ,\tag{7}$$

$$T=\sum_n\frac{(M_n^2-\hat{H}_0)\mid\Psi_n\rangle\langle\Psi_n\mid(M^2-\hat{H}_0)}{M^2-M_n^2}\ ,\tag{8}$$

$$\langle n\mid T\mid n\rangle=\sum_n\frac{(M_n^2-M_0^2)C_{nn}C_{nn}{}^*(M^2-M_0^2)}{M^2-M_n^2}\ ,\tag{9}$$

Green 函数 G 的性质：G 的极点位置 $M^2=M_n^2$ 给出系统 $\hat{H}$ 的全部本征能量 M_n^2，其留数函数给出相应的本征波函数$|\Psi_n\rangle$. 即 Green 函数包含了量子系统的全部信息.

$\langle n|T|n\rangle$在基态能量 $M^2=M_0^2$ 这一极点处的留数为

$$(M_0^2-M_{0n}^2)C_{0n}C_{0n}{}^*(M_0^2-M_{0n}^2)\ .\tag{10}$$

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按照 T 矩阵非微扰重整化，在用基态能量的实验值对 $\bar{V}_\delta$ 相互作用造成的 T 矩阵的发散进行重整化后，T 矩阵的对角元为

$$\langle n \mid T \mid n \rangle = \varphi_n^*(0) t \varphi_n(0) \ ,$$
$$\varphi_n(\boldsymbol{r}) = \langle \boldsymbol{r} \mid n \rangle \ , \tag{11}$$

$$\frac{1}{t} = \sum_n \varphi_n(0)^* \varphi_n(0) \left[\frac{1}{M^2 - M_{0n}^2} - \frac{1}{M_0^2 - M_{0n}^2} \right]$$
$$= (2\pi)^3 (M_0^2 - M^2) \cdot$$
$$\sum_n \frac{\varphi_n(0)^* \varphi_n(0)}{(M^2 - M_{0n}^2)(M_0^2 - M_{0n}^2)} \ , \tag{12}$$

$$t = \frac{1}{(2\pi)^3 \sum\limits_n \dfrac{\varphi_n(0)^* \varphi_n(0)}{(M^2 - M_{0n}^2)(M_0^2 - M_{0n}^2)}} \cdot$$
$$\frac{1}{(M_0^2 - M^2)} \ . \tag{13}$$

上述 T 矩阵重整化的(12)式能给出基态能量以外的所有能级，但不能给出波函数，这是下面要解决的问题. $\langle n|T|n\rangle$在基态能量极点处的留数为

$$\frac{\varphi_n(0)^* \varphi_n(0)}{(2\pi)^3 \sum\limits_n \dfrac{\varphi_n(0)^* \varphi_n(0)}{(M_0^2 - M_{0n}^2)^2}} \tag{14}$$

比较两个留数(10)和(14)式得归一化的展开系数：

$$C_{0n} = \frac{\varphi_n(0)}{\left[\sum\limits_n \dfrac{\varphi_n(0)^* \varphi_n(0)}{(M_0^2 - M_{0n}^2)^2}\right]^{1/2} (M_0^2 - M_{0n}^2)} \ . \tag{15}$$

$$\Psi_0 = \sum_m C_{0m} \mid \varphi_m \rangle \ , \tag{16}$$

一般，可用 a 能级的实验值对 T 矩阵重整化，用 $1/t$ 的零点

$$\frac{1}{t} = (2\pi)^3 (M_a^2 - M^2) \cdot$$
$$\sum_n \frac{\varphi_n(0)^* \varphi_n(0)}{(M^2 - M_{0n}^2)(M_a^2 - M_{0n}^2)} = 0 \tag{17}$$

确定 b 能级的能量 M_b^2. a 能级和 b 能级的波函数(展开系数)为

$$C_{an} \frac{\varphi_n(0)}{\left[\sum\limits_n \dfrac{\varphi_n(0)^* \varphi_n(0)}{(M_a^2 - M_{0n}^2)^2}\right]^{1/2} (M_a^2 - M_{0n}^2)} \ . \tag{18}$$

$$C_{bn} \frac{\varphi_n(0)}{\left[\sum\limits_n \dfrac{\varphi_n(0)^* \varphi_n(0)}{(M_b^2 - M_{0n}^2)^2}\right]^{1/2} (M_b^2 - M_{0n}^2)} \ . \tag{19}$$

确定 b 能级的能量 M_b^2 的方程(18)变为

$$\frac{1}{t} = (2\pi)^3 (M_a^2 - M_b^2) \cdot$$
$$\sum_n \frac{\varphi_n(0)^* \varphi_n(0)}{(M_b^2 - M_{0n}^2)(M_a^2 - M_{0n}^2)}$$
$$\propto (M_a^2 - M_b^2) \langle \Psi_a \mid \Psi_b \rangle$$
$$= (M_a^2 - M_b^2) \delta_{ab} = 0 \qquad (a \neq b) \ . \tag{20}$$

这正是本征波函数的正交性条件!!!

小结

(1) T 矩阵重整化的物理内容与重整化点选择无关：(12)和(17)式表明，用于重正化的能级和被决定的能级是相对的，任何能级都可用作重整化(的能级)点，重整化后的 T 矩阵所决定的能级都是一样的，与重整化点的选择无关.

(2) T 矩阵重整化的实质：用某一能级的实验值去消除 T 矩阵的发散，用本征波函的正交性去确定其他能级.

参　考　文　献：

[1] Pauli H C. Compendium of Light-Cone Quantization. Nucl Phys (Proc Suppl), 2000, **B90**: 259 -272.

[2] Pauli H C. Internal Report: Spin-orbit on the Light Cone, 2003.

[3] Frederico T, Pauli H C, Zhou S G. Phys Rev, 2002, **D66**: 054007.

[4] Frederico T, Pauli H C, Zhou S G. Phys Rev, 2002, **D66**: 116011.

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Study of High Spin States in Odd-odd Nucleus ^{90}Nb*

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Abstract: The high spin states of ^{90}Nb have been populated via reaction ^{76}Ge(^{19}F, 5n)^{90}Nb at beam energy of 80 MeV. The de-exciting γ-rays have been measured with in-beam γ-ray spectroscopy method. After γ-γ coincidence analysis, 19 new γ transitions were identified and assigned to ^{90}Nb. The new level scheme of ^{90}Nb was established. Based on the semi-empirical shell model calculations, the configurations of the levels have been suggested. In addition, the spins and parities of the new levels have been assigned according to the experimental DCO values and to the systematic comparison with the $N=49$ neighboring nuclei.

Key words: high spin state; in-beam γ-ray spectroscopy; nuclear shell model; configuration

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Nonperturbative Renormalization of Relativistic Eigen Equation for Meson Mass Spectra**

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Abstract: Nonperturbative T-matrix renormalization of the relativistic eigen equation for meson mass spectra is described and the expressions for eigen mass spectra and eigen wave functions are given.

Key words: relativistic eigen equation for meson mass spectra; nonperturbative T-matrix renormalization; eigen wave function

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