

# Proton- $^4\text{He}$ Elastic Scattering in Jastrow Description of Nuclear Structure\*

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**Abstract:** Based on the Jastrow description of nuclear structure, theoretical analysis of the experimental cross sections for small-angle elastic  $p\text{-}^4\text{He}$  scattering at the energy about 1 GeV has been performed in the framework of Glauber multiple scattering theory. Our theoretical calculations reproduce the corresponding data successfully. This agreement confirms that nuclear short range correlation seems to be important and sets up a theoretical basis for calculating nuclear halo-like phenomena which may originate from the short range correlation between nucleons in a halo-like nucleus.

**Key words:** Jastrow wave function; short range correlation; Glauber multiple scattering theory

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## 1 Introduction

Light neutron-rich nuclei located at the neutron drip line, in particular  $^6\text{He}$ ,  $^8\text{He}$ ,  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ ,  $^{17}\text{B}$  and  $^{19}\text{C}$ , have attracted much attention in recent years, because those nuclei reveal a qualitatively new type of nuclear structure, namely an extended low-density distribution of valence neutrons surrounding a compact nuclear core, so-called halo nucleus initiated in the mid-eighties by the pioneering work of Tanihata and Coworkers in Ref. [1] who observed a surprisingly steep rise of the total interaction cross sections of several nuclei in their isotope sequences. The steep rise was interpreted as being due to exceptionally large matter radii in the halo nuclei. Later on the matter radii of a series of neutron-rich nuclei were evaluated from the data on interaction cross sections.

In order to get a deeper insight into the structure of neutron-rich nuclei, the halo-like phenomenon was the subject of numerous studies during the last decade<sup>[2-8]</sup>. Various experimental methods were applied, such as beta-decay measurements following in-beam polarization by optical pumping, and the investigation of momentum distributions of the reaction products after fragmentation of the halo-like nuclei impinging on a nuclear target. The picture of the nuclear halo-like structure established after the observation of the large interaction cross sections was qualitatively confirmed. Halo-like nuclei are assumed to be composed of a nuclear core and a number (one, two or four) of valence neutrons<sup>[9-12]</sup>. Therefore, in order to attain more detail information upon the radial shape of the ha-

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lo-like nuclei and their unclear matter radii, the method of proton elastic scattering at intermediate energies, has recently been applied<sup>[13]</sup>.

For stable isotopes, probing the radial structure of the nuclear matter by exploiting the differential cross sections for proton elastic scattering at intermediate energies ( $\approx 1$  GeV) is a well proved method<sup>[14]</sup>. In this range of proton energy, elastic scattering from nuclei is described accurately by the diffractive multiple-scattering theory<sup>[15]</sup>, and related to this fact, the measured differential cross sections reflect the nuclear matter distributions of the target nuclei rather unambiguously. In an analogous manner, the structure of short-lived radioactive nuclei can be studied via proton elastic scattering experiments in inverse kinematics, using projectile nuclei from the radioactive beam facilities and hydrogen for the proton target. At present, intensities of available radioactive ion beams are generally inferior to those of the proton beams. Since the differential elastic scattering cross section  $d\sigma/dt$  decreases rapidly with the momentum transfer increasing, it is quite tedious in the case of radioactive beams to collect data in inverse kinematics over a wide range of the four-momentum transfers squared  $t$  with good statistical accuracy as is usually achieved for proton scattering from stable target nuclei. However, as recent calculations have demonstrated<sup>[16-18]</sup>, high precision data on absolute differential cross sections for proton elastic scattering in the range of small momentum transfers, where it is easier to attain good data statistics, are likewise suited for a reliable assessment of the nuclear matter radii, and for investigation of the radial distribution of the nuclear matter. It should also be stressed that the region of small momentum transfers is especially important for studying the spatial structure of halo-like nuclei since the contribution to the cross section from the scattering on the halo-nucleus is concentrated just at small scattering angles, i. e., at small momentum transfers.

In this work we want to set up a reliable basis

for studying halo-like phenomena of  ${}^6, {}^8\text{He}$  in near future, by studying  $p\text{-}{}^4\text{He}$  elastic scattering at the energy around 1 GeV. Since in the recent experiment<sup>[19]</sup>, the absolute differential cross sections  $d\sigma/dt$  for small-angle  $p\text{-}{}^4\text{He}$ ,  $p\text{-}{}^8\text{He}$ , and  $p\text{-}{}^6\text{He}$  elastic scattering have been measured with rather small statistical errors and an accuracy of  $\pm 3\%$  in the absolute normalization. The present paper is subdivided as follows. In section 2, the Glauber formalism for calculating the differential cross section of elastic scattering of the intermediate energy protons from composite nuclei is briefly recalled. In section 3, our numerical calculations of differential cross section for proton elastic scattering off the nucleus  ${}^4\text{He}$  are presented. Finally, section 4 is reserved for summary and concluding remarks.

## 2 Glauber Formalism of Hadron-nucleus Elastic Scattering at High Energies

We start by recalling the formalism of Glauber multiple scattering theory. For hadron-nucleus scattering with Coulomb effect taken into account, the relevant scattering amplitude is given by

$$F_n(q) = F_C(q)\delta_n + H_{cm}(q) \frac{ik}{2\pi} \int e^{i\phi} \Gamma_n^c(b) d^2b, \quad (1)$$

where  $F_C(q)$  is the Coulomb scattering amplitude for a point charge target and is given by Ref. [20]

$$F_C(q) = -2\xi \frac{k}{q^2} e^{i\phi_c}, \quad (2)$$

with

$$\phi_c = -2\xi \ln \frac{q}{2k} + 2\eta, \quad (3)$$

$$\xi = -Z\alpha \frac{m}{k}, \quad (4)$$

here  $Z$  is charge number of the target nucleus,  $m$  is the nucleon mass,  $\alpha=1/137$  and  $k$  denotes the initial wave number in the center of mass system. The quantity  $\eta$  is defined by

$$\eta = \arg\Gamma(1+i\xi) = \xi\psi(x=1) +$$

$$\sum_{n=0}^{+\infty} \left( 1 + \frac{\xi}{1+n} - \arctan \frac{\xi}{1+n} \right), \quad (5)$$

where  $\psi(x)$  is the digamma function and  $\psi(x=1) = -0.57721$ . The  $\Gamma(1+i\xi)$  in Eq. (5) is Euler's Gamma function. The notation  $\arg$  stands for the principal argument of a complex number  $\Gamma(1+i\xi)$ .

The  $H_{cm}(q) = \exp[q^2/4A\alpha']$  is the center-of-mass correction factor<sup>[18]</sup> with  $\alpha'$  being harmonic oscillator parameter of the target nucleus.

The Coulomb corrected profile function  $\Gamma_n^C(b)$  is determined by

$$\Gamma_n^C(b) = e^{ix_c(b)} \delta_n - e^{\tilde{x}_c(b)} \delta_n - \Gamma_n(b), \quad (6)$$

where  $x_c(b)$  and  $\tilde{x}_c(b)$  are Coulomb phase shift of a point-like charge and an extended charge distribution, respectively. They are given by

$$x_c(b) = 2\xi \ln(kb), \quad (7)$$

$$\tilde{x}_c(b) = x_c(b) + \bar{x}_c(b), \quad (8)$$

with

$$\bar{x}_c(b) = 2\xi 4\pi b^3 \int_0^1 dx \frac{1}{x^4} \rho_c \frac{b}{x} \cdot \left[ \ln \left( \frac{1+\sqrt{1-x^2}}{x} \right) - \sqrt{x-x^2} \right], \quad (9)$$

where  $\rho_c(b/x)$  is the charge distribution of the target nucleus. For instance, three parameter Fermi distribution

$$\rho_c(r) = \rho_0 \frac{1 + wr^2/c^2}{1 + e^{(r-d)/Z}}, \quad (10)$$

with parameters  $c=0.964$ ,  $Z=0.323$ ,  $w=0.517$  for <sup>4</sup>He nucleus are given by Refs. [20, 21].

Given  $\rho_c(r)$ , the  $\bar{x}_c(b)$  can be easily obtained by carrying out the integration in Eq. (9) numerically.

The  $\Gamma_n(b)$  in Eq. (6) is nuclear profile function given by

$$\Gamma_n(b) = \langle \psi_f | 1 - \prod_{j=1}^A [1 - \Gamma_j(b-s_j)] | \psi_i \rangle, \quad (11)$$

where  $\psi_f$  and  $\psi_i$  are the final and initial nuclear wave function, respectively. The  $\Gamma_j(b-s_j)$  is the two-dimensional Fourier transform of elementary two

body scattering amplitude  $f_j(q)$

$$\Gamma_j(b-s_j) = \frac{1}{2\pi i k} \int e^{-iq(b-s_j)} f_j(q) d^2q. \quad (12)$$

the  $f_j(q)$  is of spin-isospin dependence, and can be written as<sup>[22]</sup>

$$f_j(q) = f_A(q) + f_B(q)(\sigma_p \cdot n)(\sigma_t \cdot n) + f_C(q)(\sigma_p + \sigma_t) \cdot n + f_D(q)(\sigma_p \cdot m)(\sigma_t \cdot m) + f_E(q)(\sigma_p \cdot l)(\sigma_t \cdot l), \quad (13)$$

where  $\sigma_p$  and  $\sigma_t$  are spin operators of the projectile (p) and target nucleon (t), respectively. We define

$$n = \frac{k_i \times k_f}{|k_i \times k_f|} = q \times k_i, \quad (14)$$

$$q = k_i - k_f, \quad (15)$$

$k_i$  and  $k_f$  are the initial and final momentum of projectile particle,

$$m = \frac{k_i - k_f}{|k_i - k_f|} = q, \quad (16)$$

$$l = \frac{k_i + k_f}{|k_i + k_f|}. \quad (17)$$

It is obvious that  $n$ ,  $m$  and  $l$  are unit vectors. Of course, the  $f_l(q)$  are also operators in isospin space

$$f_l(q) = f_l^s(q) + f_l^v(q)\tau_p \cdot \tau_t, \quad (18)$$

with  $l=A, B, C, D, E$ . The  $\tau_p$  and  $\tau_t$  are isospin operators of projectile and target nucleon, respectively. The  $f^s(q)$  is isoscalar part of amplitude  $f_l(q)$ , while  $f^v(q)$  is the isovector part of  $f_l(q)$ . In Eq. (13) the first term  $f_A(q)$  denotes no-spin-flip amplitude, the term containing  $f_C(q)$  is spin-flip amplitude, while the terms containing  $f_B(q)$ ,  $f_D(q)$  and  $f_E(q)$  are double spin-flip amplitudes.

Inspecting Eq. (11) and Eq. (12) show that there are only two ingredients entering Glauber amplitude. One is nuclear structure represented by the initial and final nuclear wave functions,  $\psi_i(f)$ , and another is two-body elementary interaction indicated by  $f_j(q)$ .

For our present purpose, we firstly study the

effect of nuclear structure on differential cross section using the above formalism, and neglecting the spin-flip parts of two-body amplitude in Eq. (12). We also parameterize the central part  $f_A(q)$  in the form

$$f_A(q) = \frac{ik\sigma}{4\pi}(1 - i\rho)e^{-\beta^2 q^2/2}, \quad (19)$$

where  $\sigma$ ,  $\rho$  and  $\beta$  are respectively total cross section, ratio of the real-to-imaginary part of forward amplitude and slope parameter. They are of energy dependence, and are determined by experimental data.

Speaking of nuclear wave function, we use Jastrow wave function<sup>[20]</sup>

$$\psi_{JC} = \prod_{i>j=1}^A f(r_{ij}) \psi_{SD}(r_1 \cdots r_A) \quad (20)$$

to describe the nuclear short range correlation. The  $\psi_{SD}(r_1 \cdots r_A)$  in Eq. (20) is Slater determinant of target nucleus wave function which takes the Pauli effect into consideration. The  $f(r_{ij})$  represents two-body short-range correlation and satisfies

$$f(r_{ij}) = \begin{cases} 0, & r_{ij} \leq h \\ 1, & r_{ij} > h \end{cases} \quad (21)$$

where  $r_{ij}$  is internucleon separation and  $h$  is referred to the so-called recover distance.

It has been proven that the  $\psi_{JC}$  could be written as a new modified Slater determinant<sup>[20]</sup>

$$\psi_{JC} = \frac{1}{\sqrt{A!}} \|\tilde{\phi}_a\|, \quad (22)$$

with

$$\begin{aligned} \tilde{\phi}_a(r_1) = & \phi_a(r_1) \{1 - \sum_{\alpha \neq \beta} \langle \beta(2) | g(r_{12}) | \beta(2) \rangle + \\ & \sum_{\alpha \neq \beta \neq \gamma} \langle \beta(2) \gamma(3) | g(r_{12}) g(r_{13}) + \\ & \cdots | \beta(2) \gamma(3) \rangle + \cdots\}, \end{aligned} \quad (23)$$

where  $g(r_{ij})$  is the so-called "assistant factor", and

$$g(r_{ij}) = 1 - f(r_{ij}). \quad (24)$$

As usual, we take  $g(r) = J_0(q_c r)$  with  $q_c$  being correlation parameter and is taken to be 300 MeV/c in this calculation. For light nuclei,

$$\tilde{\phi}_{1s}(r) = A_{1s}\phi_{1s}(r) + A_{2s}\phi_{2s}(r) +$$

$$A_{3s}\phi_{3s}(r) + \cdots, \quad (25)$$

$$\begin{aligned} \tilde{\phi}_{1p}(r) = & A_{1p}\phi_{1p}(r) + A_{2p}\phi_{2p}(r) + \\ & A_{3p}\phi_{3p}(r) + \cdots. \end{aligned} \quad (26)$$

The Eqs. (25—26) show that the short range correlation is just an effect caused an admixture of different state with identical spin-orbital part ( $l$ ) but different principal quantum number ( $N$ ). To a good approximation, Eq. (23) can be approximated as

$$\tilde{\phi}_{1s}(r) = A_{1s}\phi_{1s}(r) + A_{2s}\phi_{2s}(r), \quad (27)$$

$$\tilde{\phi}_{1p}(r) = A_{1p}\phi_{1p}(r) + A_{2p}\phi_{2p}(r), \quad (28)$$

with  $A_{1s} = 0.9624$ ,  $A_{2s} = 0.2719$ ,  $A_{1p} = 0.9431$  and  $A_{2p} = 0.3324$ <sup>[20]</sup>.

Using the Jastrow wave function, the nuclear matrix elements in the Glauber amplitude, Eq. (11) can be expressed<sup>[18, 20]</sup> as

$$\begin{aligned} \langle \psi_{JC} | 1 - \prod_{j=1}^A [1 - \Gamma_j(\mathbf{b} - \mathbf{s}_j)] | \psi_{JC} \rangle \\ = 1 - \|\tilde{O}_{nm}\|, \end{aligned} \quad (29)$$

where

$$\tilde{O}_{nm} = \delta_{nm} - \int \tilde{\phi}_m^*(\mathbf{r}) \Gamma(\mathbf{b} - \mathbf{s}) \tilde{\phi}_n(\mathbf{r}) d^3(\mathbf{r}). \quad (30)$$

### 3 Numerical Calculations of Differential Cross Section

Our theoretical predictions by use of Jastrow wave function for p-<sup>4</sup>He elastic scattering at the incident energy of 1 GeV is shown in Fig. 1.

Inspecting the curve in Fig. 1 clearly indicates that comparing to double Gaussian nuclear density the Jastrow wave function plays an important role in improving the theoretical fit to experimental data. Namely the nuclear short range correlation effect reproduces the data successfully. This excellent fit will be possible to offer us a reliable basis for discussing the halo-like phenomena of nuclear structure of <sup>6</sup>He and <sup>8</sup>He.

For this calculation, sufficiently accurate values of  $\sigma_{pN}$ ,  $\rho_{pN}$  and  $\beta_{pN}$  are needed as input data. In the present calculations, the values of  $\sigma_{pp}$  and  $\rho_{pp}$

have been deduced by interpolation of the results from a free pp scattering phase shift analysis<sup>[23]</sup>. For the case of pn scattering, the available data on the elementary cross sections are more scarce and even partly inconsistent. For determining the required values of  $\sigma_{pn}$  at the energies of the experiment, the cross section data given in Ref. [24] have been chosen. The ratios  $\rho_{pN}$  have been deduced by interpolation from the results of Ref. [25].  $\sigma_{pp}=43.5$  mb,  $\rho_{pp}=0.095$ ,  $\rho_{pn}=-0.297$ .

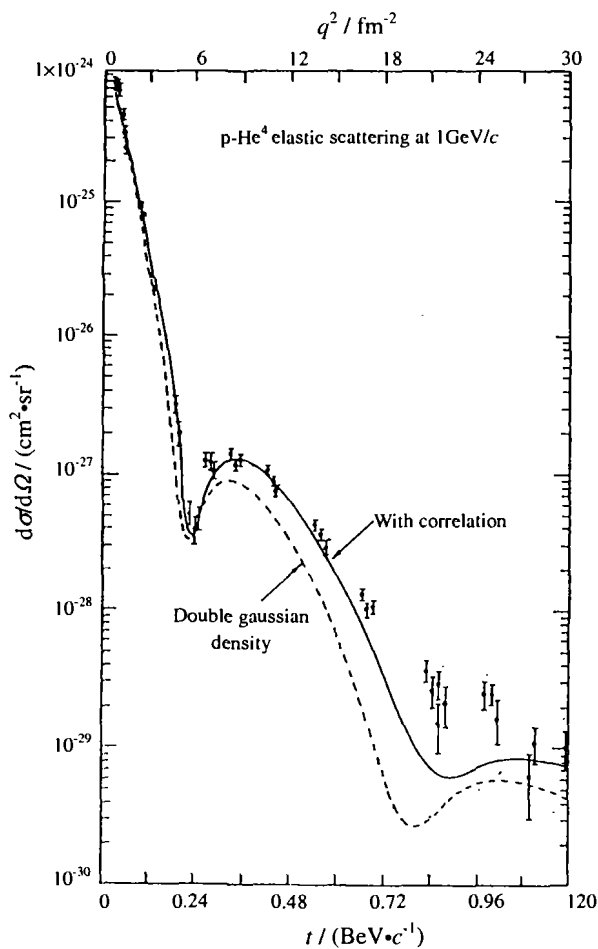


Fig. 1  $p\text{-}^4\text{He}$  elastic scattering at 1 GeV. The --- is the result given by a double Gaussian function of nuclear density. The — is given by Jastrow wave function in Eq. (20).

The slope parameters  $\beta_{pN}$  obtained in Ref. [23] for the scalar amplitudes and the  $\beta_{pN}$ -values determined directly from the experimental pN differential cross sections turn out to be slightly different since the latter describe the  $t$ -dependence of the

cross sections at forward angles including the spin-orbit and spin-spin amplitudes. In the present calculations of the  $p\text{-}^4\text{He}$  cross sections, the spin terms have been ignored. No difference has been made between  $\beta_{pp}$  and  $\beta_{pn}$ , choosing these parameters for  $^4\text{He}$  under study to be  $\beta_{pp}=\beta_{pn}=0.17$  fm<sup>2</sup>. This particular value gives the best description of the  $p\text{-}^4\text{He}$  differential cross section, with the  $^4\text{He}$  matter radius  $R_m=1.49$  fm, which corresponds to the experimentally determined  $^4\text{He}$  charge radius of 1.67 fm. Also, this choice for  $\beta_{pN}$  is consistent with the data of Ref. [26].

#### 4 Summary and Concluding Remarks

In the present work, an analysis of the differential cross sections for about 1.0 GeV small angle proton elastic scattering off the  $^4\text{He}$  has been performed. The cross section has been computed using the basic formalism of the Glauber multiple scattering theory. No simplifying approximations have been adopted in the calculations except an approximated treatment of the center-of-mass correction. The parameter of the elementary pN scattering amplitudes required in the calculations have been carefully choosing on the basis of the available free pN scattering data. The measured  $p\text{-}^4\text{He}$  cross section serves as checking the calculations and also for determining the slope parameter  $\beta_{pN}$  in the pN scattering amplitudes to be used.

The experimental data on  $^4\text{He}$  is well described with the Jastrow wave function which takes the nuclear short range correlation into account. Therefore, we conclude that nuclear short range correlation is quite important for fitting to data.

In conclusion, this work has demonstrated that small angle proton elastic scattering at intermediate energy is an efficient tool for studying the structure of light exotic nuclei. The data on proton elastic scattering can provide an accurate determination of the size and radial shape of light nuclei.

## References.

- [1] Tanihata I, Hamagaki H, Hashimoto D, *et al.* Phys Rev Lett, 1985, **55**: 2 676.
- [2] Riisager K. Rev Mod Phys, 1994, **66**: 1 105.
- [3] Hansen P G. Annu Rev Nucl Part Sci, 1995, **45**: 591.
- [4] Tanihata I. Prog Part Nucl Phys, 1995, **35**: 505.
- [5] Jonson B, Riisager K. Philos Trans R Soc, 1998, **A358**: 2 063.
- [6] Sherrill B M. Nucl Phys, 2001, **A685**: 134c.
- [7] Arnold E, Bonn J, Gegenwart, *et al.* Phys Lett, 1987, **B197**: 311; Phys Lett, 1992, **B281**: 16.
- [8] Kobayashi T, Yamakawa O, Omata K, *et al.* Phys Rev Lett, 1988, **60**: 2 599.
- [9] Hanson P G, Josen B. Eur Ohys Lett, 1987, **4**: 409.
- [10] Hanson P G. Nucl Phys, 1993, **A553**: 89c.
- [11] Tanihata I. Nucl Phys, 1990, **A520**: 411c.
- [12] Al-khalili J S, Tostevin J A, Thompson I J, *et al.* Phys Rev, 1996, **C54**: 1 843.
- [13] Neumaier S R. In: Proc Int Conf On Exctic Nuclei abd Atomic Masses, ENAM, Arles, France Jane 19—23. Editions, Frontiers, 1995, 227.
- [14] Alkhazov G D. Phys Rept, 1978, **C42**: 189.
- [15] Glauber R J. Lectures in Theoretical Physics. New york, Interscience, 1959, **1**: 315, In: Britten W E, Danham L G, ed.
- [16] Alkhazcv G D, Lobodenko A A. JETP Lett, 1992, **55**: 379.
- [17] Alkhazcv G D. Phys At Anclei, 1993, **56**: 337.
- [18] Hu Zhao-hui, Zhou Li-juan, Ma Wei-xing. HEP and NP, 2003, **27**(1): 36.
- [19] Alkhazcv G D, Dobrovolsky A V, Egelhof P, *et al.* Nucl Phys, 2002, **A712**: 269.
- [20] Ma Wei-xing, Strottman D D. Phys Rev, 1991, **D44**: 615.
- [21] Dalkarov O D, Karmaanov V A. Nucl Phys, 1985, **A445**: 579.
- [22] Alkhazov G D, Belostotsky S L, Vorobyov A A. Phys Rept, 1978, **C42**: 89.
- [23] Arndt R A, Ropa L D, Bryan R A, *et al.* Phys Rev, 1983, **D28**: 97.
- [24] Devlin T J, Johnson W, Noren J, *et al.* Phys Rev, 1973, **D8**: 136.
- [25] Grein W, Kroll P. Nucl Phys, 1982, **A377**: 505.
- [26] Silverman B H, Lugol J G, Saudinos J, *et al.* Nucl Phys, 1989, **A499**: 763.

在原子核结构的 Jastrow 描述中质子与<sup>4</sup>He 的弹性散射\*武 青<sup>1</sup>, 周丽娟<sup>2</sup>, 马维兴<sup>2, 3</sup>

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**摘 要:** 基于原子核结构的 Jastrow 描述和 Glauber 多重散射理论, 我们对 1 GeV 的质子与<sup>4</sup>He 的小角度弹性散射微分截面的实验数据进行了理论分析. 理论计算的结果成功地解说了实验观测的数据. 表明: 原子核中核子间的短程关联是重要的, 它可能是原子核晕核结构现象的动力学起源, 为理论上研究原子核的晕现象打下了基础.

**关键词:** 原子核结构的 Jastrow 波函数; 短程关联; Glauber 多重散射理论