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Low-lying Spectra and E2 Transition Rates in Even-even $^{62-76}$Zn Isotopes in Interaction Boson Model*

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Abstract: Spectra and E2 transition rates for the even-even $^{62-76}$Zn isotopes are studied in the framework of the interacting boson model. A schematic Hamiltonian capable of describing their spectra and transition is used. It is found that the even-even $^{62-76}$Zn isotopes are in the transition from $U(5)$ to $O(6)$ dynamical symmetry.

Key words: energy spectrum; electromagnetic transition; positive parity collective state
CLC number : O571.21　　**Documnt code**: A

1 Introduction

One of the most active areas in nuclear structure studies centers on the mass-60 nuclei, especially on Zn isotopes. In the early work, by means of anti-symmetric rotator model with the admixture of two quasiparticles, Petroviciet et al[1] studied the positive and negative parity states of ^{64,66}Zn , and especially gave an explanation of the forking existing in the low-lying 8^+ state. Recently, lots of experimental and theoretical studies have been done for Zn isotopes[2－7]. The superdeformed bands of Zn isotopes were studied by different theoretical models such as cranked relativistic mean-field theory and cranked Nilssion-Strutinsky model[2], Cranked shell method[3] and projected shell model[4]. These calculations reproduced many of the gross features found in Zn nuclei. All these give us an impetus to do research about Zn isotopes by other models. Therefore, the interacting boson model(IBM-1)[8] is introduced. This model considers correlated pairs of fermions outside a closed inert core coupled to states of angular momentum 0 or 2 only, the so called s and d bosons, and carries no label to distinguish neutrons from protons. It is worth mentioning that the IBM-1 has made a great success in medium-heavy nuclei. As for light nuclei, the valence neutrons and protons are in the same major shell, it is important to introduce isospin and pair correlation of proton and neutron, so IBM-1 is not often used in light nuclei. In this paper, we studied the positive parity collective states in the $^{62-76}$Zn isotopes by this model. The calculated values are in agreement with data. It is found that these even-even Zn isotopes are in the transition from $U(5)$ to $O(6)$ dynamical symmetry. The paper is organized as follows: After this introduction, we describe briefly the model Hamiltonian and the E2 transition operator in sect 2. In sect 3, we give the results and discussion on spectrum and E2 transition properties. Finally, in sect 4, a conclusion is given.

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2 The Schematic IBM Hamiltonian

The general IBM Hamiltonian contains 7 terms. For our study, we take the following schematic Hamiltonian[9]:

$$\hat{H} = \varepsilon_d \hat{n}_d + K\hat{Q} \cdot \hat{Q} + K_L \hat{L} \cdot \hat{L},$$

$$\text{Where} \quad \hat{Q}_\mu = (\hat{s}^+ \hat{\tilde{d}} + \tilde{d}^+ \hat{s})^2 + \chi(\hat{d}^+ \hat{\tilde{d}})^2_\mu,$$

$$\hat{L}_q = \sqrt{10}(\hat{d}^+ \hat{\tilde{d}})^{(1)}_q, \ \chi = 0.$$

The meaning of the symbols is the same as those in other papers about IBM. The program we used is PHINT[10], which diagonalizes the Hamiltonian on the basis of $U(5)$. Because the term K_L $(L \cdot L)$ is diagonal, it contributes the same to the energy levels with identical spin, and is a term adjusting energy level L. If the boson numbers are given, the Hamiltonian is principally determined by the two parameters ε_d and K, and is able to describe the transition from $U(5)$ to $O(6)$. If $\varepsilon_d = 0$, then the Hamiltonian reduces to an $O(6)$ limit Hamiltonian. If $K = 0$, the Hamiltonian becomes a $U(5)$ limit, describing the vibrational collective motion. Therefore, the ratio of K/ε_d is a measure of the transition from $U(5)$ to $O(6)$. If $K/\varepsilon_d = 0$, the Hamiltonian is vibrational, and if $K/\varepsilon_d = \infty$, the Hamiltonian is in γ unstable rotation. If the ratio lies in between, the Hamiltonian is in the transition between $U(5)$ and $O(6)$. The parameters in the Hamiltonian can be determined by fitting the experimental spectra. After the determination of the spectra, the wave function is determined. The electric and magnetic transition properties can then be obtained accordingly. For example, the E2 transition operator is

$$\hat{T}(\mathrm{E2})^2_\mu = e_2[(\hat{s}^+ \hat{\tilde{d}} + \hat{d}^+ \hat{s})^2_\mu + \chi(\hat{d}^+ \hat{\tilde{d}})^2_\mu].$$

Microscopically, the transition operator can be derived from shell model mapping procedure[11]. In practice, we adopted the consistent $Q \cdot Q$ Formalism. As is known, this convention is not an essential requirement of model, and sometimes, it is even necessary to use a different Q operator in E2 transition calculation to describe the E2 transitions. Noticeablely, the reduction in collectivity problem can be solved by using an operator in the transition different from that in the Hamiltonian. However, in many cases, the consistent $Q \cdot Q$ Formalism can give a good first description of the E2 transition properties. Since there are few experimental data available, we adopt the consistent $Q \cdot Q$ Formalism in the first place. When there are more experimental data on the E2 transition in the future, one can fine-tune the E2 transition operators to reproduce the details.

3 Results and Discussion

In Table 1, we give the parameters of the Hamiltonian and of the E2 transition operator in each nucleus studied. χ is taken zero when calculating the spectrum, but varies somewhat when calculating the values of B(E2). From Table 1, with the exception of ^{62}Zn, all parameters change rather smoothly. In the lighter even Zn isotopes, the value of ε_d increases with increasing mass number, until ^{66}Zn. In the heavier even-even Zn isotopes, ε_d value decreases with increasing mass number. It reflects the properties of the change of energy in excited states and of shape coexistence for Zn isotopes, in other word, so does the trend of change from oblate to prolate. With the exception of ^{62}Zn, the K_L takes a same value, while the parameter ε_d varies a little. The energy levels and E2 transition values for each nucleus were calculated with these parameters.

Table 1 Parameters of energy level and B(E2) operator for Zn isotopes

Nucleus	ε_d/MeV	K/MeV	K_L/MeV	e_2/eb
^{62}Zn	0.750	−0.090	0.005	0.072
^{64}Zn	0.900	0.028	0.020	0.098
^{66}Zn	0.950	0.028	0.020	0.078
^{68}Zn	0.900	0.013	0.020	0.070
^{70}Zn	0.620	0.028	0.020	0.095
^{72}Zn	0.570	0.150	0.020	
^{74}Zn	0.450	0.050	0.020	
^{76}Zn	0.450	0.041	0.020	

The comparisons between calculated and experimental values[12] of energy levels and $B(E2)$ for each Zn nucleus are shown in Figs. 1—8 and Table 2, respectively. In general, the agreement is good, especially for the ground-state band levels and beta-state band levels. However, there exist some discrepancies. For most Zn isotopes, the two-phonon states are slightly split in energy, about 200 keV, which may be understood by means of a small anharmonic term in the vibration. However, the three-phonon states have a much larger energy splitting, with value of about 1 MeV. It is difficult to envisage such a large energy splitting can be caused by anharmonicties alone, which may be related to the excitation of single particle.

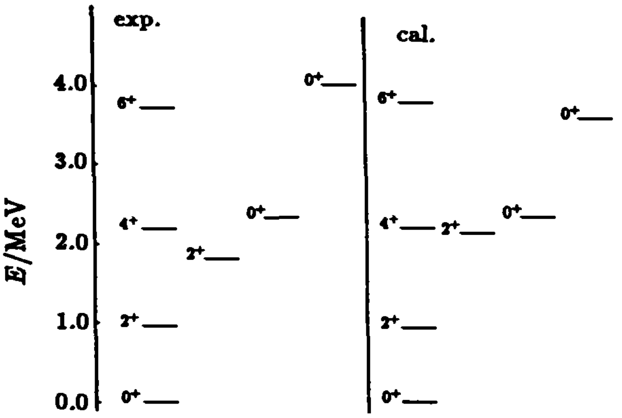

Fig. 1 Spectrum for ^{62}Zn

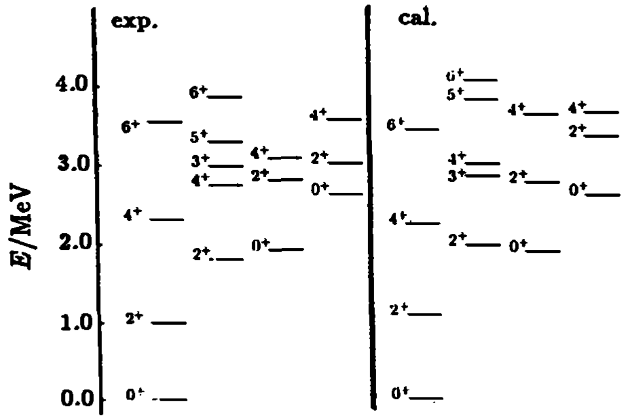

Fig. 2 Spectrum for ^{64}Zn

At first, for ^{62}Zn, results obtained in the present work are in good agreement with those of experiments. The calculated values of each energy level of gamma - state band are generally higher than experimental data, and the calculations for the excited states of the two 0^+ fit the experimental data.

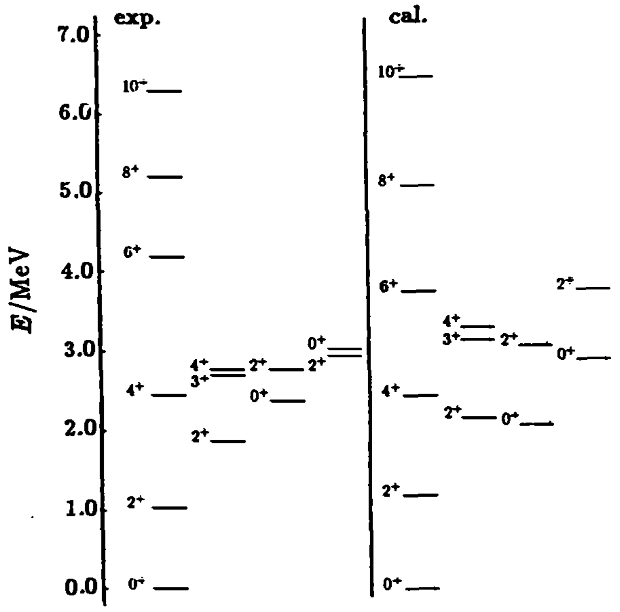

Fig. 3 Spectrum for ^{66}Zn

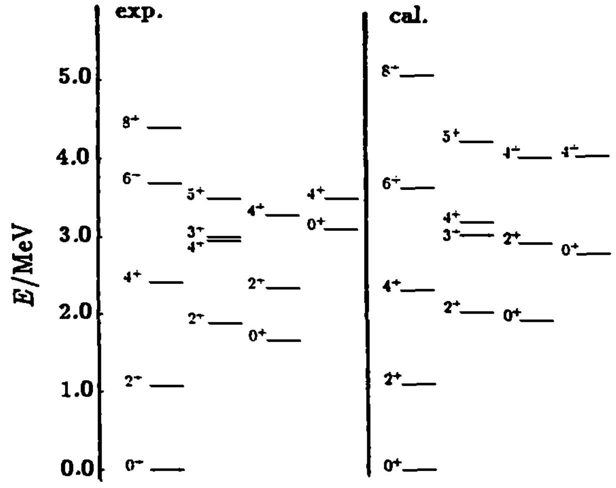

Fig. 4 Spectrum for ^{68}Zn

For the ^{66,68}Zn nuclei, especially for ^{66}Zn, the calculated values are in good agreement with experimental ones when the energy levels equal about 7 MeV. It is noticed that the two isotopes exhibit backbanding in the ground band, which can be explained by the collective backbanding raised by Long[13]. Recently, the paper[7] studied the property of the 8^+ state of ground-state band for ^{78}Zn, and considered the 8^+ state was isomer because $B(E2)$ value from 8^+ to 6^+ state is small, with 24

e^2fm^4 only. For ^{66}Zn, the B(E2) value is less than 46 e^2fm^4, so the 8^+ state of ^{66}Zn is similar to that of ^{78}Zn. In addition, the paper[14] has discussed the property of isomer by using mixed symmetric state, so we consider the 8^+ state of ^{66}Zn has the properties of isomer and of mixed symmetric state. In addition, for ^{64}Zn, we notice that there exists a small energy discrepancy between the 3_1^+ state and 4_2^+ state, and the values of B(E2) from these two states to 2_1^+ state are 1. 3 e^2fm^4 and 1. 1 e^2fm^4, respectively. In fact, it is difficult to distinguish between these two states, so it may be called that these two states have the property of exchange. From the Fig. 4 and Table 2, we find the 3_1^+ state and the 4_2^+ state in ^{68}Zn possess the property of those in ^{64}Zn. Because the values of $E(4_1^+)/E(2_1^+)$ of these four isotopes are about 2. 3, it is found that they are in the transition from U(5) to O(6).

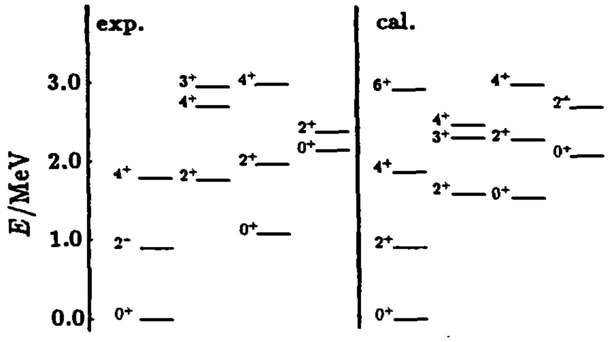

Fig. 5 Spectrum for ^{70}Zn

For ^{70}Zn, Fig. 5 shows that the calculated values are also in good agreement with the experimental data. The 3_1^+ state and 4_2^+ state are very similar with those of ^{64}Zn, and the value of $E(4_1^+)/E(2_1^+)$ is 2, so the ^{70}Zn belongs to vibration limit. Meanwhile, it is predicted that the value of B(E2) from the 4_1^+ state to the 2_1^+ state for ^{70}Zn is about 615 e^2fm^4. For ^{72}Zn, the calculated values are in quite good agreement with the experimental data. Because the value of $E(4_1^+)/E(2_1^+)$ is 4. 1, which is higher, this nucleus has obvious property of rotation that can be found from the large value of parameter K in the calculation of energy level. For the two ^{74,76}Zn, the calculated values are also in good agreement with the experimental data, especially for ^{76}Zn. For ^{74}Zn, the theoretical values of beta-state band are generally lower than experimental values. The values of $E(4_1^+)/E(2_1^+)$ of ^{74}Zn and ^{76}Zn are 1. 9 and 2. 2 respectively, so the ^{74}Zn closes to vibrational nucleus, and the ^{76}Zn is in the transition from U(5) to O(6).

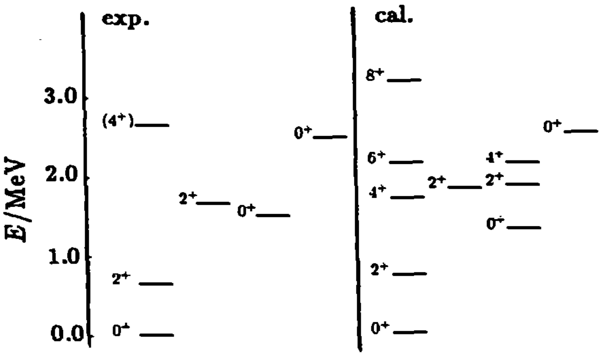

Fig. 6 Spectrum for ^{72}Zn

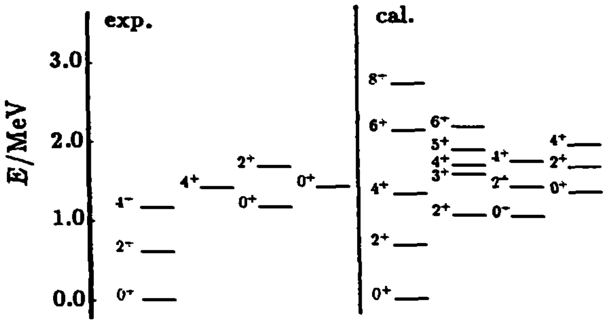

Fig. 7 Spectrum for ^{74}Zn

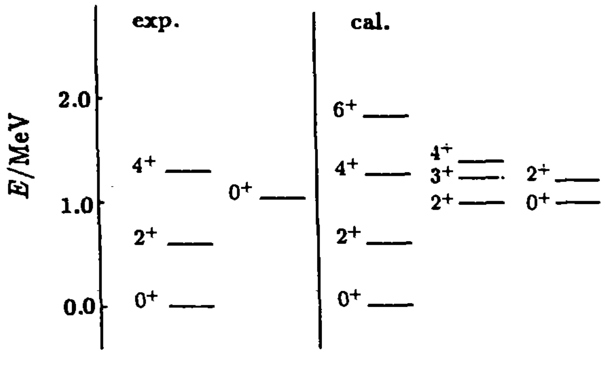

Fig. 8 Spectrum for ^{76}Zn

Moreover, distinguishing U(5) or O(6) limit can be achieved from the behavior of low-lying 0^+ states[15]. In the U(5) limit, the $\Delta n_d = 1$ selection rule allows only a decay from the 0_2^+ state to 2_1^+ state, which can be observed in the three ^{64,66,70}Zn, and these B(E2) values are larger. However, in the $O(6)$ limit, the $\Delta\sigma = 0$ and $\Delta\tau = 1$ selection rule allows only a decay from 0_2^+ to 2_2^+ state, while a decay from 0_2^+ to 2_1^+ state is forbidden. Until now, a decay from 0_2^+ to 2_2^+ state for $^{62-76}$Zn isotopes has not been found. Like Cd and Sn isotopes, these states of Zn isotopes have relation to the excitation of pairs of protons over major shell of $Z = 28$, which are equivalent to the shape coexistence between states of different deformation, or contain intruder configurations.

Table 2 Comparison of B(E2) values in Zn nuclei

Nucleus	J_i	J_f	Exp(e^2fm^4)	Cal(e^2fm^4)	Nucleus	J_i	J_f	Exp(e^2fm^4)	Cal(e^2fm^4)
^{62}Zn	2_1^+	0_1^+	170.8	200.0		4_1^+	2_1^+	554.4	408.0
	2_2^+	0_1^+	4.5	6.0		4_2^+	0_1^+	<2.1	0.0
	2_2^+	2_1^+	175.2	229.0		4_2^+	2_2^+	<633.6	251.0
	4_1^+	2_1^+	379.6	229.0		6_1^+	4_1^+	237.6	478.0
	4_2^+	4_1^+	280.8	74.0		8_1^+	6_1^+	<45.9	451.0
	4_2^+	3_1^+	102.2	35.0	^{68}Zn	10_1^+	8_1^+	205.9	304.0
^{64}Zn	6_1^+	4_1^+	277.4	156.0		2_1^+	0_1^+	258.9	255.0
	2_1^+	0_1^+	324.0	315.0		2_2^+	0_1^+	9.1	6.0
	2_2^+	0_1^+	34.5	11.0		2_2^+	0_3^+	148.4	107.0
	2_4^+	0_1^+	10.7	0.0		2_2^+	2_1^+	280.3	434.0
	2_2^+	2_1^+	600.0	493.0		2_3^+	0_1^+	0.6	0.0
	2_3^+	2_1^+	195.0	4.0		2_3^+	2_1^+	19.8	2.0
	0_3^+	2_1^+	0.9	21.0		2_4^+	0_1^+	1.8	0.0
	0_3^+	2_2^+	915.7	522.0		2_4^+	2_1^+	14.8	0.0
	0_2^+	2_1^+	255.0	629.0		3_1^+	2_2^+	98.9	381.0
	3_1^+	2_1^+	1.3	15.0		4_2^+	2_2^+		279.0
	4_1^+	2_1^+	435.0	493.0		0_3^+	2_1^+	131.9	12.0
	4_2^+	2_1^+	1.1	11.0	^{70}Zn	2_1^+	0_1^+	359.7	362.0
	4_3^+	2_1^+	15.0	0.0		0_2^+	2_1^+	638.9	795.0
	4_2^+	4_1^+	165.0	249.0		2_2^+	0_1^+	41.1	58.0
	4_3^+	4_1^+	60.6	2.0		2_2^+	2_1^+	1 182.0	615.0
	4_4^+	4_1^+	<4.5	3.0		4_1^+	2_1^+		615.0
^{66}Zn	2_1^+	0_1^+	283.5	247.0		4_2^+	2_1^+	68.5	77.0
	2_2^+	0_1^+	0.5	2.0		4_2^+	2_2^+	548.2	396.0
	2_2^+	2_1^+	5 700.0	398.0		4_2^+	2_3^+	668.1	670.0
	0_2^+	2_1^+	633.6	492.0		3_1^+	2_1^+	205.6	105.0

4 Conclusion

We have given a detailed study of the energy levels and E2 transitions in $^{62-76}$Zn isotopes in the framework of the interacting boson model. The results indicate that $^{62-76}$Zn isotopes are the $U(5)$ to $O(6)$ transitional nuclei. Meanwhile, the discrepancy between calculated and experimental data is found, which means that other factors must be introduced into Hamiltonian, such as pair interacting, isospin effect, high angular momentum boson and so on.

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$^{62-76}$Zn 核的低能谱和电磁跃迁的相互作用玻色子模型*

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摘　要：采用相互作用玻色子模型研究了$^{62-76}$Zn 核的低能正宇称态的能谱和电磁跃迁．应用一个简单的哈密顿量能够较好地描述它们的能谱和电四极跃迁．研究表明，$^{62-76}$Zn 同位素核基本上属于$U(5)$到$O(6)$的过渡核．

关键词：能谱；电磁跃迁；正宇称集体态

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