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Potential Energy Surface and Fusion Probability in DNS Model*

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Abstract: The Potential Energy Surface (PES) for particle exchange in Di-nuclear system is studied in detail. It is found that the nuclear deformation effect can change the shape of PES significantly. The dynamical deformation as a function of the reaction time in the reaction process is investigated in a simple model and we found that its variation with time is dramatic. The fusion probabilities P_{CN} of some reaction channels based on the mechanism of cold fusion are also calculated.

Key words: super heavy element; potential energy surface; fusion probability

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The production of Super Heavy Element synthesized artificially by cold fusion reaction ($Z=106$—112) in Berkley and GSI[1,2] and by hot fusion reaction ($Z=110,112,114$) in Dubna[3] has drawn wide attention. Many theoretical models have been proposed to calculate the fusion cross section [4,5,6]. In most of the models [4,7,8], it is assumed that after the capture of a projectile by a target nucleus complete fusion inevitably occurs. The competition between complete fusion and quasi-fission processes was not considered. To overcome the drawback, a new model DNS (Di-nuclear system) was put forward and the evaporation residue cross sections for the cold fusion reactions predicted in DNS model are in quite good agreement with the experimental data[6,9]. In Ref. [6,9], the potential energy surface is approximated by harmonic oscillators and two-dimensional Kramers-type expression is used to calculate the fusion probability. While in this paper, we study the potential energy surface in detail and calculate the fusion probability using a dynamical way.

In DNS model, the evaporation residue cross section can be written as a sum over all partial waves j:

$$\sigma_{ER}(E_{CM}) = \pi\lambda\!\!\!^{-2}\sum_{j=0}^{j_f}(2j+1)T(E_{CM},j)\cdot P_{CN}(E_{CM},j)W_{SUR}(E_{CM},j)\ , \qquad (1)$$

where $\lambda\!\!\!^{-}$ represents the de Broglie wavelength of relative motion of the colliding nuclei, E_{CM} is the bombarding energy in center of mass system, $T(E_{CM},j)$ is the penetration probability for the jth

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partial wave of the colliding nuclei over the Coulomb barrier. $P_{CN}(E_{CM},j)$ is the fusion probability to form a compound nucleus from the DNS configuration of the jth partial wave of the colliding nuclei. And $W_{SUR}(E_{CM},j)$ is the survive probability for the compound nucleus with angular momentum j . For the highly fissile super heavy nuclei, $W_{SUR}(E_{CM},j)$ is narrow function of j different from zero in the vicinity of $j=0$ for all energies. The effective maximal angular momentum j_{max} is about 10 $\hbar$. For small j, $P_{CN}(E_{CM},j)$ and $T(E_{CM},j)$ are not much different from $P_{CN}(E_{CM},j=0) = P_{CN}(E_{CM})$ and $T_{CN}(E_{CM},j=0) = T_{CN}(E_{CM})$. Thus, we obtain the following approximate factorization:

$$\sigma_{ER}(E_{CM}) = \sigma_c(E_{CM})P_{CN}(E_{CM})W_{SUR}(E_{CM}) \ , \quad (2)$$

where

$$\sigma_c(E_{CM}) = \pi\lambda\!\!\!^{-2}(J_{max}+1)^2T(E_{CM}) \ . \quad (3)$$

In DNS model, a key point is the calculation of PES because it governs the fusion dynamics and determines the optimum excitation energy of the compound nucleus and the optimum bombarding energy . Furthermore, it gives the information about the optimum projectile-target combination leading to a certain super-heavy nucleus. For a complete quantitative understanding of the fusion reaction leading to super-heavy nuclei, a time dependent calculation with a time dependent multidimensional potential surface should be taken into account. In the DNS model, however, the potential energy can be written as follows:

$$U(R,\ \eta) = B_1 + B_2 + V(R,\eta) - B_{12}, \quad (4)$$

where

$$V(R,\ \eta) = V_C(R,\eta) + V_N(R,\ \eta) \ , \quad (5)$$

B_1, B_2 and B_{12} are the realistic binding energies of the two fragments and compound nucleus, respectively. The shell effect is included in the binding energy already by a macroscopic method [10,11]. R is the distance between the two nuclear centers. The mass asymmetry freedom η is defined as $\eta=(A_1-A_2/A_1+A_2)$. From (4) and (5), we can see that U(or V) is a surface energy in R and η coordinates. To simplify the discussion, we first calculate PES at fixed η; then discuss it with fixed R. If we restrict ourselves to head on collisions the rotational energy of the compound nucleus can be ignored, it can be added straightforward otherwise. A suitable relative distance R_m between the two nuclei is taken to keep the interaction between them in the bottom of the quasi-fission barrier. The densities of the both nuclei are kept fixed when the interaction potential are calculated. Due to the short collision time, the frozen-density approximation (or sudden approximation) is appropriate[12].

The double folding nuclear interaction parameterized by the Morse potential [8] can be expressed as:

$$V_N(R) = D\left(\exp\left[-2\alpha\frac{R-R_0}{R_0}\right] - 2\exp\left[-\alpha\frac{R-R_0}{R_0}\right]\right) , \quad (6)$$

where

$$D = 2\pi a_1 a_2 R_{12}(10.96 - 0.8R_{12}) \ ,$$

$$R_0 = R_1 + R_2$$

and

$$\alpha = 11.47 + 2.069R_{12} - 17.32\ a_1a_2 \ ,$$

$$R_{12} = R_1R_2/R_0 .$$

If the two nuclei are both spherical, then the Coulomb interaction can be written as

$$V_C = 1.44\frac{Z_1Z_2}{R}\ (\text{in MeV}) \ ,$$

Z_1, Z_2 are the charge numbers of the two fragments, respectively, R is the distance between the centers of the two spherical nuclei.

If the ground state deformations of the two touching nuclei are taken into account, the nuclear interaction does not change much due to the short range nuclear interaction character, but the long range Coulomb interaction changes a great deal. In principle, the deformed nuclei can have different relative orientations. Some averaging over the orientations of the nuclei has to be carried out in the initial DNS, however, the orientation which gives

rise to the minimum interaction energy is in favor of the nucleon transfer. So the orientation is chosen as the one which gives rise to the minimum energy.

The Coulomb interaction can be written as

$$V_C = \int \frac{\rho_1 \rho_2}{|r_{12}|} dr_1 dr_2 , \qquad (7)$$

where ρ_1, ρ_2 are the charge density of the first and second nucleus respectively (here we assume ρ_1, ρ_2 are constants). $|r_{12}|$ denotes the distance between any two points in the two different nuclei.

For the nucleus with positive quadrupole deformation parameter β_2 (other deformation freedoms are disregarded), the dependence of the nucleus-nucleus interaction potentials on R with angular momentum $J=0$ are shown in Fig. 1 and Fig. 2 for the system of $^{54}Cr+^{208}Pb$ and $^{35}Al+^{227}Np$, respectively. ^{208}Pb nucleus is a spherical one but ^{54}Cr nucleus is a deformed one with $\beta_2=0.18$. While ^{35}Al nucleus and ^{227}Np nucleus are both deformed with positive quadrupole deformation. In Fig. 1, the minimum point on the solid line is the lowest one among those of the three curves. The same situation happens to the interaction shown in Fig. 2. Thus the pole to pole orientation has given

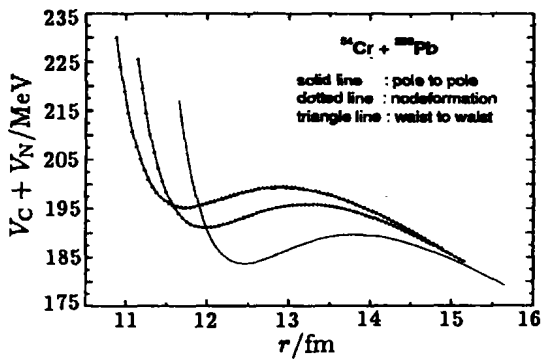

Fig. 1 The interaction between ^{208}Pb and ^{54}Cr in different orientations as a function of the distance between the centers of two nuclei.

the lower interaction between the deformed nuclei with positive quadrupole deformation. Comparing Fig. 1 with Fig. 2, we can see that the case of two nuclei both having deformation is much more affected by deformations than the case with only one nucleus being deformed.

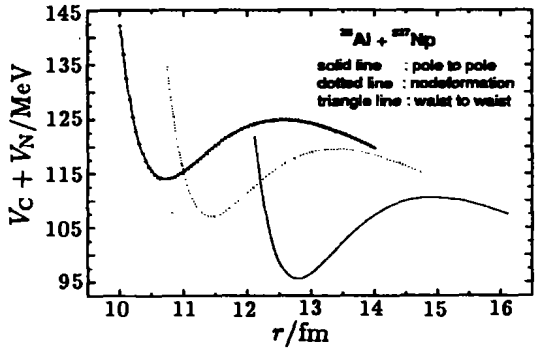

Fig. 2 The interaction between ^{227}Np and ^{35}Al in different orientations as a function of the distance between the centers of two nuclei.

For the nucleus with negative quadrupole deformation parameter β_2, such as ^{28}Al, its shape is oblate. The calculation results for the combination of $^{208}Pb+^{28}Al$ are shown in Fig. 3, in which the interaction is remarkably quenched down by the waist to waist orientation. The minimum point on the dotted line is the lowest one among those of the three curves. Thus the waist to waist orientation is the most favorable case for combination of $^{208}Pb+^{28}Al$. The deformation parameters of the above nuclei are taken from Ref. [13].

Now we turn to the effects of dynamical deformation. During nuclear contact in the collision process the composite system stretches under the influence of the repulsive Coulomb force and we assume it retains symmetry about the axis that connects the two centers. The reversible shape oscillations or other coherent modes of excitation are not considered, only the effect of the deformations which become irreversible due to the coupling with the intrinsic degrees of freedom is taken into account[14,15]. Thus it can be considered as a diffusion process. For both fragments $k=1,2$ of DNS, the deformation may be described as:

$$R_k(t,\theta) = \Lambda^{-1} R_k^0 [1+\beta_2(t) Y_{20}(\theta)] . \qquad (8)$$

Ensuring volume conservation the semi-axes of the spheroids may be expressed as:

$$a_k(t) = R_k^0[1+\varepsilon(t)]\ ,$$
$$b_k(t) = R_k^0[1+\varepsilon(t)]^{-1/2}\ . \qquad (9)$$

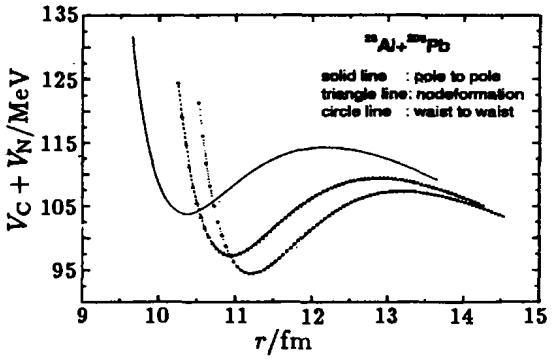

Fig. 3 The interaction between ^{208}Pb and ^{28}Al in different orientations as a function of the distance between the centers of two nuclei.

The center of mass distance can be written as $r(t) = a_1(t) + a_2(t)$, and the formation of a neck is neglected. The mean deformation

$$\langle\varepsilon(t)\rangle = \varepsilon_0\left[1+\exp\left(\frac{-t}{\tau_\varepsilon}\right)\right] \qquad (10)$$

is the solution of the Fokker-Planck equation

$$\frac{\partial P(\varepsilon,t)}{\partial t} = -\frac{\partial}{\partial\varepsilon}(V_\varepsilon P) + D_\varepsilon\frac{\partial^2 P}{\partial\varepsilon^2} \qquad (11)$$

provided the deformation process is very slow. ε_0 is the equilibrium deformation. The deformation diffusion coefficient

$$D_\varepsilon = \frac{T}{\gamma_\varepsilon\mu_\varepsilon R_0^2}\ ,\ R_0 = R_1^0 + R_2^0\ , \qquad (12)$$

where $\tau_\varepsilon \equiv \gamma_\varepsilon^{-1}$ is the deformation relaxation time commonly recognized as about 4×10^{-21} s. The drift coefficient

$$V_\varepsilon = -\frac{D_\varepsilon}{T}\frac{\partial u}{\partial\varepsilon}\ , \qquad (13)$$

where u is the driven potential corresponding to the deformation. The temperature T is provided by the energy dissipation during the collision. The Coulomb interaction of the composite system becomes

$$V_C(t) = \frac{V_{C0}}{1+\varepsilon(t)}\ , \qquad (14)$$

where V_{C0} being the Coulomb interaction without deformation. In Fig. 4 the deformation $\varepsilon(t)$ and the Coulomb interaction barrier are shown as functions of the interaction time for the reaction ^{40}Ca (188 MeV) + ^{208}Pb for nearly central collision. It can be seen that the mean deformation increases with interaction time t until it reaches an equilibrium value ε_0. Correspondingly the Coulomb interaction barrier decreases with time, and it has been reduced about 40 MeV. As a consequence 40 MeV more relative kinetic energy is converted into the intrinsic excitation energy of DNS.

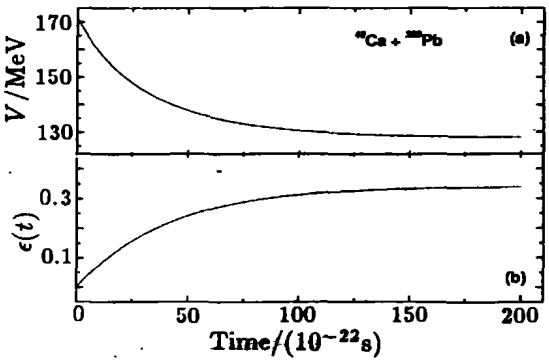

Fig. 4 The Coulomb interaction barrier (a) and the deformation $\varepsilon(t)$ (b) as a function of the interaction time for the reaction ^{40}Ca(188 MeV)+^{208}Pb.

The potential energy surface as a function of η describes how it changes with the different combinations of two fragments which all lead to the same compound nucleus. When a nucleon is transferred from one nucleus to the other, the binding energies of both nuclei will change and so does the interaction between them. The PES of the reaction ^{54}Cr+^{208}Pb $\rightarrow{}^{262}$106 as a function of η is shown in Fig. 5. The arrow points to the incident combination of nuclei with $\eta = -0.587$. If some nucleons are transferred from the small nucleus to the big one, the small one will become smaller and the big will become bigger. During this process, the mass asymmetry freedom is moving leftward. If the DNS overcomes the barrier shown in Fig. 5, the com pound nucleus will be formed inevitably. The barrier in Fig. 5 is called inner fusion barrier. To form the compound nucleus the barrier must be overcome , and the system needs some extra energy

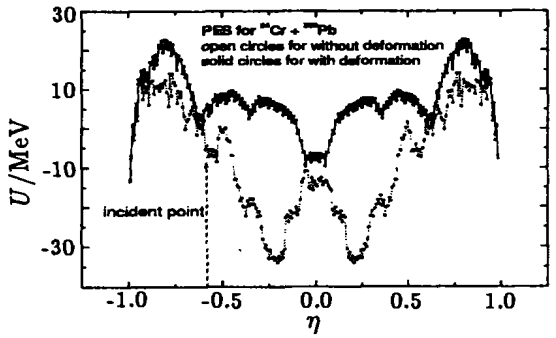

Fig. 5 Potential Energy Surface as a function of mass asymmetry freedom in reaction $^{54}Cr+^{208}\rightarrow^{262}106$.

for about 19 MeV, which must be supplied by the excitation energy obtained from the incident bombarding energy. It is seen that the deformations change the potential energy surface greatly and reduces the top of PES about 8 MeV. The variations of PES mentioned above may influence the calculation results and lead to the enhancement of the fusion probability and the lower excitation energy of compound nucleus which is of vital importance to the survival probability of the compound nucleus. Some zigzags on the curve are caused by the odd - even effects and the fluctuations, by shell corrections. Using the improved potential energy, we can get more realistic results about the fusion probability.

The diffusion process can be described by the Master's equation:

$$\frac{dP(A_1,E_1,t)}{dt}=\sum_{A'}W_{A_1A'_1}[d_{A_1}P(A'_1,E'_1,t)-d_{A'_1}P(A_1,E_1,t)]\,, \qquad (15)$$

where A_1 and A_2 are masses for the two fragments, respectively. $P(A_1,E_1,t)$ is the occupation probability of the fragment 1 with the mass A_1 and excitation energy E_1 at time t, $W_{A_1A'_1}$ is the transitional probability from state (A_1,E_1) to (A'_1,E'_1) and d_{A_1} is the dimension of the macroscopic state (A_1, E_1). Some details for the d_{A_1} and $W_{A_1A'_1}$ can be found in Ref. [16]. The fusion probabilities for some incident channels are shown in Fig. 6. It is found that the fusion probability decreases greatly with the increase of the charge number of compound nucleus. From $Z=104$ to $Z=118$, the P_{CN} decreases by about 5 orders of magnitude. The decrease of the evaporation residue cross section for the cold fusion reactions is largely attributed to the great decrease of the fusion probabilities. We also make some comparisons between our results and those from Ref. [9]. It is easy to see that for the relatively light nuclei, the two sets of results are rather close while for the relatively heavy nuclei, our results are quite different from the P_{CN} in Ref. [9]. The reason may come from the following two points. First, the potential energy surfaces in our calculations are more realistic ones. While in Ref. [9], the potential energy surfaces are approximated with the harmonic oscillators. Thus, the potential energy surfaces in their calculations are smoothed and some characteristics may get lost.

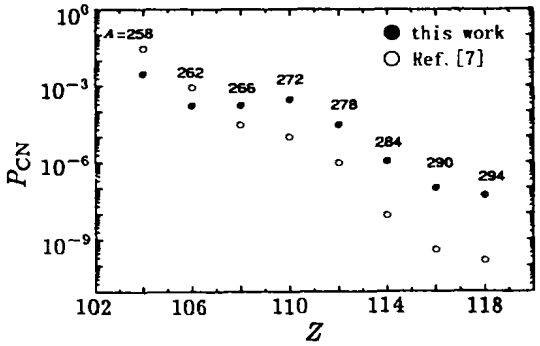

Fig. 6 Fusion probability P_{CN} as a function of charge number of compound nucleus with target ^{208}Pb.

Second, in Ref. [9], a two-dimensional Kramers-type expression (quasi-stationary solution of the Fokker-Planck equation) is used to get the fusion-probability. In this paper, a dynamical way is used to solve the Master's equation and calculate the fusion probability.

In summary, the deformation influences the potential energy surface greatly, especially to the optimum bombarding energy and the optimum projectile-target combination, both of which affect the formation cross section of the super heavy element. The dynamical deformation as a function of the reaction time in the reaction process is dramatic. Some results about the fusion probabilities for cold fusion reaction are obtained. The study about the survive probability and the evaporation residue cross section for the cold fusion reactions will appear in later papers.

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基于双核模型的粒子交换势能面与熔合几率的研究*

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摘　要：研究了基于双核模型的粒子交换势能面. 原子核的形变效应对于势能面的形状有较大的影响. 在反应过程中，作为反应时间函数的动态形变的变化是显著的. 通过求解主方程，对一些基于冷熔合机制的反应道的全熔合几率也进行了讨论.

关 键 词：超重元素；粒子交换势能面；熔合几率

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