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Majorana Interaction Dependence of Mixed Symmetric States*

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Abstract: Mixed symmetry states were studied in the framework of the neutron-proton interacting boson model (IBM2). It was found that some of the mixed symmetry states with moderate high spins changed very fast with respect to the Majorana interaction. This changes our traditional picture that the maximum F-spin states are the lowest in energy, the smaller the F-spin, the higher the energy. The minimum F-spin states or the state with F-spin next to minimum may become the yrast or yrare state. These states are difficult to decay and become very stable. This study suggests that a possible new mode of isomers may exists due to the special nature in their proton and neutron degrees of freedom.

Key words: interacting boson model; mixed symmetry state; isomeric state

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1 Introduction

The theoretical prediction[1] of the mixed symmetry states or the isovector states and their later experimental comfirmations[2,3] have been one of the great successes of the neutron-proton interacting boson model (IBM2). The studies of the mixed symmetry states have been an important subject in nuclear physics[4-7]. Recently the mixed symmetry states in nucleus with octupole deformation have been proposed[8]. Physically, the F-spin describes the relative phase of motion of the neutrons and protons[9]. A boson has F-spin 1/2, with the neutron boson having $-1/2$ 3rd component and the proton boson having the $+1/2$ 3rd component, similar to the isospin. Like the isospin, F-spin obeys the rules of the angular momentum coupling. A nucleus with N_π proton boson and N_ν neutron boson can have F-spin ranging from $F_{min}=|N_\pi-N_\nu|/2$ to $F_{max}=(N_\pi+N_\nu)/2$. In the maximum F-spin state, the neutrons and protons are moving in phase, and this state is called the F-spin symmetric state. In other F-spin state, the neutrons and protons are moving out of phase in different degree, and these states are called mixed symmetry states. The extent at which the neutrons and protons move differently is reflected in the F-spin: in the maximum F-spin state, they move in phase. In

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the minimum F-spin state, they move completely out of phase. This picture is helpful in intuitive understanding the F-spin, though some recent studies[10,11] showed that this simple interpretation was not appropriate in some cases.

Studies on F-spin have been restricted to the low-lying levels with low angular momentum, say, $L=1$, 2 and 3[2,3]. It is generally assumed that in a nucleus the lowest energy states are the states with maximum F-spin, then the states with $F_{max}-1$ lie at a higher energy, and the states with $F_{max}-2$ lie still higher, …. The states with minimum F-spin will be the highest in energy. In practice, we can only observe some low-lying levels, and nearly all the mixed F-spin states studied so far are states with $F_{max}-1$.

In this paper, we will show that under certain conditions it is possible to have high spin mixed symmetry states at very low energy. In some cases, they can even become the yrast state, i.e., the lowest state with the same angular momentum. Because their F-spin structures are different from the states below them, their transition to low-lying levels are highly retarded and make them a new mode of isomeric state.

2 Dependence of Mixed Symmetry State on the Majorana Interaction Strengths

The positions of the mixed symmetry states are closely related to the strength of the Majorana interaction. The Majorana interaction contains 3 terms:

$$M = \xi_1 (d_\nu^\dagger d_\pi^\dagger)^1 (\tilde{d}_\pi \tilde{d}_\nu)^1 + \frac{\xi_2}{2} (s_\pi^\dagger d_\nu^\dagger - s_\nu^\dagger d_\pi^\dagger)^2 \cdot (\tilde{s}_\nu \tilde{d}_\pi - \tilde{d}_\nu \tilde{s}_\pi)^2 + \xi_3 (d_\nu^\dagger d_\pi^\dagger)^3 \cdot (\tilde{d}_\pi \tilde{d}_\nu)^3 \ . \quad (1)$$

The Majorana interaction is part of the neutron-proton interaction. Among the vast number of literatures on IBM2, the choices of the parameters used can be classified into 3 categories[12-20]: 1) the 3 strengths in the Majorana interaction are equal and positive, for instance in Ref. [9]; 2) two Majorana parameters, namely ξ_1 and ξ_3 are kept equal and ξ_2 is determined differently; 3) the 3 Majorana interactions are varied freely to give a best fit to the spectrum of the nucleus studied. We now study the dependence of mixed symmetry states on the strengths of the Majorana interactions. In this paper, we limit ourselves to the vibrational limit of the IBM2. For simplicity, the following Hamiltonian is chosen,

$$H = 0.12\, C_{2U(5)} + 0.01 C_{2O(5)} + M\ , \quad (2)$$

where the expressions of the Casimir operators can be found in Ref. [9]. In the following discussion, a system with $N_\pi=2$ and $N_\nu=3$ is discussed. The results obtained here can be generalized into systems with more boson numbers.

Two choices of the Majorana parameters are used: 1) $\xi_1=\xi_2=0.8$ MeV, and let ξ_3 changeable; 2) $\xi_1=\xi_3=0.8$ MeV and let ξ_2 changeable. When $\xi_1=\xi_2=\xi_3=0.8$ MeV, the system is in the $U(5)$ limit, both the wave functions and the energy levels can be calculated analytically. Then we vary the parameter and trace the change of the states by looking at the overlapping integrals of each state at neighboring Majornana interactions. In this way, we can study the dependence of the mixed symmetry states on the Majorana interactions. It is worth pointing out here that even though the $U_{\pi+\nu}(6)$ is no longer a good quantum number, the F-spin is still a good quantum number. Under changes of the Majorana interaction, the states within each F-spin change.

In the first case, all the states with maximum F-spin do not change as ξ_2 changes. One remarkable feature for this set of calculations is that the $U(5)$ limit labels are still conserved. This may imply a particular symmetry that has not been found yet.

In the second case, all the maximum F-spin states do not change with respect to ξ_3 either.

However, different from the first case, the $U(5)$, $O(5)$ labels are no longer good quantum numbers. They admix with one another for some states. Another feature of this case is that the energies of the F-spin mixed symmetric states are no longer linear with respect to ξ_3 in a few cases, though in most cases linearity still exists.

3 High-spin Mixed Symmetry States

Usually it is thought that the states with the same F-spin will depend on the Majorana interaction identically, i. e., they change the same amount as the Majorana interaction changed. This is true when the Majorana interaction takes the special form that the 3 parameters are equal. However, when the Majorana interaction parameters are not limited to that form, mixed symmetry states change with the Majorana interaction quite differently, even though they have the same F-spin. Some of the mixed symmetry states remain constant. Some change slowly while the others change very fast. For instance, for $L=8$, as ξ_3 decreases from 0.8 MeV to -0.2 MeV in step of 0.2 MeV, and keeping $\xi_1=\xi_2=\xi_3=0.8$ MeV, the state [32]⟨32⟩ with $F=1/2$ decreases by 0.8 MeV, much faster than state [41]⟨41⟩ with $F=3/2$. Here we have used the $U(5)$, $O(5)$ group irreducible representations to label the states. Other $L=8$ states do not change. When ξ_3 is reduced to 0.0 MeV, the [32]⟨32⟩ state becomes the lowest state among the $L=8$ states. Similarly, when ξ_2 changes while keeping $\xi_1=\xi_3=0.8$ MeV, similar result is obtained. The state [41]⟨4⟩ with $F=3/2$ has the biggest change in energy as the Majorana parameter changes. It should be mentioned that when $\xi_3\neq0.8$ MeV, the state labelling is approximate, just like the case in Nilsson model where the state label in the polar coordinate is used to represent the biggest component when the deformation is not infinite.

The result presented here is surprising because it is contrary to our usual understanding of the mixed symmetry state: the smaller the F-spin of the states, the higher the energy which is true for F-spin symmetric Hamiltonian where $\xi_1=\xi_2=\xi_3$. When the 3 parameters are not equal as is being studied in this work, the common picture is in doubt. For states with low spin, the mixed symmetric states have not come down so low to become the lowest state in energy, the common picture is right for a few low-lying states. However for states with high spins, the picture is quite different. Some mixed symmetry states may come very low, and even become the yrast state, or the yrare (the state next to yrast) state for a reasonable range of the Majorana interaction. The wave function structures of these mixed symmetry states are quite different from the states below, and they are difficult to decay. To see this more clearly, E2, M1 transition calculations. The following transition operators are used for simplicity,

$$T(\mathrm{E2}) = e_\pi[(s_\pi^+\tilde{d}_\pi + d_\pi^+ s_\pi) + \chi_\pi(d_\pi^+\tilde{d}_\pi)^2 + (s_\nu^+\tilde{d}_\nu + d_\nu^+ s_\nu) + \chi_\nu(d_\nu^+\tilde{d}_\nu)^2], \quad (3)$$

$$T(\mathrm{M1}) = \sqrt{\frac{30}{4\pi}}(d_\nu^+\tilde{d}_\nu)^1 . \quad (4)$$

We have also put $e_\nu=e_\pi=e$. The result for some interested states are

$$B(\mathrm{E2};\ 8_1 \to 6_2) = 0.000\,2e^2 \quad (5)$$

and the transition from 6_2 state are zero. The 8_1^+ and 6_2^+ states have no M1 transitions to states down below. Thus these states are very difficult to decay. In particular, the 6_2^+ state has no transitions to states down below. It is very stable and has a long lifetime. It is a new mode of isomers. These isomers arise from the different mode of motions of the neutrons and the protons. It is collective.

4 8^+ State in ^{94}Sr, a Possible Example

Recently, the yrast state of ^{94}Sr has been extended to 14^+. The structure of Sr isotopes have been studied in the neutron-proton interacting boson model[21-23]. The peculiar feature in this figure is the backbending at $L=8$ in the yrast band. To describe the spectrum in the IBM2, the following Hamiltonian has been used

$$H = \varepsilon_d \hat{n}_d + \kappa Q_\nu \cdot Q_\pi + M \ , \qquad (6)$$

where

$$Q_\rho = (s_\rho^+ \tilde{d}_\rho + d_\rho^+ s_\rho) + \chi_\rho (d_\rho^+ \tilde{d}_\rho)^2 \ . \qquad (7)$$

The parameters ε_d, κ, χ_ν and χ_π are determined by the low-lying levels. As for the Majorana interaction, we have no knowledge of the 1^+ state. We put $\xi_1=\xi_2=0.15$ MeV so that the 1^+ state lies at about 3.0 MeV. The ξ_3 is chosen such that the 8_1^+ state best fit the experimental data. It is found that the 8_1^+ state is mainly from $F=2$ mixed symmetric configuration, while 6_1^+ comes from $F=4$, the maximum F-spin configuration. The electronagnetic transition properties are calculated by using the following transition operators,

$$T(\mathrm{E2}) = e\,|Q_\nu + Q_\pi| \ , \qquad (8)$$

$$T(\mathrm{M1}) = \sqrt{\frac{3}{4\pi}} L_\pi \ , \qquad (9)$$

where Q is the quadrupole operator and L is the angular momentum operator. We have adopted the consistent $Q\cdot Q$ formalism, where we have used the Q operators used in the Hamiltonian. It is found from these calculation that 8_1^+ has very small E2 and M1 transition rates to the low-lying levels. It is a kind of isomeric state. Similarly the states 10_1^+ and 12_1^+ are slow to decay, and are of similar nature. There are experimental indication that the 8_1^+ state might be an isomeric state.

5 Discussions

The Majorana interaction dependence of mixed symmetry states have been studied numerically. It is shown that under certain parameterization of the Majorana interaction, some high spin mixed symmetry states may become very low, even become the yrast or yrare state. This contrasts to previous assumption that mixed symmetry states should be very high in energy. In some case, these high-spin mixed symmetry states are slow to decay and become a kind of isomeric states. A possible candidate nucleus ^{94}Sr is analyzed using a usual IBM2 scheme. However, in order to confirm the example in ^{94}Sr, more experimental data are needed.

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混合对称态对 Majorana 相互作用的依赖性*

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摘 要：在IBM2中研究了混合对称态. 发现某些较高自旋的混合对称态随着Majorana相互作用很快变化. 这个结果改变了传统的关于混合对称态的观点：最大F旋态的能量最低，F旋越小，能量越高. F_{min}或$F_{min}-1$的态会成为yrast或yrare态. 这些态很难向下衰变，因而很稳定. 这个研究的结果表明可能存在一种由于质子和中子自由度的特殊性质而引起的新的同质异能态.

关 键 词：相互作用玻色子模型；混合对称态；同质异能态

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