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Integrable Systems——Trace Formulas and Inverse-$\hbar$ Spectroscopy*

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Abstract: Periodic orbits of two dimensional uncoupled quartic oscillator were calculated by integrating Hamiltonian equations of motion on reasonable tori, and several classical quantities were also computed. Inserting them into Berry-Tabor trace formula, a trace, i. e., the semiclassical density of states of the corresponding quantum system, was obtained. Finally, Fourier transform was adopted to verify the contribution of each periodic orbit. Good agreement between the semiclassical action function and the quantum action function indicates the validity of Berry-Tabor trace formula.

Key words: integrable system; trace formula; inverse-$\hbar$ spectroscopy

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Trace formulas relate the quantum density of states to the properties of the periodic orbits of the underlying classical system[1-4]. They provide us with a possible way to erase the great gap between the microscopic quantum mechanics and the macroscopic classical mechanics. From Gutzwiller's pioneer work[2] to now, several trace formulas have been developed for systems with different dynamics such as integrable, near-integrable, mixed, chaotic dynamics. One of them is Berry-Tabor trace formula, which is powerful for the integrable systems[3],

$$\rho_M^{sc} = \frac{T_M}{\pi\hbar^{3/2} M_2^{3/2} \left| g_E'' \right|^{1/2}} \cos\left(\frac{S_M}{\hbar} - \frac{\eta_M \pi}{2} - \frac{\pi}{4} \right) , \tag{1}$$

where the subscript M denotes the reasonable torus M with topology $M(M_1, M_2)$. S_M, T_M, η_M, ρ_M^{sc} are the periodic orbit's action function, period, Maslov index and semiclassical contribution to the density of states, respectively. g_E'' is the curvature on the energy surface.

A two-dimensional uncoupled quartic oscillator is selected as the paradigm because the system is simple and integrable. The Hamiltonian is written as following[4]:

$$H = \frac{p_1^2 + p_2^2}{2} + a\left| \frac{q_1^4}{b} + b q_2^4 \right| . \tag{2}$$

In the equation, p_1, p_2, q_1, q_2 are momenta and coordinates on each dimension. respectively. a. b are two constants. Because the system has the scaling property, we can choose an arbitrary energy value such as $E = 1$ (the systems with other energies can be mapped onto the system with energy equal to 1 through a scaling parameter[5]).

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Fig. 1 shows its Poincare section surface. One can see that it is regular in the whole phase space. Integrating Hamiltonian equations of motion, the classical quantities

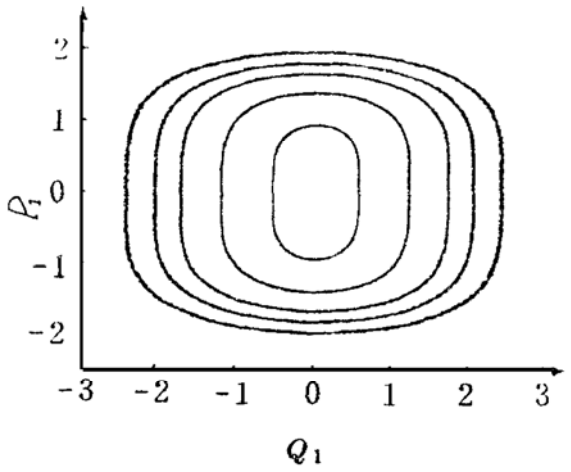

Fig. 1 Poincare section surface of two-dimensional uncoupled quartic oscillator.

which are indispensible to calculate the semiclassical density of states, as well as one periodic orbit on each reasonable torus are obtained (for this system, there is a family of one-parameter continuous periodic orbits with identical topology on each reasonable torus). The curvature on energy surface is calculated by using the way suggested in reference [5]. Inserting S_M, T_M, η_M, g''_E, M_2 into Berry-Tabor trace formula, the semiclassical density of states is achieved. To discriminate the contribution of each periodic orbit, Denis Ullmo[4] et al. took action S and inverse-$\hbar$ (denoted by t) as a Fourier-transform pair and transformed the density of states into action domain. The final semiclassical action function $R^{sc}(S,E)$ has the form[4]:

$$R^{sc}(S,E) = \sum_M \int_0^\infty \mathrm{d}t\, t^{-1/2} \mathrm{e}^{-(\alpha + \mathrm{i}S)t} \rho_M^{sc}(t,E)$$
$$= \sum_M \frac{T_M \exp(\mathrm{i}\eta_M \pi/2 + \mathrm{i}\pi/4)}{2\pi |M_2^3 g''_E|^{1/2} [\alpha - \mathrm{i}(S - S_M)]^2}, \quad (3)$$

$\mathrm{e}^{-\alpha}$ is an exponential dumpling added to the transforming kernel for the uncertainty principle of Fourier transform, $t^{-1/2}$ is the factor used to get a compact action function. The R-S configuration is graphed in Fig. 2.

We can see each periodic orbit contributes a δ-like peak to the density of states. To test the accuracy of this semiclassical method, the first peak in Fig. 2 is expanded in Fig. 3 and compared with the

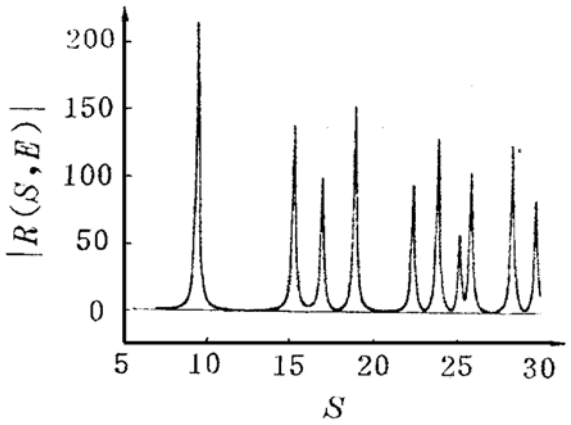

Fig. 2 Absolute value of action function given by the Fourier transform of Berry-Tabor trace formula. X-axial shows actions of periodic orbits.

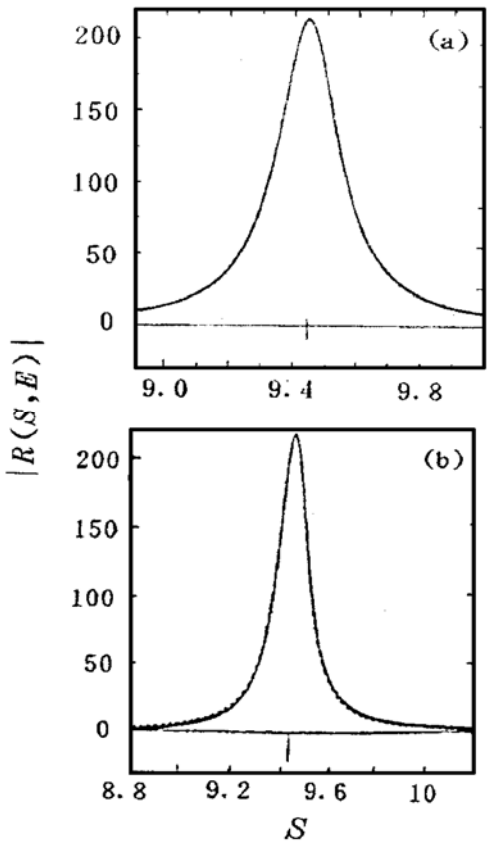

Fig. 3 The action functions
(a) The expanded (1, 1) peak given by the semiclassical method, (b) The peak given by the quantum density of states[4].

peak given by the quantum action function[4]:

$$R^{QM}(S,E) = \int_0^\infty \mathrm{d}t\, t^{-1/2} \mathrm{e}^{-(\alpha + \mathrm{i}S)t} \rho^{QM}(t,E)$$

which is the Fourier transform of the quantum density of states:

$$\rho^{QM}(t,E) = \sum_{0}^{\infty} \delta(E - E_n(t)) ,$$

with t denoting $\hbar^{-1}$.

It can be seen that the two curves are almost identical. It indicates the validity of Berry-Tablor trace formula.

In a word, the two-dimensional uncoupled quartic oscillator is calculated. The good agreement between the semiclassical action function and the quantum action function for the integrable system shows both the validity of the trace formula and the correspondence between quantum states and classical periodic orbits. But the concrete correspondence rule is still a problem unsolved.

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可积系统——求迹公式和 $\hbar$ 逆谱分析*

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摘 要: 研究了二维无关联四次振子系统, 有理环面上积分 Hamiltonian 运动方程给出了系统一系列周期轨道和经典物理量, 使用半经典近似下的 Berry-Tabor 求迹公式, 得到了半经典的态密度. 应用 Fourier 变换分析了每条周期轨道对态密度的贡献, 并与量子态密度的 Fourier 变换结果比较证实了半经典求迹公式的有效性.

关 键 词: 可积系统; 求迹公式; $\hbar$ 逆谱分析

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