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Effects of Skyrme Tensor Interactions on the β Decay Half-lives in Possible Waiting Point Nuclei

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Abstract: The β^- decay half-lives of $N \sim 80$ and 126 isotonic chains are calculated with HFB+QRPA models based on Skyrme force. In the calculations, the well constrained Skyrme tensor interaction and isoscalar spin-triplet (IS) pairing interaction are included so that to study their effects on the half-lives. The effects of tensor interaction and IS pairing interaction on the half-lives are compared. The IS pairing interaction with strength similar to that of isovector (IV) one produces only a trivial effect in $N \sim 82$ nuclei, and $N \sim 126$ nuclei with big neutron excess. While the tensor interaction with presently constrained strengths produces an obvious effect.

Key words: β decay half-life; tensor interaction; isoscalar pairing interaction

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1 Introduction

The synthesis of heavy elements in the universe is one of the most important topics in modern physics, which account for the isotopic abundances. In this synthesis, the astrophysical r-process gives rise to heavy elements beyond iron in our solar system. While the β-decay rate is an important ingredient in the simulation of the astrophysical r-process. Unfortunately, part of the nuclei on the r-process path are still out of the reach of experiment. It is up to theoretical calculations to evaluate the rates for many neutron-rich nuclei.

Recently, many theoretical efforts have been paid to evaluate the β decay half-lives of nuclei relevant to the r-process path. In Refs. [1, 2] the shell model was used, and very good results were provided. While in Hartree-Fock-Bogliubov (HFB) plus Quasiparticle-Random-Phase-Approximation (QRPA), different kinds of nucleon-nucleon forces are applied. In Refs. [3–5], the realistic interactions were adopted in QRPA calculation. In Ref. [6], the self-consistent relativistic HFB+QRPA with meson-exchange interactions was employed. In Ref. [7] the HF+RPA models based on effective Skyrme force were applied. Very recently, the non-relativistic HFB plus finite-amplitude-method (FAM)-QRPA calculations based on Skyrme interactions have been done to produce a global description of β^- decay in even-even nuclei with axially deformation taken into account[8]. However, the calculations of β decay half-life have never been a closed question, due to some uncertainties in the above models.

What are the factors which may produce obvious strong effects on the β decay half-life in QRPA calculation? The IS pairing interaction might be the most involved factor, since the residual central force usually push the excited states upwards to the high energy region, one need to take the IS pairing into account in particle-particle channel so that to draw the excited states downwards to low energy region and induce the β decay[9, 10]. But the problem existed is that there has been no widely accepted IS pairing strength, and different group use different IS pairing strength. The second one might be the tensor interaction, which might shift the low energy excited states, especially the Gamow-Teller (GT) states obviously upwards or downwards, and produce strong effect on the β decay half-life[7, 11–13]. The third one might be the reasons

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beyond QRPA model, such as the pariticle vibration couppling, or quasi-particle vibration coupling, which also shifts the low energy states downwards[14].

In this paper, we will apply the HFB+pnQRPA model based on Skyrme force to study the effects of Skyrme tensor interaction together with the IS pairing on the β decay half-life in the possible r-process path for nucleis with $N\sim$82, and 126. And this paper also aims to demonstrate to what an extent the two interactions will affect the β decay half-life . In Sec. 2, a brief description on QRPA models with canonical basis and the formula about GT and First-Forbidden (FF) transitions will be given. In Sec. 3, the parameters of tensor interactions and IS pairing will be discussed. In Sec. 4, the results and discussions will be presented. The summary is given in Sec. 5.

2 Formalism

As explained in the introduction, we aim to compare the effects of tensor interaction and IS pairing interaction on the β decay half-lives of nuclei in the r-process path.The zero-range two-body tensor inteactions we employed was originally proposed by Skyrme[15, 16]:

$$V^T = \frac{T}{2}\left\{[(\sigma_1\cdot\boldsymbol{k}')(\sigma_2\cdot\boldsymbol{k}')-\frac{1}{3}(\sigma_1\cdot\sigma_2)\boldsymbol{k}'^2]\delta(r)+ \delta(\mathbf{r})[(\sigma_1\cdot\boldsymbol{k})(\sigma_2\cdot\boldsymbol{k})-\frac{1}{3}(\sigma_1\cdot\sigma_2)\boldsymbol{k}^2]\right\}+ \frac{U}{2}\left\{(\sigma_1\cdot\boldsymbol{k}')\delta(\boldsymbol{r})(\sigma_2\cdot\boldsymbol{k})+(\sigma_2\cdot\boldsymbol{k}')\delta(\boldsymbol{r})(\sigma_1\cdot\boldsymbol{k})- \frac{2}{3}[(\sigma_1\cdot\sigma_2)\boldsymbol{k}'\cdot\delta(\boldsymbol{r})\boldsymbol{k}]\right\}. \tag{1}$$

The parameters T and U denote the strengths of triplet-even (TE) and triplet-odd (TO) tensor terms, respectively. And the density-dependent, contact (*i.e.*, zero-range) surface pairing interactions[17, 18] are

$$V_{IV}(\boldsymbol{r}_1,\boldsymbol{r}_2)=V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\boldsymbol{r})}{\rho_o}\right)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2), \tag{2}$$

$$V_{IS}(\boldsymbol{r}_1,\boldsymbol{r}_2)=fV_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\boldsymbol{r})}{\rho_o}\right)\delta(\boldsymbol{r}_1,\boldsymbol{r}_2), \tag{3}$$

where $\boldsymbol{r}=(\boldsymbol{r}_1-\boldsymbol{r}_2)/2$, ρ_0 is taken to be ρ_0=0.16 fm^{-3}, and P_σ is the spin exchange operator. Since the IS pairing strength is not yet well constrained, we take f as a free parameter.

We start the calculation with HFB to solve the HFB equation in coordinate-space[19, 20]. The Skyrme force applied is SKO′[21], which adopted the s-wave time-odd Landau-Migdal parameter g_0' to be 0.79, and on this interaction, the tensor interaction was added perturbatively[22] with constraints come from the GT and SD main peak energies in ^{90}Zr and ^{208}Pb, together with the energy differences between $1h11/2$ and $1g7/2$ single-proton states along the Z=50 isotopes[23]. In HFB, the maximum angular momentum $J_{\max}=19/2$, for $N\sim 126$ nucleis, and $J_{\max}$ =15/2 for other nuclei, respectively. And the maximum quasi-particle energy cut-off 180 MeV, are taken. In HFB, the IS pairing is not included as long as the proton-neutron mixing is neglected, and the strength of the IV pairing is separately determined to reproduce the odd-even mass staggering of proton and neutron, *i.e.* we actually use different IV pairing strength $V_{0\rm p}$ and $V_{0\rm n}$, respectively for proton and neutron so that to get better fit.

After HFB calculations, in order to obtain the canonical basis, we diagonalize the density matrix by using an intermediate basis, that is, by orthonormalizing the set of functions $\{\phi_1^k+\phi_2^k\}$ for each $(l,\ j)$ and charge, ϕ_1^k and ϕ_2^k being the upper and lower components of the quasi-particle state k[24]. After the canonical basis is obtained, we solve the QRPA equation:

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}=E_{\rm QRPA}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{4}$$

in which,

$$A_{\rm pn,p'n'}=E_{\rm pp'}\delta_{\rm nn'}+E_{\rm nn'}\delta_{\rm pp'}+ V^{\rm ph}_{\rm pn,p'n'}(u_{\rm p}v_{\rm n}u_{\rm p'}v_{\rm n'}+v_{\rm p}u_{\rm n}v_{\rm p'}u_{\rm n'})+ V^{\rm pp}_{\rm pn,p'n'}(u_{\rm p}u_{\rm n}u_{\rm p'}u_{\rm n'}+v_{\rm p}v_{\rm n}v_{\rm p'}v_{\rm n'})\ , \tag{5}$$

$$B_{\rm pn,p'n'}=V^{\rm ph}_{\rm pn,p'n'}(v_{\rm p}u_{\rm n}u_{\rm p'}v_{\rm n'}+u_{\rm p}v_{\rm n}v_{\rm p'}u_{\rm n'})- V^{\rm pp}_{\rm pn,p'n'}(u_{\rm p}u_{\rm n}v_{\rm p'}v_{\rm n'}+v_{\rm p}v_{\rm n}u_{\rm p'}u_{\rm n'})\ . \tag{6}$$

In p-p channel, both T=0 and T=1 pairing interactions are included, with $V_0=(V_{0\rm n}+V_{0\rm p})/2$ in our calculations. After diagonalizing the QRPA matrix, transition amplitude in t_- channel with the operator $\hat{O}$ can be calculated by

$$B_-^\nu=-\sum_{\rm pn}(X^\nu_{\rm pn}u_{\rm p}v_{\rm n}+Y^\nu_{\rm pn}v_{\rm p}u_{\rm n})\langle{\rm p}||\hat{O}_-||{\rm n}\rangle\ , \tag{7}$$

The β^- decay rates λ as well as the partial half-lives $t_{1/2}$ of the transitions are defined by the following formulas[28–31]:

$$\lambda=\ln 2/t_{1/2}=f/8896({\rm s}^{-1}), \tag{8}$$

$$f=\int_1^{w_0}C(w)F(Z,w)pw(w_0-w)^2{\rm d}w, \tag{9}$$

$$C(w)=k+Kaw+kb/w+kcw^2, \tag{10}$$

where w is the electron energy in units of the electron mass, w_0 is the maximum electron energy which can be received from the decay in units of the electron mass, $F(Z,w)$ is the Fermi function as expressed in Ref. [28], and k, ka, kb, and kc are the nuclear matrix elements depending on the nuclear structure[1, 2].

3 Parametrization of the tensor interaction and IS pairing

The IS pairing p-p interaction was the most concerned interaction in the previous calculations of β decay half-life. But due to the fact that $T=0$ boson like proton-neutron pairing is very difficult to be observed, the IS pairing strength is still not well constrained, hence different IS pairing strengths are applied in different works. One obvious constraint might exist in the low-energy super GT states observed in $N=Z+2$ pf nuclei, especially in ^{42}Ca, ^{46}Ti, and ^{50}Cr, where the IS pairing strength plays a dominant role to form the low-energy super GT states[25, 26], which provides a constraint that $f=1.0\sim1.05$[18].

Moreover, the tensor interaction was reported to has a strong effect on the β decay for closed-shell nuclei, since the tensor interaction produces strong effect on the low energy GT states. And in Ref. [8], the tensor interaction was included in the FRM-QRPA models to do a global calculations in even-even nuclei. In which, the tensor interaction was very carefully fitted by GT and SD main peak energies and β decay half-lives in some chosen nuclei. In our present work, the tensor interaction are constrained by the GT and SD main peak energies as was done in Ref. [27], together with the energy differences between $1h11/2$ and $1g7/2$ single-proton states along the $Z=50$ isotopes[23]. As one can see in Ref. [22], since SKO′ is a very good interaction for spin-isospin excitations with reasonable g_0' value, the acceptable range of tensor interaction is rather wide, while the strong constraint comes from the single-particle energies. The constrained acceptable strength area for TE and TO tensor interaction is presented in Fig. 1[22].

As a result, the strength of TO tensor term is constrained to varies from −350 to −270 MeVfm^5, and that of TE tensor term varies from about 270 to 600 MeVfm^5. In this section, we choose (T,U) = (500.00, −320.00)MeVfm^5, which is close to the center of the acceptable region in our study. We have also checked the effect of this tensor interaction on the single-particle energies in ^{48}Ca and ^{208}Pb, together with the binding energies in ^{48}Ca, ^{56}Ni, ^{90}Zr, and ^{208}Pb, which produce some improvements.

4 Results and discussion

In this section, we present the β decay half-lives for nuclei in $N\sim82$, and 126 isotonic chains calculated by HFB+QRPA models, in which many waiting point nuclei in r-process path are included. In the calculations, the Skyrme interaction SKO′ was employed, the strength of IV pairing are about 500 MeVfm^5, and the strength of IS pairing interaction are chosen with f=0.90 and 1.15 just for studying its effect on the half-life. In addition, the strength of TE and TO tensor interaction are adopted to be $(T,U)=(500,-320)$ MeVfm^5, which lies in the center of the acceptable area in Fig. 1.

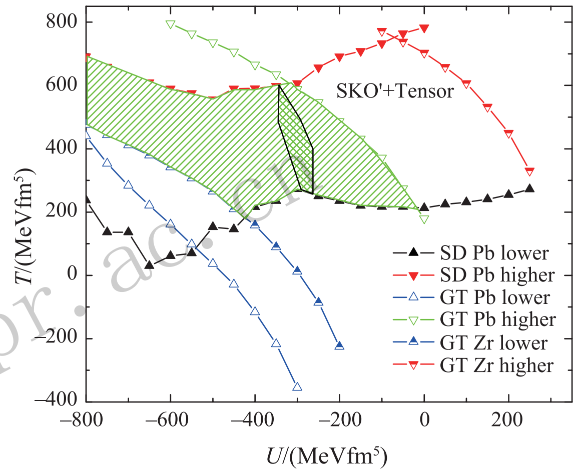

Fig. 1 (color online) The strengths of TE and TO tensor terms constrained by the GT and charge-exchange SD main peak energies in ^{90}Zr and ^{208}Pb, together with the energy differences between $1h11/2$ and $1g7/2$ single-proton states along the $Z=50$ isotopes[23]. Figure cited from Ref. [22].

The β decay half-lives for nuclei in $N=82$ isotonic chain are presented in Fig. 2. For $N=82$ isotonic chain, the tensor interactions obviously decrease the decay half-lives, and better reproduction for the experimental results is attained, which is as expected as the discussion in Ref. [7]. Moreover, it is quite unexpected that the changing of IS pairing strength by about 20% around the strength of the corresponding IV pairing makes almost no change, compared with that produced by the tensor interaction. The calculations for $N=80$ and 82 isotonic chains are also done, and the similar results are produced except for ^{130}Sn and ^{134}Sn, where the tensor interactions increase the half-lives, which might be due to that the tensor interaction may shift the low energy GT states upwards , as was shown in Ref. [11] for the case of ^{90}Zr.

The β decay half-lives for nuclei in $N=126$ isotonic chain are presented in Fig. 3. The effect produced by the tensor interaction in the $N\sim126$ isotonic chains is weaker than that in $N\sim 82$ chains. We have also done the same calculations for the $N=124$ and 128 isotonic chains. The tensor interactions may increase half-lives in the region close to the β stability line in the chains. But in the region far away from the β stability, the tensor interaction certainly decrease the half-lives. In addition, the effect of IS pairing was only appeared in the region close to the β stability.

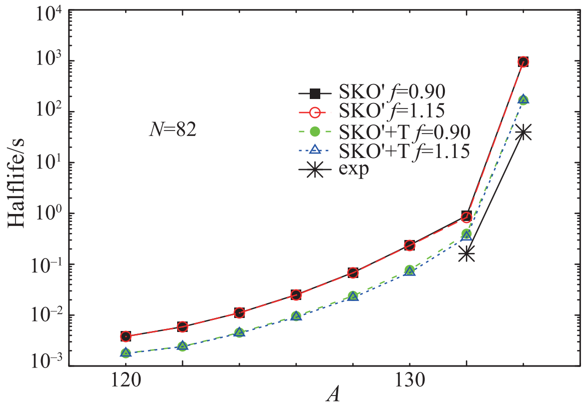

Fig. 2　(color online) The β decay half lives for $N=82$ chain, calculated by HFB+QRPA with SKO′. SKO′ labels the results calculated without including tensor interaction, while SKO′+T labels the results calculated with tensor interaction $(T,U)=(500,-320)$ MeVfm^5. In the calculations, IS pairing strength is chosen to be $f=0.90$ and 1.15 times of V_0.

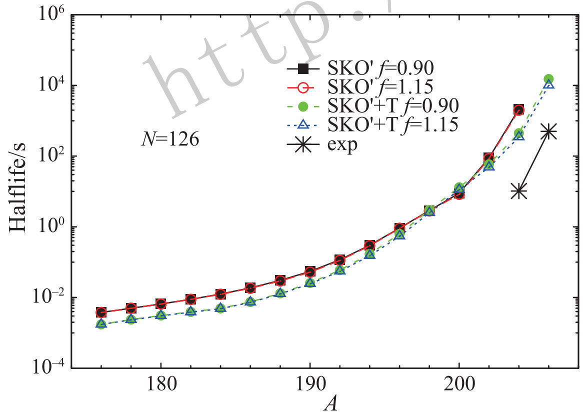

Fig. 3　(color online) Same as Fig. 2, for $N=126$ isotonic chain.

5　Summary

The effects of Skyrme tensor interaction and surface δ IS pairing on the β decay half-lives in $N\sim 82$, and 126 isotonic chains are studied by the HFB+QRPA models. The tensor interaction determinately decrease the half-lives in the nuclei with big neutron rich, while it might increase the half-lives in the region close to the β stability line. Furthermore, we changed the IS pairing strength by about 20% around the strength of corresponding IV pairing, and the results show that this changing can make some small changes only in the region close to the β stability line, which means the IS pairing with strength similar to the IV pairing produces only negligible effect on the half-lives and it only make small differences in the region close to β stability. Therefore, the tensor interactions play a very important role in the half-lives in the neutron rich region, and the strength of TE and TO tensor interaction should be well constrained.

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Skyrme 张量相互作用对可能的等待点原子核 β 衰变半衰期的影响

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摘要： 使用基于Skyrme相互作用的HFB+QRPA模型研究 $N\sim82$ 和126的同中子异位素链的 β^- 衰变的半衰期。在计算中所使用的张量相互作用和同位旋标量(IS)对相互作用都是很好地被约束了的。比较了张量相互作用和IS对相互作用对半衰期的影响。IS对相互作用的强度与相应的同位旋矢量(IV)对相互作用的强度相当时，对 $N\sim82$ 和126的同中子异位素链中有大的中子过剩的原子核的半衰期影响很微弱。而张量相互作用采用最近约束的强度时对半衰期的影响非常显著。

关键词： β 衰变半衰期；张量相互作用；同位旋标量对相互作用

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