Article ID: 1007-4627(2017) 01-0016-04

Effect of Fermion Velocity on Fermion Chiral Condensate in QED_3 at Finite Temperature

LI Jianfeng

(*Department of Physics, Nantong University, Nantong 226019, Jiangsu, China*)

Abstract: Analogous to Quantum QCD, QED_3 has two interesting features: dynamical chiral symmetry breaking (DCSB) and confinement. By adopting the rainbow approximation, we numerically solve the fermion self-energy equation at finite temperature in the framework of Dyson - Schwinger equations and discuss the relation between chiral condensate and fermion flavor for several fermion velocities in the finite temperature QED_3. It is found that the fermion chiral condensate decreases monotonically with the fermion velocity increasing for a fixed N at finite temperature.

Key words: QED_3; fermion velocity; fermion chiral condensate

CLC number: O413.1　　**Document code:** A　　**DOI:** 10.11804/NuclPhysRev.34.01.016

1 Introduction

Quantum electrodynamics in two space and one time dimensions(QED_3) has been studied widely in the past. QED_3 displays a rich structure of nonperturbative phenomena such as dynamical chiral symmetry breaking (DCSB) in chiral limit and confinement[1–6]. The fermions are gapped and confined out of the physical spectrum when DCSB takes place. Moreover, QED3 is superrenormalizable and never suffer from the ultraviolet divergence. Therefore QED_3 can be regarded as a toy model of Quantum Chromodynamics (QCD). It is well known that there is a critical N_c up to which there is dynamical fermion mass generation due to DCSB. In addition, QED_3 model with $N=2$ has be employed as an effective field theories for 2D condensed matter systems, including high temperature superconductors[7–13] and graphene[14, 15].

DCSB occurs when the massless fermion acquires a dynamical mass through nonperturbative effects at low energy, but the Lagrangian keeps chiral symmetry[16]. The Dyson-Schwinger equation is a very powerful tool to study nonperturbative phenomena. Appelquist first studied DCSB in massless QED_3 with N fermion flavors by solving the Dyson-Schwinger equation in the lowest-order of $1/N$ expansion, and found that a fermion mass was dynamically generated when N is less than a critical number $N_c = 32/\pi^2$[17]. Later, Nash found that the gauge-invariant critical number of fermion flavor still exists by considering higher order corrections to the gap equation[18]. At finite temperature, Aitchison *et al.*[19] found that N_c is temperature-dependent and the chiral symmetry can be restored above T_c. However, the effect of fermion velocity on DCSB have not been discussed in their papers. Recently, the fermion velocity is found to play a important role for the non-Fermi liquid behaviors in high temperature superconductors. Thus the exact value of N_c is of some physical interest. In this paper, we study the effect of the fermion velocity on chiral condensate in thermal QED_3.

2 Dynamical gap in QED_3 at finite temperature

In Euclidean space, the Lagrangian of QED_3 with N massless fermion flavors is

$$\mathcal{L} = \sum_{\sigma=1}^{N} \bar{\psi}_\sigma v_{\sigma,\mu}(\mathrm{i}\partial_\mu - eA_\mu)\gamma^\mu \psi^\sigma + \frac{1}{4}F_{\rho\nu}^2 , \qquad (1)$$

where the spinor ψ_i is a set of N 4-component fermion fields, $v_{\sigma,1}$ is fermi velocity, and $v_{\sigma,2}$ is related to the amplitude of the superconducting order parameter. In general $v_{\sigma,1} \neq v_{\sigma,2}$. It was reported that the fermionic anisotropy $\frac{v_{\sigma,1}}{v_{\sigma,2}} = 14$ for optimally doped $YBa_2Cu_3O_7$ compound[20]. However, for simplicity, it is assumed that $v_{\sigma,1} = v_{\sigma,2} = c = 1$ in previous papers. In this paper, we neglect velocity anisotropy and

Received date: 21 Sep. 2016

Foundation item: National Natural Science Foundation of China (11105112, 11347212)

Biography: LI Jianfeng(1978–), male, Nantong, Jiangsu, Associate Professor, working on quantum field theory;
E-mail: ljf169@ntu.edu.cn.

http://www.npr.ac.cn

set $v_{\sigma,1} = v_{\sigma,2} = v_{\mathrm{F}}$. In rainbow approximation, we obtain the Dyson-Schwinger equation for the fermion propagator:

$$G^{-1}(p) = G_0^{-1}(p) - \frac{\alpha}{N}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\gamma^{\rho} G(k)\gamma^{\nu} D_{\rho\nu}(q) \ , \quad (2)$$

where $q = p–k$ and $\alpha = e^2 N$. The free propagator of the massless Dirac fermion is $G_0(p_0, \boldsymbol{p}) = (\gamma_0 p_0 - v_{\mathrm{F}}\gamma\cdot\boldsymbol{p})^{-1}$ and the full fermion propagator is:

$$G(p) = G(p_0, \boldsymbol{p}) = (\gamma_0 p_0 A_0(p) - v_{\mathrm{F}}\gamma\cdot\boldsymbol{p}\boldsymbol{A}(p) - B(p))^{-1} \ , \quad (3)$$

where $A(p)$ is the wave function renormalization factor and $B(p)$ is fermion self-energy. At finite T, by adopting a similar strategy as the authors of Ref. [21] and then the boson propagator is

$$D_{\mu\nu}(q_0, \boldsymbol{q}) = \frac{A_{\mu\nu}}{\frac{q^2}{v_{\mathrm{F}}^2} + \boldsymbol{q}^2 + \Pi_A(q_0, \boldsymbol{q})} + \frac{B_{\mu\nu}}{\frac{q^2}{v_{\mathrm{F}}^2} + \boldsymbol{q}^2 + \Pi_B(q_0, \boldsymbol{q})} \ , \quad (4)$$

the tensor of boson polarization can be decomposed in terms of two independent tensors

$$\Pi_{\mu\nu}(q_0, \boldsymbol{q}) = \Pi_A(q_0, \boldsymbol{q}) A_{\mu\nu} + \Pi_B(q_0, \boldsymbol{q}) B_{\mu\nu} \quad (5)$$

where

$$A_{\mu\nu} = \left(\delta_{\mu 0} - \frac{q_\rho q_0}{q^2}\right)\frac{q^2}{v_{\mathrm{F}}^2 \boldsymbol{q}^2}\left(\delta_{0\nu} - \frac{q_0 q_\nu}{q^2}\right) \quad (6)$$

and

$$B_{\mu\nu} = \delta_{\mu i}\left(\delta_{ij} - \frac{q_i q_j}{\boldsymbol{q}^2}\right)\delta_{j\nu} \ , \quad (7)$$

They are orthogonal and related by the relationship

$$A_{\mu\nu} + B_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \ . \quad (8)$$

The value of boson polarization Π_A and Π_B are related with the boson polarization tensor $\Pi_{\mu\nu}$ by the following

$$\Pi_A = \frac{q^2}{\boldsymbol{q}^2}\Pi_{00}, \ \Pi_B = \Pi_{ij}\left(D_{ij} - \frac{q_i q_j}{q^2}\right) \ . \quad (9)$$

A number of different approximations have been proposed to study the dynamical gap at finite temperature. The most frequently used one is the so-called instantaneous approximation[21], which remain only the $\mu = \nu = 0$ component of the gauge boson propagator at zero frequency and thus obtains the simplified boson propagator

$$D_{\mu\nu}(q_0, \boldsymbol{q}, \beta) = \frac{\delta_{\mu 0}\delta_{\nu 0}}{v_{\mathrm{F}}^2[\boldsymbol{q}^2 + \Pi_A(\boldsymbol{q}, \beta)]} \ . \quad (10)$$

where

$$\Pi_A(\beta, \boldsymbol{q}) = \frac{16\alpha}{\pi\beta}\int_0^1 \mathrm{d}x \ln\left\{2\cosh\left[\frac{1}{2}v_{\mathrm{F}}|\boldsymbol{q}|\beta\sqrt{x(1-x)}\right]\right\} \ . \quad (11)$$

It is very convenient to adopt an excellent analytic approximation[19, 21] to Eq. (11), so the polarization is provided by the expression

$$\Pi_A(\boldsymbol{q}, \beta) = \frac{\alpha}{8 v_{\mathrm{F}}^2}\left[v_{\mathrm{F}}|\boldsymbol{q}| + \frac{16\ln 2}{\pi\beta}\exp\left(-\frac{\pi\beta}{16\ln 2}v_{\mathrm{F}}|\boldsymbol{q}|\right)\right] \ . \quad (12)$$

Eq.(12) turns to the form in Ref. [19] when one sets the $v_{\mathrm{F}} = c = 1$. It is convenient to keep fixed the dimensionful quantity $\alpha \equiv Ne^2 = 1$ and thus every quantity with mass dimension is scaled and becomes dimensionless. We finally obtains the temperature- and momentum-dependent gap function

$$B(\beta, P) = \frac{\alpha}{8N\pi^2}\int \mathrm{d}^2 K \frac{B(\beta, K)}{\sqrt{K^2 + B^2(\beta, K)} \times [v_{\mathrm{F}}^2 Q^2 + v_{\mathrm{F}}^2 \Pi_0(\beta, Q)]} \times \tanh\left[\frac{\beta}{2}\sqrt{K^2 + B^2(\beta, K)}\right] \ , \quad (13)$$

where $Q = |\boldsymbol{q}| = |\boldsymbol{p} - \boldsymbol{k}|$.

3 Results and discussion

If Eq. (13) has a nontrivial solution(Nambu solution), which implies a nonzero condensate and signals dynamical mass generation. It is well known that the fermion chiral condensate is the order parameter for dynamical chiral transition. The fermion chiral condensate at zero temperature is given by the integral

$$<\bar{\psi}\psi> = 4\int \frac{\mathrm{d}^3 k}{(2\pi)^3} Re\left\{\frac{B(k^2)}{A^2(k^2)k^2 + B^2(k^2)}\right\} \ , \quad (14)$$

when one set the function renormalization factor $A(p^2) = 1$ and the fermion chiral condensate at nonzero temperature is derived by

$$\langle\bar{\psi}\psi\rangle = 2\int \frac{\mathrm{d}^2 P}{(2\pi)^2}\frac{B(P^2)}{\sqrt{P^2 + B^2(P^2)}}\tanh\frac{\sqrt{P^2 + B^2(P^2)}}{2T} \ , \quad (15)$$

we obtain the relation between chiral condensate and fermion flavor by employing the iteration algorithm to numerically solve Eq. (13). This is shown in Fig. 1 where it is easy to see that the fermion chiral condensate is suppressed with the fermion flavor N increasing for a fixed T and v_{F}. It is found that the fermion chiral condensate will disappear when N exceeds a critical fermion flavor N_{c} for the same fermi velocity, which agree with the result at zero fermi velocity[21]. Our result show that the fermion chiral condensate increases

http://www.npr.ac.cn

monotonically with the fermion velocity decreasing for a fixed N at finite temperature.On the other hand, it is found that the less fermi velocity $v_{\rm F}$ is, the more critical fermion flavor $N_{\rm c}$ will be.

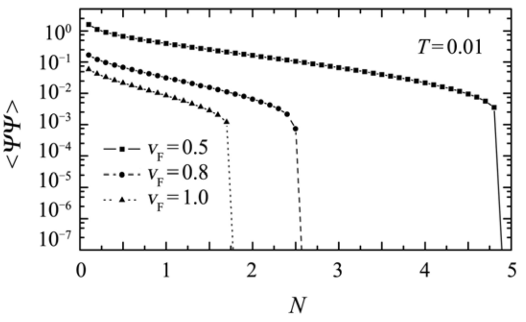

Fig. 1 The dependence of chiral condensate on the fermion flavor for several different values of fermion velocities.

So far, we only focus on the situation of isotropy $\rm QED_3$ from the perspective of Dyson-Schwinger equation. Recently the impact of anisotropic fermion velocities in $\rm QED_3$ on dynamical mass generation has been studied. The authors of Ref. [22, 23] found that the weak velocity anisotropy never change the value of $N_{\rm c}$. If one take the $v_{\sigma,1} \to 0$, the system show the strong velocity anisotropy. The extreme anisotropic $\rm QED_3$ become similar to the $\rm QED_2$ (Schwinger model[24]) where the gauge field obtain a finite mass. However, there is no DCSB in Schwinger model, which relies on the fact that any dynamically broken symmetry must produce Goldstone modes, *i.e.* massless bosons, and then establishes that Goldstone bosons cannot exist in $\rm QED_2$. On the other hand, it is indicated that the gauge field will suppress the DCSB and alter the critical $N_{\rm c}$[10, 13, 25]. One should expect qualitatively different physics in the extreme anisotropic $\rm QED_3$.

4 Conclusions

In this work we numerically solve the coupled fermion self energy equation and calculate the chiral condensate at finite temperature. We find that the critical $N_{\rm c}$ increases with the fermion velocity decreasing. Numerical result also show that the fermion condensate enlarge with the fermion velocity decreasing for a fixed N in thermal $\rm QED_3$. We hope that our results will contribute to better understand the connection between $\rm QED_3$ model and high-temperature superconductor.

References:

[1] PISARSKI R D. Phys Rev D, 1984, **29**: 2423.
[2] BURDEN C J, PRASCHIFKA J, ROBERTS C D. Phys Rev D, 1992, **46**: 2695.
[3] MARIS P. Phys Rev D, 1995, **52**: 6087.
[4] MARIS P. Phys Rev D, 1996, **54**: 4049.
[5] BASHIR A, RAYA A, CLOET I C, *et al.* Phys Rev C, 2008, **78**: 055201.
[6] FENG H T, WANG B, SUN W M, *et al.* Phys Rev D, 2012, **86**: 105042.
[7] RANTNER W, WEN X G. Phys Rev Lett, 2001, **86**: 3871.
[8] RANTNER W, WEN X G. Phys Rev B, 2002, **66**: 144501.
[9] FRANZ M, TESANOVIC Z. Phys Rev Lett, 2001, **87**: 257003.
[10] LIU G Z, CHENG G. Phys Rev D, 2003, **67**: 065010.
[11] HERBUT I F. Phys Rev Lett, 2002, **88**: 047006.
[12] HERBUT I F. Phys Rev B, 2002, **66**: 094504.
[13] LEE P A, NAGAOSA N, WEN X G. Rev Mod Phys, 2006, **78**: 17.
[14] KHVESHCHENKO D V. Phys Rev Lett, 2001, **87**: 246802.
[15] KHVESHCHENKO D V, LEAL H. Nucl Phys B, 2004, **687**: 323.
[16] NAMBU Y, LASINIO G J. Phys Rev, 1961, **122**: 345.
[17] APPELQUIST T W, BOWICK M, KARABALI D, *et al.* Phys Rev D, 1986, **33**: 3704.
[18] NASH D. Phys Rev Lett, 1989, **62**: 3024.
[19] AITCHISON I J R, DOREY N, KLEIN-KREISLER M, *et al.* Phys Lett B, 1992, **294**: 91.
[20] CHIAO M, HILL R W, LUPIEN C, *et al.* Phys Rev B, 2000, **62**: 3554.
[21] DOREY N, MAVROMATOS N E. Nucl Phys B, 1992, **386**: 614.
[22] CONCHA A, STANEV V, TESANOVIC Z, Phys Rev B, 2009, **79**: 214525.
[23] LEE D J, HERBUT I F. Phys Rev B, 2002, **66**: 094512.
[24] SCHWINGER J. Phys Rev, 1962, **128**: 2425.
[25] LI J F, HOU F Y, CUI Z F, *et al.* Phys Rev D, 2014, **90**: 073013.

http://www.npr.ac.cn

有限温下三维 QED 中费米速度对费米子手征凝聚的影响

李剑锋[1)]

(南通大学理学院，江苏 南通　226019)

摘要： 三维 QED 具有两个和 QCD 类似的性质：动力学手征对称破缺和禁闭。为了研究动力学手征对称破缺，基于彩虹近似，在 Dyson - Schwinger 方程框架下，通过迭代求解有限温下的费米子自能方程，讨论了不同的费米速度下费米子手征凝聚与费米子味数之间的关系。发现在有限温下，对于固定的费米子味数，费米手征凝聚随费米速度的变大而单调减小。

关键词： 三维量子电动力学；费米速度；费米子手征凝聚

http://www.npr.ac.cn

收稿日期： 2016-09-21

基金项目： 国家自然科学基金资助项目(11105112, 11347212)

1)　E-mail: ljf169@ntu.edu.cn。