第 33 卷 第 2 期 原 子 核 物 理 评 论 Vol. 33, No. 2
2016 年 6 月 Nuclear Physics Review June, 2016

Article ID: 1007-4627(2016) 02-0167-06

Proton Radioactivity Within a Hybrid Method

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Abstract: The proton radioactivity half-lives are investigated theoretically within a hybrid method. The potential barriers preventing the emission of protons are determined in the quasimolecular shape path within a generalized liquid drop model (GLDM). The penetrability is calculated with the Wentzel-Kramers-Brillouin (WKB) approximation. The spectroscopic factor has been taken into account in half-life calculation, which is obtained by employing the relativistic mean field (RMF) theory combined with the Bardeen-Cooper-Schrieffer (BCS) method. The half-lives within the present hybrid method reproduced the experimental data very well. Some predictions for proton radioactivity are made for future experiments.

Key words: proton radioactivity; generalized liquid drop model; WKB approximation; BCS method; relativistic mean field theory

CLC number: O571.3 **Document code:** A **DOI:** 10.11804/NuclPhysRev.33.02.167

1 Introduction

Proton radioactivity was first observed in an isomeric state of ^{53}Co[1] by the reaction ^{16}O(^{40}Ca, p2n)^{53}Co and confirmed by different nuclear reaction ^{54}Fe(p, n)^{53}Co[2] in 1970. The ground-state proton radioactivity was observed by the reaction ^{96}Ru(^{58}Ni, p2n)^{151}Lu up to 1982[3]. Two-proton radioactivity was reported in the new century[4]. With the development of experimental facilities and radioactive beams, proton emissions from ground state or low lying isomeric states have been identified and more proton-emitting nuclei will be observed in future[5]. Proton drip line represents one of the fundamental limits of the existence of nuclei and the nucleus with a large excess of protons undergoes spontaneous proton emission towards stability. This rapid proton capture process which plays a very important role in nuclear astrophysics is the inverse reaction of proton radioactivity from the nuclear ground state or low isomeric states. The discovery of new isotopes expands the chart of nuclei. It is the first step necessary to explore and understand new nuclei. Therefore the study of the proton emission will shed the new light on nuclear physics.

Several approaches have been employed to study the half-lives of proton emitters, such as the distorted-wave Born approximation[6], the coupled-channels approach[7–9], the density-dependent M3Y effective interaction[10–11] which was based on the G-matrix elements of the Reid-Elliott nucleon-nucleon potential, the effective interaction of Jeukenne, Lejeune and Mahaux (JLM)[11], and the unified fission model[12]. However, the calculations in Ref. [10–12] did not take account the spectroscopic factor which is very important from the viewpoint of the nuclear structure. In this study, we calculate the spectroscopic factor by employing the relativistic mean field (RMF) model in combination with the Bardeen-Cooper-Schrieffer (BCS) method as proposed by Delion[7], then we use a quantum mechanical method, including the structure of the parent nucleus to estimate the assault frequency, at last we determine the partial proton emission half-lives of proton emitters using Wentzel-Kramers-Brillouin (WKB) penetrability through the potential barrier constructed by the generalized liquid drop model (GLDM). The calculations are consistent with the experimental data and other theoretical results. The purpose of this paper is to summarize the main results reported earlier on one-proton radioactivity[13], give some new predictions and outlook about the next work related to the development of the hybrid method and the potential applications including proton radioactivity.

Received date: 15 Apr. 2016; **Revised date:** 20 May 2016
Foundation item: National Natural Science Foundation of China(11175074, 11475050, 11265013); Knowledge Innovation Project of Chinese Academy of Sciences(KJCX2-EW-N02)

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2 The hybrid method

In the present hybrid method for proton radioactivity, the decay constant is defined as

$$\lambda = S_p \nu_0 P \ , \tag{1}$$

where S_p is the spectroscopic factor, ν_0 is the assault frequency, and P is the proton penetration probability from tunneling through the potential barrier.

The spectroscopic factor comes from the spin and parity conservation laws: $J_{\mathrm{parent}} = J_{\mathrm{d}} + J_{\mathrm{p}} + l$ and $\pi_{\mathrm{parent}} = \pi_{\mathrm{d}} \pi_{\mathrm{p}} (-1)^l$, where l is the angular moment transferred by the emitted proton. It should be noted that in the case of α-decay the four nucleons get clustered on the nuclear surface through an elaborated interplay among the nucleons inside the parent nucleus, then the preformed cluster is ready for emission. For proton radioactivity, the preformation factor is not needed since there exist protons in the parent nucleus actually. Thus the spectroscopic factor becomes effective due to the spin and parity conservation laws. The decay constant due to an unified understanding of α-decay should be $\lambda = P_\alpha S_\alpha \nu_0 P$, where P_α is the preformation factor, S_α is the spectroscopic factor, ν_0 the assault frequency, and P is the α penetration probability. The hinderance factor should be $H = P_\alpha S_\alpha$ for α-decay. This concept can be extended for heavy cluster radioactivity.

For proton radioactivity, the spectroscopic factor can be estimated by $S_p = u_j^2$, where u_j^2 is the probability that the orbit of the emitted proton is empty in the daughter nucleus[6–7]. Fortunately, the daughter nuclei of spherical proton emitters are all in ground states[7]. Thus, it is relatively easy to determine this spectroscopic factor by RMF+BCS methods. The RMF theory is now a standard tool in low energy nuclear physics. It has received much attention due to its great success in describing the structure of the stable nuclei, neutron-rich nuclei[14], proton-rich nuclei[15], superdeformed nuclei[16] and superheavy nuclei[17–18]. In this work, the effective Lagrangian is written as following:

$$\begin{aligned}\mathcal{L} = & \overline{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - M)\psi + \\ & \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \left(\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4\right) - g_\sigma \overline{\psi}\psi\sigma - \\ & \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - g_\omega \overline{\psi}\gamma^\mu \psi \omega_\mu - \\ & \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - g_\rho \overline{\psi}\gamma^\mu \tau \psi \rho - \\ & \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\overline{\psi}\gamma^\mu \left(\frac{1+\tau_3}{2}\right)\psi A_\mu \ ,\end{aligned} \tag{2}$$

the nucleon field interacts with σ, ω, ρ meson fields and photon field. Self-coupling terms for the σ meson are introduced. The isospin dependence of the nuclear interaction is provided by the isovector ρ-meson. Varying the effective Lagrangian, one gets the Dirac equation for the nucleons and Klein-Gordon equations for the mesons. In the present calculation, an axially symmetric deformation has been assumed. The deformation of nuclei is described by the oscillator expansion method [19], which means that the basis of the Dirac equation and Klein-Gordon equations are taken as eigen functions of the axially symmetric harmonic oscillator. The major shell is chosen as $N_{\mathrm{f}} = 16$, $N_{\mathrm{b}} = 16$ for the nucleons and mesons, respectively. The pairing correlation is treated by the BCS method. The strength of the pairing force is adopted by the following forms for neutrons and protons, respectively[18, 20]:

$$G_{\mathrm{n}} = \frac{21}{A}\left(1 - \frac{N-Z}{2A}\right) \ \mathrm{MeV} \ , \tag{3}$$

$$G_{\mathrm{p}} = \frac{27}{A}\left(1 + \frac{N-Z}{2A}\right) \ \mathrm{MeV} \ , \tag{4}$$

which depend on the proton number Z and neutron number N. A is the total mass number. A blocking method is used to deal with the unpaired nucleons, the states of which are chosen in such a way that the total single particle energy is the minimum. If the state of the last odd nucleon is k with energy ε_k, the total single particle energy can be written as:

$$E_{\mathrm{part}} = \varepsilon_k + 2\sum_{i \neq k} \varepsilon_i v_i^2 - \varDelta \sum_{i \neq k} u_i v_i \ , \tag{5}$$

then the state k is determined by the variation of the total single particle energy E_{part} with respect to k. Therefore, the Fermi energy λ, pairing energy gap Δ as well as E_{part} all depend on the choice of the unpaired odd nucleon state k. The NL3 parameter set, which is able to provide a very good description not only for the properties of stable nuclei but also for those far from the valley of β stability[21–22], is used here. Unlike the situation near the neutron drip line, for proton-rich nuclei the Coulomb barrier confines the protons in the interior of the nucleus. As a consequence, the effects of the coupling to the continuum are weaker and therefore for nuclei close to proton drip line, the RMF+BCS model could still be considered as a reasonable approximation and can provide sufficiently accurate results[15].

We assume that the proton, which will be emitted, vibrates nearby the surface of the parent nucleus in a harmonic oscillator potential $V(r) = -V_0 + \frac{1}{2}\mu\omega^2 r^2$ with a classical frequency ω and the reduced mass μ. By employing the virial theorem, we obtain

$$\mu\omega^2\overline{r^2} = \left(2n_{\rm r}+\ell+\frac{3}{2}\right)\hbar\omega\ , \tag{6}$$

where n_r and the ℓ are the radial quantum number (corresponding to the number of nodes) and angular momentum quantum number, respectively. $<\psi|r^2|\psi>^{1/2}$ is the root mean square (rms) radius of the outermost proton distribution. It is assumed to be equal to the rms radius $R_{\rm n}$ of the nucleus here. We take the oscillation frequency ν_0 as the assault frequency, which is related to the oscillation frequency ω:

$$\nu_0 = \frac{\omega}{2\pi} = \frac{(2n_{\rm r}+\ell+\frac{3}{2})\hbar}{2\pi\mu R_{\rm n}^2} = \frac{(G+\frac{3}{2})\hbar}{1.2\pi\mu R_0^2}\ . \tag{7}$$

The relationship of $R_{\rm n}^2 = \frac{3}{5}R_0^2$ [23] is used here. $G = 2n_{\rm r}+\ell$ is the principal quantum number. For proton emission we choose $G = 4$ or 5 corresponding to 4 or 5 $\hbar\omega$ oscillator shell depending on the individual nucleus.

Gamov proposed in his seminal paper to explain the α-decay as a quantum process, thus starting the probabilistic interpretation of quantum mechanics as well as theoretical nuclear physics[24]. He conceived the α particle as a small ball moving in the parent nucleus which is bounded upon the nuclear surface, and eventually penetrates the surrounding Coulomb barrier. This picture is actually more suitable for proton emission than α-decay since the protons exist as building blocks inside the nuclear mean field. The penetration probability of the proton emitter is calculated using the WKB approximation,

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2B(r)(E(r)-E(sphere))}{\rm d}r\right]\ , \tag{8}$$

where $R_{\rm in}$ and $R_{\rm out}$ are the two turning points of the WKB action integral and mass inertia $B(r)$ is approximated to reduced mass μ. The barriers are constructed by the GLDM, which can be used to describe the processes of fusion, fission, light nucleus, α and proton emission [25–29] successfully. For a deformed nucleus, the macroscopic GLDM energy is defined as [25–29],

$$E = E_{\rm V}+E_{\rm S}+E_{\rm C}+E_{\rm Rot}+E_{\rm Prox}. \tag{9}$$

When the nuclei are separated:

$$E_{\rm V} = -15.494\left[(1-1.8I_1^2)A_1+(1-1.8I_2^2)A_2\right]\ {\rm MeV}\ , \tag{10}$$

$$E_{\rm S} = 17.9439\left[(1-2.6I_1^2)A_1^{2/3}+(1-2.6I_2^2)A_2^{2/3}\right]\ {\rm MeV}\ , \tag{11}$$

$$E_{\rm C} = 0.6e^2Z_1^2/R_1+0.6e^2Z_2^2/R_2+e^2Z_1Z_2/r\ , \tag{12}$$

where A_i, Z_i, R_i and S_i are the mass number, charge number, radii and relative neutron excesses of the two nuclei, respectively. r is the distance between the mass centres. The radii R_i are given by

$$R_i = (1.28A_i^{1/3}-0.76+0.8A_i^{-1/3})\ {\rm fm}\ . \tag{13}$$

This formula allows to follow the experimentally observed increase of the ratio $r_i = R_i/A_i^{1/3}$ with the mass; for example, $r_0 = 1.13$ fm for ^{48}Ca and $r_0 = 1.18$ fm for ^{248}Cm.

For one-body shapes, the surface and Coulomb energies are defined as:

$$E_{\rm S} = 17.9439(1-2.6I^2)A^{2/3}(S/4\pi R_0^2)\ {\rm MeV}, \tag{14}$$

$$E_{\rm C} = 0.6e^2(Z^2/R_0)\times 0.5\int(V(\theta)/V_0)(R(\theta)/R_0)^3\sin\theta{\rm d}\theta\ . \tag{15}$$

S is the surface of the one-body deformed nucleus. $V(\theta)$ is the electrostatic potential at the surface and V_0 the surface potential of the sphere.

The additional centrifugal energy $E_{\rm Rot}$ coming from the angular momentum of the emitted proton has been introduced:

$$E_{\rm Rot} = \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\ . \tag{16}$$

The dimensionless quantity of l is the angular momentum carried by the emitted proton (angular momentum transfer). μ is the reduced mass and r is the distance between the mass centers.

The surface energy results from the effects of the surface tension forces in a half space. When there are nucleons in regard in a neck or a gap between separated fragments an additional term called proximity energy must be added to take into account the effects of the nuclear forces between the close surfaces. This term is essential to describe smoothly the one-body to two-body transition and to obtain reasonable fusion barrier heights. It moves the barrier top to an external position and strongly decreases the pure Coulomb barrier.

$$E_{\rm Prox}(r) = 2\gamma\int_{h_{\rm min}}^{h_{\rm max}}\varPhi\left[\frac{D(r,h)}{b}\right]2\pi h{\rm d}h\ , \tag{17}$$

where h is the distance varying from the neck radius or zero to the height of the neck border. D is the distance between the surfaces in regard and $b = 0.99$ fm the surface width. $\varPhi$ is the proximity function of Feldmeier[25]. The surface parameter γ is the geometric mean between the surface parameters of the two nuclei or fragments. The combination of the GLDM and of a quasi-molecular shape sequence has allowed to reproduce the fusion barrier heights and radii, the fission, the α, the proton and cluster radioactivity data.

For proton emission the formula was adopted to simulates the proximity energy [31]:

$$E_{\mathrm{prox}}(r)=(4\pi\gamma)\mathrm{e}^{-1.38(r-R_{\mathrm{p}}-R_{\mathrm{d}})}\times \left[0.6584A^{2/3}-\left(\frac{0.172}{A^{1/3}}+0.4692A^{1/3}\right)r- 0.02548A^{1/3}r^2+0.01762r^3\right] . \quad (18)$$

To obtain the proton decay barrier from the contact point between the nascent proton particle and daughter nucleus it is sufficient to add this proximity energy to the Coulomb repulsion.

3 Results and discussions

The decay constant λ can be obtained from the experimental half-life $\lambda_{\mathrm{expt.}}=\ln 2/T_{\mathrm{expt.}}$. After the calculation of the assault frequency ν_0 by quantum mechanical method and penetrability P from WKB approximation, the experimental spectroscopic factor can be extracted using $S_{\mathrm{p}}^{\mathrm{expt.}}=\lambda_{\mathrm{expt.}}/\nu_0 P$. At the same time, we can obtain the spectroscopic factor $S_{\mathrm{p}}^{\mathrm{theo.}}$ using the RMF+BCS theoretically.

The values of transferred angular momentum l, decay energy Q, experimental and theoretical $\log_{10}T_{\mathrm{p}}$, the assault frequency ν_0, the penetrability P, the spectroscopic factor from RMF+BCS ($S_{\mathrm{p}}^{\mathrm{RMF}}$) and from experiment ($S_{\mathrm{p}}^{\mathrm{expt.}}$) are presented, and the relative deviation(RD) of $|T_{\mathrm{p}}^{\mathrm{expt.}}-T_{\mathrm{p}}^{\mathrm{theo.}}|/T_{\mathrm{p}}^{\mathrm{expt.}}\times 100\%$ is shown in the last column in Table 1 for the spherical proton radioactivity. The order of magnitude of assault frequency ν_0 are 10^{20}, and the values stand between 7.5 to 10. So it is a good approximation if the value of assault frequency is fixed as a constant. The penetrability P stays between 10^{-23} and 10^{-16} which is relatively very large while the range is narrow, compared with $10^{-39}\sim 10^{-14}$ for α-decay[30]. Hence it is easy for proton to escape from the proton emitter, confirming that the proton-emitting nuclei are weakly bound. The standard deviation between $\log_{10}T_{\mathrm{p}}^{\mathrm{expt.}}$ and $\log_{10}T_{\mathrm{p}}^{\mathrm{theo.}}$ can be calculated by the below formula $\sqrt{\sigma^2}=\sqrt{\sum_{i=1}^{N}\frac{(\log_{10}T_{\mathrm{p}}^{\mathrm{expt.}}-\log_{10}T_{\mathrm{p}}^{\mathrm{theo.}})^2}{N}}$, whose value is 0.247 for the spherical proton radioactivity in Table 1. It should be noticed that the proton radioactivity half-lives are estimated by a simple formula and the standard deviation $\sigma=0.26$ in Ref. [33]. Recently Half-life measurements for both ground-state and isomeric transitions in proton radioactivity are systematized by using a semiempirical, one-parameter model based on tunneling through a potential barrier, and the standard deviation $\sigma=0.311$ and $\sigma=0.333$ for isomeric and ground-state transitions respectively[34]. The average relative deviation between extracted experimental spectroscopic factors and those from RMF+BCS is 56% (the value will be 27% if the largest relative deviation for ^{185}Bi is neglected). These good agreements show the present method works quite well for the spherical proton radioactivity. For nuclei ^{156}Ta, ^{156}Ta(isomeric state) ^{177}Tl and ^{177}Tl(isomeric state), the DDM3Y and JLM not considering the spectroscopic factor can not provide good explanation[10–11], while the present concise method could give a very excellent results with the deviation 13%, 4.6%, 33%, and 25% respectively. For ground state of ^{159}Re and isomeric state of ^{165}Ir, ^{166}Ir, ^{167}Ir and ^{171}Au, the agreement between our theoretical calculations and experimental data is excellent, and relative deviations of spectroscopic factor are not more than 10%. These indicate that the introducing of the spectroscopic factors in calculations is necessary. The quantitative agreement with the experimental data are better than other theoretical ones which demonstrates that the GLDM with the proximity effects, centrifugal potential energy, the mass asymmetry and spectroscopic factor could be used to investigate the proton emission successfully when the right Q values are given. In this work, the assault frequency is estimated via the quantum mechanics by considering the structure of the parent nucleus. The penetrability is calculated in the WKB approximation. No additional parameters are introduced. The physical process is clear and the methods are concise. Checking the results in detail, one can find that the RMF+BCS underestimates the spectroscopic factor of ^{185}Bi by a factor of seven. The reason is probably due to the uncertainty of the Q value, or the emitted proton being not from the $s_{1/2}$ state. This requires further theoretical investigations and measurements with high accuracy. Recently, the spherical proton emitter ^{156}Ta was observed again and its emitted proton energy and half-life have been measured[32]. With the transferred angular momentum $l=5$[33] and proton energy $E_{\mathrm{p}}=(1.444\pm 0.015)$ MeV ($Q=(1.453\pm 0.015)$ MeV), we obtained spectroscopic factor of $S_{\mathrm{p}}^{\mathrm{RMF}}=0.761$ by using the RMF+BCS, which is in agreement with the extracted experimental value of $S_{\mathrm{p}}^{\mathrm{expt.}}=0.660$. The accurate consistency indicates that the experimental data should be reliable. The new spherical proton emitter ^{159}Re was synthesized in the reaction ^{106}Cd (^{58}Ni, p4n) ^{159}Re[35] and its proton emission Q value along with half-life has been measured recently. The experimental spectroscopic factor value of $S_{\mathrm{p}}^{\mathrm{expt.}}=0.314$ is very close to the calculated one with RMF+BCS ($S_{\mathrm{p}}^{\mathrm{RMF}}=0.308$) if and only if $l=5$, which indicates that the proton is emitted from $\pi h_{11/2}$ state, which agrees with the conclusions in Ref. [35].

Table 1 The Comparison between experimental and calculated microscopic factors for spherical proton radioactivity.

Parent	l	Q/MeV expt.	$\log_{10}T_\mathrm{p}$/s expt.	$\log_{10}T_\mathrm{p}$/s theo.	$\nu_0(\times10^{20})$ Eq. (6)	P WKB	$S_\mathrm{p}^\mathrm{RMF}$ RMF	$S_\mathrm{p}^\mathrm{expt.}$ expt.	RD(%)
^{105}Sb	2	0.491	2.049	1.740	9.853	1.280×10^{-23}	0.999	0.491	51
^{155}Ta	5	1.791	-2.538	-2.452	8.816	5.280×10^{-19}	0.422	0.514	22
^{156}Ta	2	1.028	-0.620	-0.682	8.776	4.994×10^{-21}	0.761	0.660	13
^{156}Ta*	5	1.130	0.949	0.951	8.776	1.793×10^{-22}	0.493	0.495	4.6
^{157}Ta	0	0.947	-0.523	-0.208	8.736	1.608×10^{-21}	0.797	1.644	107
^{159}Re	5	1.816	-4.678	-4.670	8.657	1.216×10^{-16}	0.308	0.314	1.9
^{160}Re	2	1.284	-3.046	-3.143	8.619	2.204×10^{-18}	0.507	0.406	20
^{161}Re	0	1.214	-3.432	-3.349	8.581	2.024×10^{-18}	0.892	1.079	21
^{161}Re*	5	1.338	-0.488	-0.707	8.581	1.419×10^{-20}	0.290	0.175	40
^{164}Ir	5	1.844	-3.959	-4.239	8.468	7.542×10^{-17}	0.188	0.099	48
^{165}Ir*	5	1.733	-3.469	-3.482	8.432	1.335×10^{-17}	0.187	0.181	2.9
^{166}Ir	2	1.168	-0.824	-1.120	8.395	2.624×10^{-20}	0.415	0.210	49
^{166}Ir*	5	1.340	-0.076	-0.046	8.395	4.887×10^{-21}	0.188	0.201	7.2
^{167}Ir	0	1.086	-0.959	-1.093	8.359	1.126×10^{-20}	0.912	0.670	27
^{167}Ir*	5	1.261	0.875	0.839	8.359	6.559×10^{-22}	0.183	0.168	8.0
^{171}Au	0	1.469	-4.770	-4.884	8.219	7.608×10^{-17}	0.848	0.652	23
^{171}Au*	5	1.718	-2.654	-2.626	8.219	4.101×10^{-18}	0.087	0.093	6.7
^{177}Tl	0	1.180	-1.174	-1.050	8.020	1.324×10^{-20}	0.733	0.975	33
^{177}Tl*	5	1.986	-3.347	-3.472	8.020	1.166×10^{-16}	0.022	0.016	25
^{185}Bi	0	1.624	-4.229	-3.379	7.771	1.942×10^{-16}	0.011	0.078	608

The asterisk(*) symbols in parent nuclei denote isomeric states.

4 Predictions for proton radioactivity

Due to extreme interesting by the nuclear experimentalists[36], we give the predictions for one proton radioactivity half-lives of the nuclei ^{189}At and ^{195}Fr. The angular momentum of the outgoing proton was supposed to be 2, 3, 5 $\hbar$ respectively, because we did not know the necessary structure properties (*e.g.* spin and parity) for the these proton-rich nuclei up to now. The spectroscopic factor was fixed as $S_\mathrm{p}=1.0$, and the assault frequency ν_0 was adopted 8.0×10^{20} from the reference[29] in these predictions. One can see from Table 2 that the order of the magnitude of proton radioactivity half-lives is millisecond for nucleus ^{189}At. For nucleus ^{195}Fr, the proton radioactivity half-lives change from 1.9633×10^{-1} s to 66.608 s when angular momentum change from 2 $\hbar$ to 5 $\hbar$. The proton radioactivity for nuclei ^{189}At and ^{195}Fr should be observed in the future experiments.

Table 2 Predictions for the proton radioactivity half-life. l is the supposed angular momentum of the outgoing proton.

Parent	Q/MeV	$l/\hbar$	$\log_{10}T_\mathrm{p}$/s
^{189}At	1.597	2	1.1114×10^{-4}
		3	5.0275×10^{-4}
		5	3.7415×10^{-2}
^{195}Fr	1.330	2	1.9633×10^{-1}
		3	8.8440×10^{-1}
		5	6.6088×10^{1}

5 Summary and outlook

A hybrid method was proposed for proton radioactivity: the spectroscopic factor is obtained by the RMF theory combined with BCS method; the assault frequency is estimated by a quantum mechanical method considering the structure of the parent nucleus; the penetrability is calculated by WKB approximation and the penetration barrier is constructed by the GLDM. In the whole process, there are no additional parameters introduced. The extracted experimental spectroscopic factors are compared with those from calculations by RMF+BCS and the agreement is very good, implying that the present method works quite well. Predictions are provided for some nuclei ^{189}At and ^{195}Fr, which can serve for future experiments. In the next work, we will develop the macro-microscopic model (MMM) to deal with the shell and pairing corrections simultaneous[38] including pauli blocking for the odd nucleon firstly. The spectroscopic factor for proton radioactivity can be given by the MMM. Then, the quasimolecular shapes will be introduced in the framework of MMM, and the potential barrier can be constructed by the generalized macro-microscopic model (GMMMM). At last, the proton radioactivity process can be described by the generalized macro-microscopic model.

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