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Nuclear Symmetry Energy Extracted from the (p,n) Reaction Experimental Data

WANG Hongyuan, XU Chang

(*Department of Physics, Nanjing University, Nanjing 210008, China*)

Abstract: The isovector term of the real part of the nucleon optical potential is obtained by analysing differential cross section data of (p,n) charge exchange reaction. The magnitude of the differential cross section of (p,n) experiment is proportional to the square of the isovector term of the real part of the nucleon optical potential v_1, so the extraction of the value of v_1 is straightforward by using the plane wave Born approximation. Based on the Hugenholtz-Van Hove theorem, the nuclear symmetry energy $E_{\rm sym}(\rho_0)$ and its density slope $L(\rho_0)$ are estimated by using the extracted value of v_1. The calculated results $E_{\rm sym}(\rho_0)=(28.5\pm2.0)$ MeV, $L(\rho_0)=(67.0\pm5.0)$ MeV agree reasonably with those extracted by analysing nuclear masses and other experimental data.

Key words: nuclear symmetry energy; (p,n) reaction; optical model; plane wave Born approximation

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1 Introduction

The nuclear symmetry energy $E_{\rm sym}(\rho)$ is the energy related to neutron-proton asymmetry in the equation of state of nuclear matter. For asymmetric nuclear matter, the energy per nucleon can be well approximated by[1]

$$E(\rho,\alpha)=E(\rho,\alpha=0)+E_{\rm sym}(\rho)\alpha^2+O(\alpha^4)\ , \qquad (1)$$

in terms of density $\rho=\rho_{\rm p}+\rho_{\rm n}$ and isospin asymmetry $\alpha=(\rho_{\rm n}-\rho_{\rm p})/(\rho_{\rm n}+\rho_{\rm p})$, where $\rho_{\rm p}$ and $\rho_{\rm n}$ are the densities of proton and neutron, respectively. The nuclear symmetry energy $E_{\rm sym}(\rho)$ plays an important role in both nuclear physics and astrophysics. Not only its magnitude but also its density dependence is important to understand the structure of neutron-rich nuclei[2−3], the reaction mechanism of heavy-ion collisions[1,4−7], the structure of neutron stars[8−10] and the dynamics of the supernova collapse[11]. Near the nuclear matter saturation density ρ_0, the symmetry energy can be described by using the value of $E_{\rm sym}(\rho_0)$ and the slope parameter $L(\rho_0)$[12]

$$E_{\rm sym}(\rho)=E_{\rm sym}(\rho_0)+\frac{L(\rho_0)}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right)+O\left[\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right]. \qquad (2)$$

Conventionally, the information about $E_{\rm sym}(\rho_0)$ can be obtained from analysing nuclear masses with liquid-drop models[13−15]. In this empirical way the value of $E_{\rm sym}(\rho_0)$ is estimated to be around $28.0\sim34.0$ MeV. The uncertainties of the symmetry energy mainly come from $L(\rho_0)$. Because of the importance of the density slope in nuclear physics and astrophysics, there has been much effort to investigate the density slope $L(\rho_0)$ based on various phenomena in terrestrial nuclear laboratory experiments such as neutron/proton ratio of pre-equilibrium nucleon emissions in heavy-ion reactions[16−17], neutron skin of heavy nuclei[18], giant dipole as well as pygmy dipole resonances[19] and so on. The extracted value of $L(\rho_0)$ spreads between 20.0 and 115.0 MeV, although each individual study gives a small uncertain region.

Based on the Hugenhultz-Van Hove theorem, it has been shown[12] analytically that both $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$ are determined by the nucleon optical potential including the isoscalar and isovector terms which can be extracted from nucleon-nucleus reactions such as the (p,n) charge-exchange reaction. The (p,n) experimental results have been used to obtain the value of the isovector term of the optical potential v_1 since the 1960s[20]. Later, the precision of the measured

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Biography: WANG Hongyuan(1990–), male, Nanjing, Jiangsu, Master Candidate, majoring on the field of theoretical nuclear physics; E-mail: wanghongyuan_nju@163.com.

data in the reactions has been improved with the fast development of the equipment and the experimental technique. It is interesting to make use of the accumulated new experimental data[21−23] to extract the value of the isovector term of the optical potential v_1. In this work, through analysing the (p,n) reaction experimental data with the plane wave Born approximation, we obtain the value of the isovector term of the nucleon optical potential v_1 as well as the values of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$. The outline of this paper is as follows. In section 2 the derivation of the relation between the differential cross section of the (p,n) experiment and v_1 is presented. In section 3 the detailed results of v_1 is obtained by using the differential cross section data of the (p,n) experiments, the values of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$ are given, and the approximations used in our calculation are discussed. In section 4 a brief summary is given.

2 Formulism of the differential cross section of (p,n) reaction

The Lane potential is widely used in nuclear reaction calculations with the form[20]

$$V = v_0 + A^{-1}(\mathbf{t}\cdot\mathbf{T})v_1 \ . \tag{3}$$

This form explicitly shows that the optical potential contains a dependence on the scalar product of the incident nucleon isospin $\mathbf{t}$ and the target nucleus isospin $\mathbf{T}$. A is the mass number of the target nucleus. v_0 and v_1 are the isoscalar and isovector terms of the optical potential, respectively, which are independent of neutron number N or proton number Z.

With the formula of V in Eq. (3), the (p,n) reaction is described with the Schrödinger equation[20]

$$\begin{aligned}&\{\mathscr{T} + v_0 + \frac{1}{A}(\mathbf{t}\cdot\mathbf{T})v_1 + (\frac{1}{2} - t_3)v_{\rm C}\}\psi \\ &= \{E - (\frac{1}{2} + t_3)\varDelta_{\rm C}\}\psi \ ,\end{aligned} \tag{4}$$

where $\mathscr{T}$ is the kinetic energy term, t_3 is the isospin projection of the incident nucleon, and E is the incident proton energy. $v_{\rm C}$ is the Coulomb potential term, and $\varDelta_{\rm C}$ is the energy loss of proton due to the Coulomb effect. The solution of the Schrödinger equation has the form[20]

$$\psi = g_{\rm p}\alpha_{\rm p}\psi_{T_0T_0} + g_{\rm n}\alpha_{\rm n}\psi_{T_0(T_0-1)} \ , \tag{5}$$

where $\alpha_{\rm p}$, $\alpha_{\rm n}$, $\psi_{T_0T_0}$, $\psi_{T_0(T_0-1)}$ are isobaric spin states of proton, neutron, target nucleus and residual nucleus, respectively. T_0 is the isospin projection of the target nucleus which equals to $\frac{1}{2}(N-Z)$. $g_{\rm p}$ and $g_{\rm n}$ are wave functions with asymptotic forms:

$$g_{\rm p} \sim {\rm e}^{ik_{\rm p}z} + f_{\rm pp}(\theta)\frac{{\rm e}^{ik_{\rm p}r}}{r} \ , \ g_{\rm n} \sim f_{\rm pn}(\theta)\frac{{\rm e}^{ik_{\rm n}r}}{r} \ , \tag{6}$$

where $k_{\rm n}$ and $k_{\rm p}$ are wave numbers of neutron and proton. $f_{\rm pn}(\theta)$ and $f_{\rm pp}(\theta)$ are the scattering amplitudes of proton-neutron and proton-proton reactions, respectively. Insertion of the solution into the Schrödinger equation leads to the coupled equations[20]

$$\begin{aligned}&(\mathscr{T} + v_0 - E + v_C - \frac{1}{2A}T_0v_1)g_{\rm p} + \\ &\frac{1}{2A}(2T_0)^{\frac{1}{2}}v_1g_{\rm n} = 0 \ , \\ &(\mathscr{T} + v_0 - E + \varDelta_C + \frac{1}{2A}(T_0-1)v_1)g_{\rm n} + \\ &\frac{1}{2A}(2T_0)^{\frac{1}{2}}v_1g_{\rm p} = 0 \ .\end{aligned} \tag{7}$$

The differential cross section and the scattering amplitude $f_{\rm pn}(\theta)$ have the relation

$$\sigma = \frac{k_{\rm n}}{k_{\rm p}} \mid f_{\rm pn}(\theta) \mid^2 . \tag{8}$$

To get the form of $f_{\rm pn}(\theta)$, we treat the terms containing v_1 as perturbation here. We also ignore the Coulomb terms $v_{\rm C}$ and $\varDelta_{\rm C}$. Then the scattering amplitude $f_{\rm pn}(\theta)$ is given approximately by the form[20]

$$f_{\rm pn}(\theta) = \frac{M}{2\pi\hbar^2}\langle g_{\rm n0}^{\theta-} \big| \frac{1}{2A}(2T_0)^{\frac{1}{2}}v_1 \big| g_{\rm p}\rangle \ , \tag{9}$$

where M is the mass of a nucleon. $g_{\rm n0}^{\theta-}$ and $g_{\rm p}$ are wave functions with unit amplitude (plane waves) in the final and initial directions. Over the nuclear volume, we consider that v_1 is essentially constant. In principle, v_1 should have both density dependence and energy dependence. Here v_1 is approximately taken as a constant in our calculation. It will be interesting to include its density/energy dependence and improve the present calculation. We take v_1 out of the integral and have

$$f_{\rm pn}(\theta) = \frac{M(N-Z)^{\frac{1}{2}}v_1R^3}{3\hbar^2A}x(\theta), \tag{10}$$

where $x(\theta)$ is the inner product of the wave functions over the radial direction, which indicates the probability amplitude of reaction:

$$x(\theta) = (\frac{4}{3}\pi R^3)^{-1}\langle g_{\rm n0}^{\theta-} \mid g_{\rm p}\rangle_{|r|<R} \ . \tag{11}$$

The relation $R = 1.45\ A^{\frac{1}{3}}$ is made use of, which is commonly used to describe the nuclear radius in nuclear physics[24]. Here we do not modify the parameter in the radius and use the same value 1.45 as Lane[20]. And we have

$$f_{\rm pn}(\theta) = \frac{(N-Z)^{\frac{1}{2}}}{42}v_1x(\theta) \ . \tag{12}$$

It is assumed that g functions are made negligible through absorption in the interior and are approximately plane waves in the outer skin of the nucleus of thickness $\triangle R = R_2 - R_1$. The value of $\triangle R$ is taken as 1 fm according to Ref. [20], which can be approximately considered as the thickness of density decreasing from 90% to 50% of the saturation density. And here we give the detailed form of $x(\theta)$:

$$x(\theta) = (\frac{4}{3}\pi R^3)^{-1} 4\pi \int_{R_1}^{R_2} \mathrm{e}^{-i\Delta k R'} R'^2 \mathrm{d}R' \ . \tag{13}$$

After expanding the exponent and simplifying the integral, we have

$$x(\theta) = 3\frac{\Delta R}{R}\left(\frac{\sin\Delta k R}{\Delta k R}\right) \ , \tag{14}$$

where $\Delta k = |\boldsymbol{k}_{\mathbf{p}} - \boldsymbol{k}_{\mathbf{n}}|$. Taking $\Delta R = 1$ fm, the scattering amplitude and the differential cross section can both be calculated. It is noted that the angular distribution for small angles is given by $(\frac{\sin\Delta kR}{\Delta kR})^2$.

In many (p,n) reactions, the observed angular distribution shows a peak[20] around $\theta \approx 30^\circ$, which corresponds to the first maximum of spherical Bessel function j_0 that is included in the form of $x(\theta)$ in Eq. (14). The maximum of the angular distribution implies an interference phenomenon[25]. In principle, the values of the neutron spectra at other angles can also be used to extract the value of v_1. However, the value of the peak of the neutron spectra from the (p,n) reaction is the largest relative to the values at other points of the spectra, therefore this value is used to obtain the value of the isovector term of optical potential v_1 for the best precision. The final and initial states of the nucleus of the (p,n) reaction are called Isospin Analog States (IAS), which means that the residual nucleus is at its excited state that has the same isospin as the ground state of the target nucleus. The 30° peak of the spectra of the (p,n) reaction differential cross section is named as IAS peak[21], which has the form

$$\sigma_{\mathrm{IAS}} = \frac{N-Z}{A^{\frac{2}{3}}} v_1^2 \times c \ , \tag{15}$$

where the parameter c is given by the form $c = \frac{k_{\mathrm{n}}}{k_{\mathrm{p}}}[MR^2\Delta R(\frac{\sin\Delta kR}{\Delta kR})]^2/(9\hbar^4 A^{\frac{4}{3}})$. The relation $R = 1.45\ A^{\frac{1}{3}}$ fm[24] is used, the value of $(\frac{\sin\Delta kR}{\Delta kR})$ is about 0.22 at the first maximum, and the neglect of the Coulomb terms means that k_{n} is equal to k_{p}. Thus the value of c is 1.13×10^{-3} mb/[sr·(MeV)^2]. The isovector term of optical potential v_1 is closely related to the (p,n) reaction. The relation

$$v_1 = A^{\frac{1}{3}}(\frac{\sigma_{\mathrm{IAS}}}{N-Z})^{\frac{1}{2}} \times \frac{1}{\sqrt{c}} \ , \tag{16}$$

is used to obtain the value of v_1.

3 Results and discussions

Based on the HVH theorem, the symmetry energy $E_{\mathrm{sym}}(\rho_0)$ and its density slope $L(\rho_0)$ have the form[12]

$$E_{\mathrm{sym}}(\rho_0) = \frac{1}{3}\frac{\hbar^2 {k_{\mathrm{F}}}^2}{2m_0^*} + \frac{1}{2}U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}}) \ ,$$
$$L(\rho_0) = \frac{2}{3}\frac{\hbar^2 {k_{\mathrm{F}}}^2}{2m_0^*} + \frac{3}{2}U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}}) + \frac{\partial U_{\mathrm{sym}}}{\partial k}\Big|_{k_{\mathrm{F}}} k_{\mathrm{F}} \ , \tag{17}$$

where k_{F} is the Fermi momentum, and m_0^* is the effective mass weighing about 0.7 times as much as the mass of a nucleon M. ρ_0 is the saturation density of nuclear matter, and $U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}})$ is the symmetry potential which has the relationship with the single nucleon potential[12] $U_{\mathrm{n/p}}(\rho_0, \alpha, k_{\mathrm{F}}) = U_0(\rho_0, k_{\mathrm{F}}) \pm U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}})\alpha$ for neutron and proton, respectively. $U_0(\rho_0, k_{\mathrm{F}})$ is the isoscalar nucleon potential. The relation of v_1 and $U_{\mathrm{n/p}}$ in Ref. [20] is $U_{\mathrm{n/p}} = v_0 \pm \frac{1}{4}\alpha v_1$. The difference between $U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}})$ and v_1 is only a constant $\frac{1}{4}$, which means $U_{\mathrm{sym}}(\rho_0, k_{\mathrm{F}}) = v_1/4$. The value of the term $\frac{\hbar^2 {k_{\mathrm{F}}}^2}{2m_0^*} = 54.8$ MeV is obtained directly with the values $k_{\mathrm{F}} = 1.36\ \mathrm{fm}^{-1}$ and $m_0^* = 0.7$ M. Here we assume that U_{sym} is independent of the momentum k, which means that the differential term $\frac{\partial U_{\mathrm{sym}}}{\partial k}\Big|_{k_{\mathrm{F}}} k_{\mathrm{F}}$ is 0. To obtain the value of $E_{\mathrm{sym}}(\rho_0)$ and $L(\rho_0)$, we need the value of U_{sym} (or the value of v_1).

The value of the isovector term of optical potential v_1 is obtained with the values of σ_{IAS} of the (p,n) reaction experiments by using Eq. (16), and the values of σ_{IAS} of the recent (p,n) charge-exchange reaction experiments are taken from Refs. [21–23]. In Table 1 we show the results obtained from the data of the (p,n) charge-exchange reactions in Ref. [21]. The first column and the second column are the sequence number and the detailed reaction, respectively. The values of the differential cross sections at the IAS peak and the corresponding values of v_1 obtained through the present analysis are given in the third column and the fourth column , respectively.

In Ref. [21], there are eight sets of data of different target nuclei reacting with the incident protons in the (p,n) charge-exchange reaction. The mass numbers of the target nuclei range from $A = 14$ to $A = 42$, and they are all even-even nuclei. The values of σ_{IAS} range from 1.13 to 2.51, and most of the values spread between 1.0 to 2.0. The value of σ_{IAS} is proportional to $(N-Z)$ and reciprocal to $A^{\frac{2}{3}}$, which is explicitly given in Eq. (15). The differences between proton and neutron numbers of all the target nuclei are $(N-Z) = 2$, so the value of σ_{IAS} decreases with the increase of the mass number of the target nucleus. We can see from Table 1 that the extracted values of v_1 range from 72.2 to 84.6 MeV, and the averaged value of of all these values is 78.1 MeV.

Table 1 Extracted values of the isovector term of nucleon optical potential v_1 by using the IAS peak of the neutrons from the (p,n) reaction experiment in Ref. [21]. The incident energy of the reaction is $E_\mathrm{p} = 35$ MeV.

No.	Reaction	σ_IAS/(mb/sr)	v_1/MeV
1	^{14}C(p,n)^{14}N	2.51	80.2
2	^{18}O(p,n)^{18}F	1.72	72.2
3	^{22}Ne(p,n)^{22}Na	1.82	79.5
4	^{22}Mg(p,n)^{26}Al	1.85	84.6
5	^{30}Si(p,n)^{30}P	1.52	80.5
6	^{34}S(p,n)^{34}Cl	1.23	75.5
7	^{38}Ar(p,n)^{38}K	1.10	74.1
8	^{42}Ca(p,n)^{42}Sc	1.13	77.6

The results obtained from the experimental data of Ref. [22] and Ref. [23] are shown in Table 2 and Table 3, respectively. The mass numbers of the target nuclei of these reactions scatter in a large range, *i.e.* $A = 27$, 48, 51 and 90. Compared with Table 1, the target nuclei include not only even-even nuclei but also odd-A nuclei. In Table 2, the differences between N and Z of the target nuclei are 1, 5 and 8 while the corresponding mass numbers are 27, 51, and 90. Considering the influence of the term $\frac{N-Z}{A^{\frac{2}{3}}}$ on the value of σ_IAS given in Eq. (15), it increases with the larger mass number. In Table 3, the differences between N and Z of the target nuclei are both 8. The value of σ_IAS of the reaction ^{48}Ca(p,n) ^{48}Sc is as large as 5.6 mb/sr because the factor $\frac{N-Z}{A^{\frac{2}{3}}}$ is large, and the value of σ_IAS in Table 3 decreases with the increasing mass number as in Table 1. The averaged values of the obtained values of v_1 in Table 2 and 3 are 85.3 and 82.8 MeV, respectively. These two values are slightly larger than the averaged value of $v_1 = 78.1$ MeV in Table 1.

Table 2 Extracted values of the isovector term of nucleon optical potential v_1 by using the IAS peak of the neutrons from the (p,n) reaction experiment in Ref. [22]. The incident energy of the reaction is $E_\mathrm{p} = 40$ MeV.

No.	Reaction	σ_IAS/(mb/sr)	v_1/MeV
9	^{27}Al(p,n)^{27}Si	1.47	108.1
10	^{51}V(p,n)^{51}Cr	2.39	76.2
11	^{90}Zr(p,n)^{90}Nb	2.92	71.5

Table 3 Extracted values of the isovector term of nucleon optical potential v_1 by using the IAS peak of the neutrons from the (p,n) reaction experiment in Ref. [23]. The incident energy of the reaction is $E_\mathrm{p} = 45$ MeV.

No.	Reaction	σ_IAS/(mb/sr)	v_1/MeV
12	^{48}Ca(p,n)^{48}Sc	5.60	90.2
13	^{90}Zr(p,n)^{90}Nb	3.21	75.3

To test the validity of the relation given by Eq. (15), in Fig. 1 we plot the experimental results of σ_IAS in Table 1 and 3 as a function of $A^{-2/3}$ where A is the mass number. The theoretical lines are obtained from Eq. (15) where the values of v_1 are taken as the average values, respectively. The values of $N - Z$ are constant in Table 1 and Table 3, respectively. Because the values of $N - Z$ are not a constant in Table 2, the data in it are not shown in Fig. 1. From Fig. 1 we can see that the experimental data match reasonably with the lines given by Eq. (15).

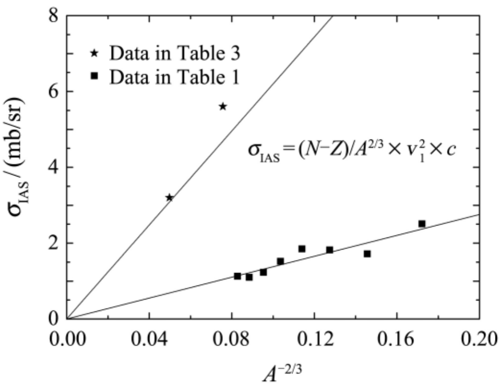

Fig. 1 Comparison between the experimental results of σ_IAS in Table 1 and Table 3 and the theoretical lines obtained from Eq. (15). The vertical axis is the Isospin Analog State (IAS) peak of the differential cross section of the (p,n) charge exchange reactions. The horizontal axis is $A^{-2/3}$ where A is the mass number.

It is noted that the present extraction of v_1 is based on some approximations and there are some factors which can be further improved. One possible factor of the difference of the average value of v_1 is the difference of the incident energy that may change the extracted value of v_1. The energy dependence of the angular distribution of the neutron from the (p,n) reaction is contained in the terms $k_\mathrm{n}/k_\mathrm{p}$ and $(\frac{\sin \Delta kR}{\Delta kR})^2$ [see Eq. (8) and Eq. (14)], which are both taken as constants approximately. Besides, the Coulomb scattering of proton process with the reaction energy is not taken into account, which means that the Coulomb terms Δ_C and v_C are neglected in the derivation of the formula of the differential cross section. Taking ^{14}C(p,n)^{14}N as an example, its Coulomb barrier $\frac{1}{4\pi\epsilon_0}\frac{Z_\mathrm{p}Z_\mathrm{C}e^2}{r}$ (the charge distribution radius $r = 1.1\ A^{\frac{1}{3}}$ fm)[24] is approximately 4.2 MeV, which is smaller compared to the incident protons energies of 35, 40 and 45 MeV. Thus, it is expected that the neglect of the Coulomb terms will not significantly influence the value of v_1.

To show the extracted values of v_1 more clearly, in Fig. 2 we plot the values of v_1 as a function of the corresponding mass number A of the target nuclei of the (p,n) reaction. It is clearly shown in Fig. 2 that the dependence of the value of v_1 on the mass number A is weak. We directly obtain the value $v_1 = 81.0$ MeV by averaging all the values obtained with the (p,n) reaction experimental data. Most of the obtained values spread around the averaged value of v_1. There is only one exception that the value of v_1 obtained with the data of the reaction ^{27}Al(p,n) ^{27}Si is larger compared to the other values. One possible reason may be that the differential cross section becomes large due to the shell effect of the $Z = 14$ subshell of ^{27}Si.

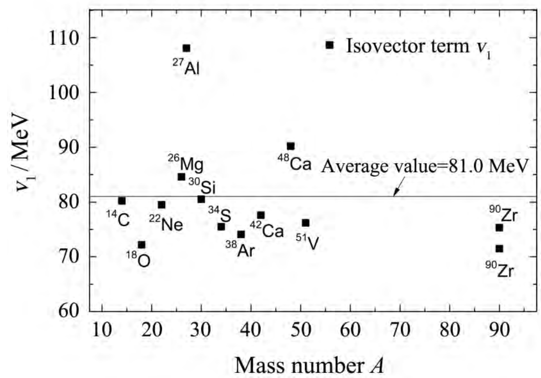

Fig. 2 Extracted values of the isovector term of optical potential v_1 as a function of mass number A of the target nucleus of the (p,n) reactions in Refs. [21-23]. The averaged value of v_1 is 81.0 MeV.

Through analysing the data of the (p,n) charge exchange reaction experiments, we have obtained the value of the isovector term of the optical potential $v_1 = 81.0$ MeV, so the corresponding value of the symmetry potential is $U_{\rm sym}(\rho_0, k_{\rm F}) = 20.2$ MeV. By inputting this value into Eq. (17)

$$E_{\rm sym}(\rho_0) = \frac{1}{3}\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*} + \frac{1}{2}U_{\rm sym}(\rho_0, k_{\rm F}),$$

$$L(\rho_0) = \frac{2}{3}\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*} + \frac{3}{2}U_{\rm sym}(\rho_0, k_{\rm F}) + \frac{\partial U_{\rm sym}}{\partial k}|_{k_{\rm F}} k_{\rm F},$$

we obtain the symmetry energy at saturation density $E_{\rm sym}(\rho_0) = 28.5$ MeV and its density slope $L(\rho_0) = 67.0$ MeV, respectively. According to Eq. (17), the uncertainty of the symmetry energy arises from both the kinetic energy term $\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*}$ and the symmetry potential term $U_{\rm sym}(\rho_0, k_{\rm F})$. The uncertainty of the kinetic energy term $\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*}$ mainly comes from the effective mass m_0^*. With the constraint $\frac{m_0^*}{M} = 0.70 \pm 0.05$ we adopted[12], an error bar of ± 3.0 MeV is obtained for $\frac{\hbar^2 {k_F}^2}{2m_0^*}$. For the symmetry potential, the variance of the obtained values of v_1 in Fig. 2 is $s^2 = [\sum_{i=1}^{13}(v_1^i - \overline{v_1})^2]/13 = 88.4\ {\rm MeV}^2$, and based on the standard statistical margin error method[26] the error bar of v_1 is estimated to be ± 7.9 MeV and the error bar of $U_{\rm sym}(\rho_0, k_{\rm F})$ is about ± 2.0 MeV.

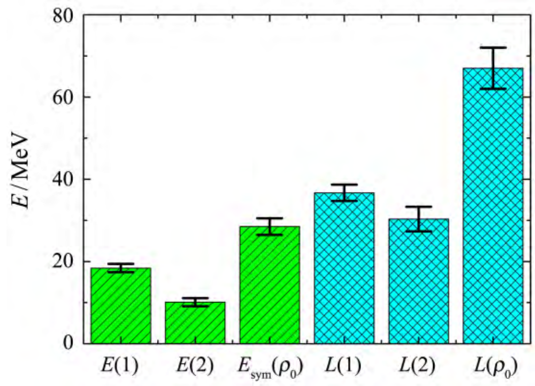

Fig. 3 (color online) The magnitude of each term in the nuclear symmetry energy $E_{\rm sym}(\rho_0)$ and its density slope $L(\rho_0)$ at the saturation density. The corresponding estimated error bars of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$ are also given.

The magnitudes of different terms contributing to the nuclear symmetry energy $E_{\rm sym}(\rho_0)$ and its density slope $L(\rho_0)$ as well as the obtained values of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$ with their error bars are shown in Fig. 3. As shown in Fig. 3, $E(1) = \frac{1}{3}\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*}$ is the kinetic energy term of $E_{\rm sym}(\rho_0)$, and $E(2) = \frac{1}{2}U_{\rm sym}(\rho_0, k_{\rm F})$ denotes the symmetry potential contribution to $E_{\rm sym}(\rho_0)$. Similarly, $L(1) = \frac{2}{3}\frac{\hbar^2 {k_{\rm F}}^2}{2m_0^*}$ and L(2)=$\frac{3}{2}U_{\rm sym}(\rho_0, k_{\rm F})$ are the kinetic energy term and the symmetry potential term of $L(\rho_0)$, respectively. The error bar of $E_{\rm sym}(\rho_0)$ is $\pm(1.0 + 1.0) = \pm 2.0$ MeV which is the sum of the error bars of $E(1)$ and $E(2)$, and the error bar of $L(\rho_0)$ is $\pm(2.0 + 3.0) = \pm 5.0$ MeV which is the sum the error bars of $L(1)$ and $L(2)$.

In the (p,n) charge exchange reaction the residual nucleus state is the isobarc analog of the ground state of the target nucleus, thus the magnitude of the cross section is large[27]. The value of $\sigma_{\rm IAS}$ is proportional to the square of v_1 [see Eq. (15)], so the extraction of the value of v_1 is straightforward. And the symmetry energy is directly dependent on the isovector term of the optical potential, namely, the symmetry potential. The present results agree reasonably with the results obtained by analysing nuclear masses[13−15] and other experimental data[16−19]. The plane wave Born approximation is used to analyse the data of the (p,n) reaction as a preliminary step to obtain the present results. These approximations could be further improved by using the distorted wave Born approximation by taking into account the neglected terms in the future. Moreover, if there will be more (p,n) experimental results, we can constrain the values of both nuclear symmetry energy at saturation density $E_{\rm sym}(\rho_0)$ and its density slope $L(\rho_0)$ better.

4 Conclusions

Through the relation between the differential cross section of the (p,n) reaction and the isovector term of optical potential v_1 derived by using the form of optical potential by A. M. Lane, the isovector term v_1 is obtained by using three sets of experimental data. The approximations such as the neglect of the Coulomb terms are expected to be reasonable. The value of the isovector term v_1 is used to obtain the values of the symmetry energy $E_{\rm sym}(\rho_0) = 28.5\pm2.0$ MeV and its density slope at the saturation density $L(\rho_0) = 67.0\pm5.0$ MeV, which agree reasonably with those extracted by analysing nuclear masses and other experimental data. Still, more (p,n) experimental results and more detailed calculations such as including the more realistic form of the optical potential are required, which will help to better constrain the values of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$.

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通过(p,n)反应实验数据提取对称能

王洪辕[1)]，许 昌

（南京大学物理学院，南京 210008）

摘要： 通过分析(p,n)电荷交换反应的微分散射截面数据，得到了核子光学势实部的同位旋矢量项。(p,n)反应的微分散射截面的大小正比于核子光学势实部的同位旋矢量项 v_1 的平方，利用平面波玻恩近似可以直接提取 v_1 的值。再根据HVH理论，利用 v_1 的值能够得到对称能 $E_{\rm sym}(\rho_0)$ 和它的密度依赖斜率 $L(\rho_0)$ 的值。得到的结果 $E_{\rm sym}(\rho_0)=(28.5\pm2.0)$ MeV, $L(\rho_0)=(67.0\pm5.0)$ MeV 与分析原子核质量和其他核实验数据得到的结果符合较好。

关键词： 对称能；(p,n)反应；光学模型；平面波玻恩近似

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1) E-mail: wanghongyuan_nju@163.com。