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Freeze-out Concept in Antisymmetrized Molecular Dynamics

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Abstract: As a part of statistical freeze-out study in antisymmetrized molecular dynamics (AMD), an analysis of ^{112}Sn+^{112}Sn at 50 MeV/nucleon is presented. Using the self-consistent method combined with the Modified Fisher model, the fragmenting source density, $\rho/\rho_0 = 0.69\pm0.03$, temperature, $T = (6.1\pm0.2)$ MeV, are extracted. By comparing the maximum density in the system during the time evolution of AMD calculation, a significantly lower density is found for the fragmenting source density. The extracted fragmenting source density and temperature are very similar to those for ^{40}Ca + ^{40}Ca at 35 MeV/nucleon and ^{64}Zn + ^{112}Sn at 40 MeV/nucleon. These indicate that there is a common statistical freeze-out volume at the time of the formation of intermediate mass fragments (IMFs) in AMD transport model.

Key words: statistical freeze-out; AMD; heavy ion reaction; self consistent method; density; temperature

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1 Introduction

In violent heavy ion collisions in the intermediate energy regime ($20 \leqslant E_{\rm inc} \leqslant$ a few hundred MeV/nucleon), intermediate mass fragments (IMFs) are copiously produced through a multi-fragmentation process. Nuclear multi-fragmentation was predicted a long time ago[1] and has been studied extensively following the advent of 4π detectors. In general the nuclear multi-fragmentation process, can be divided into three stages, *i.e.*, dynamical compression and heating, expansion and production of primary fragments, and finally the separation and cooling of the primary fragments by evaporation.

The freeze-out in equilibrated low density nuclear matter at an intermediate stage of a heavy ion collision is frequently used as a basic assumption of the statistical multi-fragmentation models such as MMMC[2] and SMM[3]. In contrast, the transport models, such as FMD[4], AMD[5−7], CoMD[8], ImQMD[9], QMD[10], BNV[11], SMF[12], BUU[13], among others, do not assume any chemical or thermal equilibration a priori. In the transport models, nucleons travel in a mean field experiencing nucleon-nucleon collisions subject to the Pauli principle. The mean-field parameters and the in-medium nucleon-nucleon cross sections are the main physical ingredients. Such models are appropriate for testing to what extent thermodynamic equilibrium is actually achieved.

Furuta *et al.* demonstrated in Ref. [14] that, in AMD calculations of ^{40}Ca+^{40}Ca at 35 MeV/nucleon, IMFs are formed in a wide range of time interval (100 ∼ 300 fm/c) and the isotope yield distribution changes with time. However the yield and excitation energy distributions as a function of mass at a given time can be identified as one of statistically equili-

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brated ensembles generated by the same model separately. In one of our previous works, we presented that IMFs with mass $\geqslant 15$ show a power law distribution with the critical exponent, $A^{-2.3}$, in experimental data and AMD simulations[16]. These studies suggest that the variety in the dynamical fragmentation process originates from the fluctuation of a statistical ensemble (a freezeout ensemble) in time, density and temperature and that there is a statistical "freeze-out" volume for the production of IMFs.

In a series of our recent works[15−19], the isotopic yield ratio method has been applied to extract the density and temperature of the fragmenting source. In the present study, we apply the same method to AMD events of ^{112}Sn + ^{112}Sn at 50 MeV/nucleon with three different density dependent interactions, *i.e.* the standard Gogny interaction which has an asymptotic soft symmetry energy (g0), the Gogny interaction with an asymptotic stiff symmetry energy (g0AS), and the Gogny interaction with an asymptotic super-stiff symmetry energy (g0ASS)[6,25], using the AMD code of Ono[5−6]. More than one thousand of AMD events with impact parameter range from 0 to 3 fm are generated for each interaction.

2 Self consistent method

In order to extract the characteristics of the emitting source of these isotopes, we first determined the ratio of the symmetry energy coefficient relative to the temperature, $a_{\rm sym}/T$, employing the improved method[17−18] in which all the available isotope yields are included. Based on the Modified Fisher Model (MFM)[20], which has been used to study the properties of the hot nuclear matter in previous works[15−19,21−23], the free energy relative to the temperature, $-F(N,Z)/T$, can be extracted using isotope yields in single reaction system as

$$\begin{aligned}-\frac{F(N,Z)}{T} &= \ln[\frac{Y(N,Z)A^{\tau}}{Y_0}] \\ &= \frac{\widetilde{a_{\rm v}}}{T}A-\frac{a_{\rm s}}{T}A^{2/3}-\frac{a_{\rm c}}{T}\frac{Z(Z-1)}{A^{1/3}}- \\ &\quad \frac{a_{\rm sym}}{T}\frac{(N-Z)^2}{A}-\frac{a_{\rm p}}{T}\frac{\delta}{A^{1/2}}+ \\ &\quad \frac{\Delta\mu}{2T}(N-Z)+S_{\rm mix}\ , \qquad (1)\\ S_{\rm mix} &= N\ln(\frac{N}{A})+Z\ln(\frac{Z}{A})\ ,\end{aligned}$$

where $\widetilde{a_{\rm v}}=a_{\rm v}+\frac{1}{2}(\mu_{\rm n}+\mu_{\rm p})$, $\Delta\mu=(\mu_{\rm n}-\mu_{\rm p})$. $a_{\rm v}$, $a_{\rm s}$, $a_{\rm c}$, $a_{\rm p}$, $\mu_{\rm n}$ and $\mu_{\rm p}$ are volume, surface, Coulomb, paring coefficients, neutron and proton chemical potential respectively. $\delta=\frac{1}{2}[(-1)^N+(-1)^Z]$. τ is the critical exponent. In this work, the value of $\tau=2.3$ is adopted from the previous studies[24].

For isotopes with $N=Z=A/2$, Eq. (1) can be simplified as[17]

$$\begin{aligned}-\frac{F(A/2,A/2)}{T} &= \ln\left[\frac{Y(A/2,A/2)A^{\tau}}{Y_0}\right] \\ &= \frac{\widetilde{a_{\rm v}}}{T}A-\frac{a_{\rm s}}{T}A^{2/3}-\frac{a_{\rm c}}{T}\frac{A(A-2)}{4A^{1/3}}-\frac{a_{\rm p}}{T}\frac{\delta}{A^{1/2}}+A\ln(\frac{1}{2})\ .\end{aligned} \qquad (2)$$

To eliminate the constant Y_0, all isotope yields are normalized to that of ^{12}C. Fig. 1(a) shows the $-F(A/2,A/2)/T$ values of AMD calculation with g0 interaction (blue solid squares) as a function of fragment mass number A. The open circles represent the fitting results of Eq. (2) using $\widetilde{a_{\rm v}}/T$, $a_{\rm s}/T$, $a_{\rm c}/T$ and $a_{\rm p}/T$ as free parameters. Similar quality results are obtained for the events generated using the g0AS and

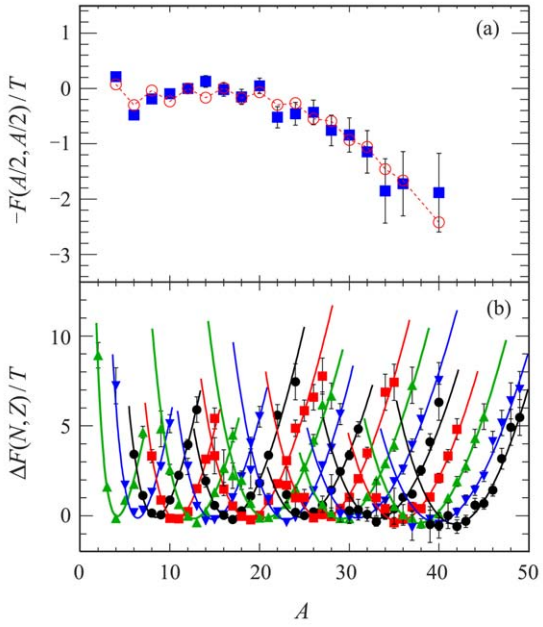

Fig. 1 (color online) (a) $-F(A/2,A/2)/T$ of AMD calculation with g0 interaction (blue solid squares) as a function of fragment mass number. The red open circles are those fit with Eq. (1). (b) Calculated $\Delta F(N,Z)/T$ values for the g0 interaction and quadratic fits with Eq. (3) for $Z=2$ to 20.

g0ASS interactions. The extracted parameters are shown in Table 1. Then the modified free energy, $\Delta F(N,Z)/T$, which is reduced by subtracting the volume, surface, Coulomb and paring terms from Eq. (2), is expressed as[17],

$$\frac{\Delta F(N,Z)}{T}=\frac{a_{\rm sym}}{T}\frac{(N-Z)^2}{A}-\frac{\Delta\mu}{2T}(N-Z)\ . \qquad (3)$$

Resultant $\Delta F(N,Z)/T$ values are shown in Fig. 1(b). Different symbols and colors represent different Z iso-

topes. The lines are the quadratical fits with Eq. (3) using $a_{\rm sym}/T$ and $\Delta\mu/T$ as free parameters. The extracted $a_{\rm sym}/T$ values are plotted in Fig. 2(a) as a function of fragment Z number for g0(solid circles), g0AS(solid squares) and g0ASS(solid triangles) respectively. Due to the large error bars in the data points, we don't see clear increasing trend for $a_{\rm sym}/T$ values as fragment Z number increases. This may also caused by the large system size for ^{112}Sn + ^{112}Sn and the finite size effect can be neglected. Therefore the mass dependence of the temperature is neglected in this analysis[15−17].

Table 1 Extracted parameters.

Interaction type	$a_{\rm v}/T$	$a_{\rm s}/T$	$a_{\rm c}/T$	$a_{\rm p}/T$	Ratio	ρ/ρ_0	$a_{\rm sym}$ /MeV	T/MeV
g0	1.59567	1.99951	0.15038	0.33993	—	—	26.8±0.4	
g0AS	1.34955	1.30642	0.13704	0.76676	1.12±0.17	0.75±0.08	23.3±0.6	
g0ASS	1.63599	1.98818	0.17671	0.87565	1.41±0.41	0.65±0.07	19.8±0.8	
Source						0.69±0.03		6.1±0.2

The ratios of $a_{\rm sym}/T$ values for g0/g0AS(solid squares) and g0/g0ASS(solid triangles) are shown in Fig. 2(b) for each Z isotopes respectively. The red and blue lines are the constant fits for g0/g0AS and g0/g0ASS. Fig. 2(c) shows the symmetry energy coefficient as a function of density for g0(solid line), g0AS(dashed line) and g0ASS(dotted line) interactions used in the AMD model and the ratios of symmetry energy coefficient in Fig. 2(c) are plotted in Fig. 2(d) as a function of density. The shaded horizontal lines indicate the ratios extracted in Fig. 2(b) and the vertical shaded areas show the density region corresponding to these ratios. Two different shaded areas are used for the two ratio values. We take the overlap region of two different shading as fragmenting source density, $\rho/\rho_0 = 0.69 \pm 0.03$. Then the symmetry energy coefficients of g0, g0AS and g0ASS are extracted. The ratio, density and symmetry energy coefficient values for g0, g0AS, g0ASS are shown in Table 1.

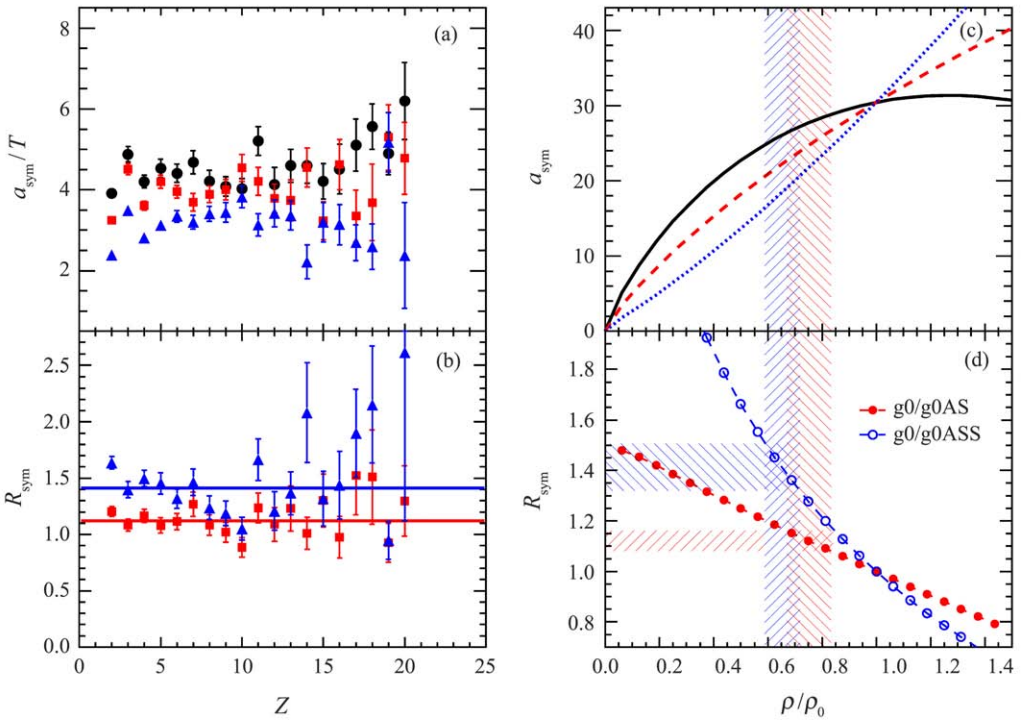

Fig. 2 (color online)(a) Extracted $a_{\rm sym}/T$ values from Fig. 1(b) for g0(solid circles), g0AS(solid squares), g0ASS(solid triangles) as a function of fragment charge number Z. (b) Ratios of $a_{\rm sym}/T$ from (a) for g0/g0AS(solid squares) and g0/g0ASS(solid triangles). (c) Symmetry energy coefficient vs density used in the AMD model. Solid curve(g0), dashed (g0AS) and dotted (g0ASS). (d) The ratio of the symmetry energy coefficient in (c) vs density. The shaded horizontal lines indicate the ratios extracted in (b) and the vertical shaded area shows the density region corresponding to these ratios. Two different shadings are used for the two ratio values.

Once the symmetry energy coefficient value is determined for a given interaction, the temperature, T, can be calculated as $T = a_{\rm sym}/(a_{\rm sym}/T)$. Fig. 3 shows the temperature as a function of fragment charge number Z for three interactions. Within the error bars, a flat trend is found for all the three interactions. The solid line represents the constant fit for all available points in the figure and the fragmenting source temperature is extracted as $T = (6.1 \pm 0.2)$ MeV.

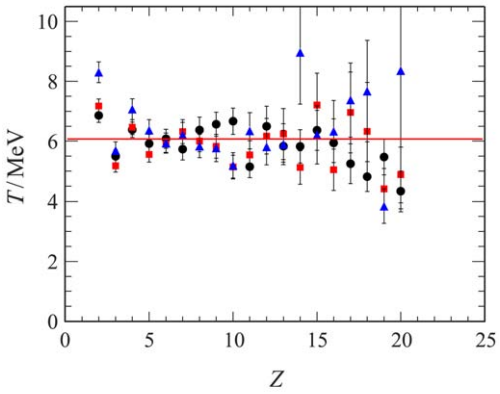

Fig. 3 (color online) The extracted temperatures vs fragment Z number for g0(solid circles), g0AS(solid squares), g0ASS(solid triangles) respectively. The red line represents the constant fit for all the available data in the figure.

3 Results and discussion

The fragmenting source density and temperature of three different systems, *i.e.* System ID 1 (^{40}Ca+^{40}Ca at 35 MeV/nucleon from Ref. [17]), System ID 2 (^{64}Zn+^{112}Sn at 40 MeV/nucleon from Refs. [15, 16]), System ID 3 (^{112}Sn+^{112}Sn at 50 MeV/nucleon), are summarized in Figs. 4(a) and (b), respectively. The open circles in Fig. 4(a) are the maximum density in the system during the time evolution at the origin of the center of the mass system for central collisions ($b < 1$ fm) while the solid circles represent the density extracted from self-consistent method. The extracted temperature values for self-consistent method are summarized in Fig. 4(b). Though the system size are different, the maximum density increases from $\rho/\rho_0 \sim 1.30$ at 35 MeV/nucleon for ^{40}Ca+^{40}Ca to $\rho/\rho_0 \sim 1.46$ at 50 MeV/nucleon for ^{112}Sn+^{112}Sn. On the contrary, the extracted density for the fragmenting source shows a flat trend and the values are around $\rho/\rho_0 \sim 0.67$. The temperature extracted for the fragmenting source are very close to each other for all the three systems and slightly increases as the incident energy increases from 35 to 50 MeV/nucleon. Due to the impact parameter range from 0 to 8 fm was used in Refs. [15, 16], a slight deviation of density and temperature values for the fragmenting source is found for the ^{64}Zn+^{112}Sn system (System ID=2). This suggests that a statistical freeze-out volume with density around $\rho/\rho_0 \sim 0.67$ and temperature $T \sim 6$ MeV is formed for this three systems when the IMFs are produced in the AMD transport model.

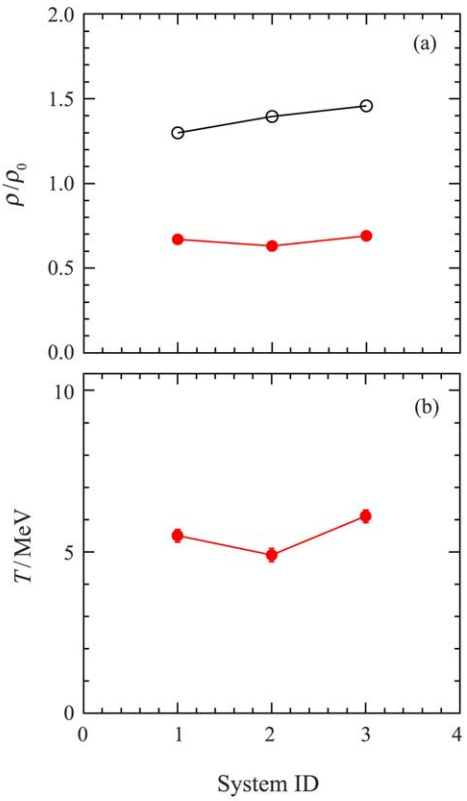

Fig. 4 (color online)(a) Maximum density(open circles) in Z-axis during the time evolution with coordinate Z range from $-1 \leqslant Z \leqslant 1$ fm and the density extracted from self-consistent method (close circles). The System ID 1(^{40}Ca+^{40}Ca at 35 MeV/nucleon[17]), 2(^{64}Zn+^{112}Sn at 40 MeV/nucleon[15−16]), 3(this work). (b) The extracted temperature, T, for 3 systems.

Summarizing, the density and temperature of the fragmenting source are extracted for ^{112}Sn+^{112}Sn at 50 MeV/nucleon as $\rho/\rho_0 = 0.69 \pm 0.3$ and $T = (6.1 \pm 0.2)$ MeV, using self-consistent method with MFM to extracted the ratio of symmetry energy coefficient to the temperature, $a_{\rm sym}/T$. The maximum density in the system at the origin of the center of mas system during the time evolution are compared to the fragmenting source density for three different systems. The fragmenting source temperature are also shown for these three systems. The lower density for the fragmenting source, comparing to the maximum density, and the very similar density and temperature of the fragmenting source for all the three systems indicate that

a statistical freeze-out is realized at the time of the formation of the IMFs in the AMD transport model.

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反对称分子动力学模型中的冻结概念

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摘要： 给出了反对称分子动力学模型 (AMD) 计算的 50 MeV/nucleon ^{112}Sn+^{112}Sn 反应的分析结果。该研究是反对称分子动力学模型中统计冻结概念的部分研究结果。利用自洽法结合修正的 Fisher 模型，提取了发射源的温度和密度分别为 $T=(6.1\pm0.2)$ MeV，$\rho/\rho_0=0.69\pm0.03$。通过与 AMD 模型计算的系统在时间演化过程中的最大密度比较，得出碎片发射源的密度远小于系统的最大密度。利用自洽法提取的温度和密度与 35 MeV/nucleon的^{40}Ca+^{40}Ca 反应系统及 40 MeV/nucleon 的^{64}Zn + ^{112}Sn 反应系统所提取的温度和密度非常接近。该结果表明反对称分子动力学模型中，系统在中等质量碎片形成时刻处于统计冻结体积。

关键词： 统计冻结；反对称分子动力学；重离子核反应；自洽法；密度；温度

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