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Structure of Nonlocal Quark Vacuum Condensate in Non-perturbative QCD Vacuum

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Abstract: Based on the Dyson-Schwinger Equations (DSEs) with the rainbow truncation, and Operator Product Expansion, the structure of nonlocal quark vacuum condensate in QCD, described by quark self-energy functions A_f and B_f given usually by the solutions of the DSEs of quark propagator, is predicted numerically. We also calculate the local quark vacuum condensate, quark-gluon mixed local vacuum condensate, and quark virtuality. The self-energy functions A_f and B_f are given by the parameterized quark propagator functions $\sigma_v^f(p^2)$ and $\sigma_s^f(p^2)$ of Roberts and Williams, instead of the numerical solutions of the DSEs. Our calculated results are in reasonable agreement with those of QCD sum rules, Lattice QCD calculations, and instanton model predictions, although the resulting local quark vacuum condensate for light quarks, u, d, s, are a little bit larger than those of the above theoretical predictions. We think the differences are caused by model dependence. The larger of strange quark vacuum condensate than u, d quark is due to the s quark mass which is more larger than u, d quark masses. Of course, the Roberts-Williams parameterized quark propagator is an empirical formulism, which approximately describes quark propagation.

Key words: quark vacuum condensate; Non-perturbative QCD; Dyson-Schwinger Equations

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1 DSEs for fully dressed quark propagator in QCD Vacuum

The Dyson-Schwinger Equations(DSEs) are a nonperturbative means of analyzing a quantum field theory. Derived from a theory's Euclidean space generating functional, they are an enumerable infinity of coupled integral equations whose solutions are the n-point Euclidean Green functions, which are the same matrix elements estimated in numerical simulations of lattice-QCD. In theories with elementary fermions, the simplest of the DSEs is the gap equation, which is basic to studying dynamical symmetry breaking in systems. Its solution is a 2-point function, that is, the fermion propagator while its kernel involves higher n-point functions.

The 2-point Green function, quark propagator, is usually given by the solutions of DSEs at zero temperature $T=0$ under rainbow truncation $\Gamma^\nu=\gamma^\nu$. An important observation is that the general form of the inverse quark propagator $S_f^{-1}(p^2)$ can be rewrit-

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ten in momentum space[1] as

$$S_f^{-1}(p^2) = \mathrm{i}\not{p}A_f(p^2) + B_f(p^2)\ , \tag{1}$$

in a covariant gauge, with $\not{p} = \gamma^\mu p_\mu$. The propagator is renormalized at space-like point μ_p^2 according to $A_f(\mu_p^2) = 1$ and $B_f(\mu_p^2) = m_f(\mu_p^2)$ with $m_f(\mu_p^2)$ being the current quark mass whose value is empirically about $m_{\mathrm{u,d}} \simeq 5.1$ MeV for u, d quarks, and $m_{\mathrm{s}} = 127.5$ MeV for s quark.

Except for the current quark mass and perturbation corrections, the functions $A_f(p^2)$ and $B_f(p^2)$ are non-perturbative quantities, and satisfy the new set of DSEs in the Feynman gauge[1−2].

$$\left[A_f(p^2) - 1\right]p^2 = \frac{8}{3}g_s^2 \int \frac{\mathrm{d}^4 q}{(4\pi)^4} G(p-q) \times \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p\cdot q \tag{2}$$

$$B_f(p^2) = \frac{16}{3} g_s^2 \int \frac{\mathrm{d}^4 q}{(4\pi)^4} G(p-q) \times \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)}\ . \tag{3}$$

Where $G(p-q)$ is the gluon fully dressed propagator for which we know nothing since non-perturbative effect in QCD. Therefore, the numerical solutions of the DSEs are dependent on gluon propagator and truncation models.

2 Non-perturbative QCD vacuum state: quark condensates

Quark vacuum condensate is very important property of nonperturbative QCD vacuum state since QCD vacuum is densely populated by long-wave fluctuations of gluon and quark fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of gluon and quark fields. Vacuum condensates $\langle 0 |: \bar{q}(0)q(0) :| 0\rangle$, $\langle 0 |: G^a_{\mu\nu}G^a_{\mu\nu} :| 0\rangle$, $\langle 0 |: \bar{q}(0)[\mathrm{i}g_s G^a_{\mu\nu}\sigma_{\mu\nu}\frac{\lambda^a}{2}]q(0) :| 0\rangle$. the nonzero quark vacuum condensate $\langle 0 |: \bar{q}(0)q(0) :| 0\rangle$ is responsible for the spontaneous breakdown of chiral symmetry. Nonlocality of the quark condensate is characterized by the vacuum virtuality[2], $\lambda_q^2 = \frac{1}{2}\langle 0 |: \bar{q}D^2 q :| 0\rangle / \langle 0 |: \bar{q}q :| 0\rangle$, where $D_\mu = \partial_\mu - ig_s A^a_\mu \lambda^a/2$ is a covariant derivative with A^a_μ being gluon field. All of the above physical quantities are important and fundamental properties of QCD vacuum state.

2.1 Structure of nonlocal quark vacuum condensate and local quark vacuum condensate in DSEs

Upon on Ref. [3], One knows that the structure of the nonlocal quark vacuum condensate in nonperurbative QCD vacuum can be written as

$$\langle 0 |: \bar{q}(x)q(0) :| 0\rangle = (-4N_{\mathrm{c}}) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{B_f(p^2)\mathrm{e}^{\mathrm{i}px}}{p^2 A_f^2(p^2) + B_f^2(p^2)} = -\frac{3}{4\pi^2}\int_0^\infty s\mathrm{d}s \frac{B_f(s)}{sA_f^2(s) + B_f^2(s)} \frac{2J_1(\sqrt{sx^2})}{\sqrt{sx^2}}\ , \tag{4}$$

where the color number $N_{\mathrm{c}} = 3$. Using the definitions of the quark vacuum condensates given by us[3], one finds that the local quark vacuum condensate ($x = 0$) is

$$\langle 0 |: \bar{q}(0)q(0) :| 0\rangle_f = -\frac{3}{4\pi^2}\int_0^{s_0} \mathrm{d}s \frac{sB_f(s)}{sA_f^2(s) + B_f^2(s)}\ , \tag{5}$$

while the local quark-gluon mixed vacuum condensate, $\langle 0 |: \bar{q}(0)[\mathrm{i}g_s G(0)\sigma]q(0) :| 0\rangle_f$ is

$$\langle 0 |: \bar{q}(0)\left[\mathrm{i}g_s G(0)\sigma\right]q(0) :| 0\rangle = -\frac{3}{8\pi^2}\int_0^{s_0} \mathrm{d}s s^2 \frac{B_f(s)}{sA_f^2(s) + B_f^2(s)}\ . \tag{6}$$

Eq. (6) is an important quantity in describing quark virtuality. On principle, solving the DSEs of Eqs. (2∼3) with the gluon propagator, we can obtain the solutions A_f and B_f, in QCD vacuum state. But since we really know nothing about gluon propagator at the present and have to truncate the DSEs when we solve the infinite coupled equation, the numerical solutions of the DSEs are still approximate answers to real reality.

2.2 Roberts and Williams parameterized quark fully dressed propagator

The DSEs can not be solved rigorously due to the unknowing gluon propagator and the infinite series of DSEs itself, and have to do truncation as well as to solve Bethe-Salpeter Equations for getting the vertex Γ^ν. Roberts and Williams[4] have proposed a parameterized quark propagator S_f using σ_v^f and σ_s^f which have used extensively in the literatures.

$$A_f(p^2)=\frac{\sigma_v^f}{(\sigma_s^f)^2\left[p^2(\sigma_v^f/\sigma_s^f)^2+1\right]}\ , \qquad (7)$$

$$B_f(p^2)=\frac{1}{\sigma_s^f\left[p^2(\sigma_v^f/\sigma_s^f)^2+1\right]}\ , \qquad (8)$$

where $\sigma_s^f=\bar{\sigma}_s^f/\Lambda$, and $\sigma_v^f=\bar{\sigma}_v^f/\Lambda^2$. They are expressed in terms of the following forms

$$\bar{\sigma}_s^f(x)=\frac{\left[1-\exp(-b_1^f x)\right]}{b_1^f x}\cdot\frac{\left[1-\exp(-b_3^f x)\right]}{b_3^f x}\times \left[b_0^f+b_2^f\frac{1-\exp(-\Lambda' x)}{\Lambda' x}\right]+ \bar{m}_f\frac{1-\exp\left[-2(x+\bar{m}_f)\right]}{x+\bar{m}_f^2}\ , \qquad (9)$$

$$\bar{\sigma}_v^f(x)=\frac{2(x+\bar{m}_f^2)-1+\exp\left[-2(x+\bar{m}_f^2)\right]}{2(x+\bar{m}_f^2)^2}\ . \qquad (10)$$

Where $\bar{m}_f=m_f/\Lambda$, $x=p^2/\Lambda^2$, $\Lambda'=10^{-4}$. $\Lambda=0.566$ GeV. The parameters b_i^f ($i=0$, 1, 2, 3) and m_f ($f=$u, d, s) are listed in Table 1.

Table 1 Parameters b_i^f and m_f (MeV) of the light quarks: u, d and s[2].

flavor	b_0^f	b_1^f	b_2^f	b_3^f	m_f/MeV
u	0.131	2.90	0.603	0.185	5.1
d	0.131	2.90	0.603	0.185	5.1
s	0.105	2.90	0.740	0.185	127.5

To overcome the difficulty of solving DSEs, we use Roberts-Williams parameterized quark propagator to predict the structure of nonlocal quark vacuum condensate, $\langle 0 |: \bar{q}(x)q(0) :| 0\rangle$, and calculate local quark vacuum condensate $\langle 0 |: \bar{q}(0)q(0) :| 0\rangle$, quark-gluon mixed local vacuum condensate $\langle 0 |: \bar{q}(0)[\mathrm{i}g_sG\sigma]q(0) :| 0\rangle$ as well as quark virtuality for the light quarks. The propagator of the Roberts-Williams formulism has no Lehmann representation and hence there are no quark production thresholds in any calculations of observable. The absence of such thresholds admits the interpretation that the Roberts-Williams quark propagator describes the propagator of a confined quark. We present our theoretical predictions in the Sect. 3.

3 Local quark vacuum condensate and structure of nonlocal condensate of light quarks

We put the Roberts-Williams parameterized functions A_f and B_f of Eq. (7) and Eq. (8) into Eq. (4) and Eqs. (5∼6) with formulae in Eqs. (9∼10) and parameters given in Table 1, to obtain nonlocal quark vacuum condensate structure function, local quark vacuum condensate and quark-gluon mixed vacuum condensate in Eqs. (4∼6). With the up limit of the integration over s is taken to be 10 GeV^2, the calculating results are given in Table 2 and Fig. 1 in the following subsections 3.1, and 3.2.

3.1 Local quark vacuum condensate and quark gluon mixed vacuum condensate

According to Eqs. (4∼6), the local quark vacuum condensate and quark-gluon mixed vacuum condensate are numerically produced and are given in Table 2.

Table 2 Local quark condensate and local quark-gluon mixed condensates in QCD vacuum state.

quarks	$\langle 0 \|: \bar{q}(0)q(0) :\| 0\rangle$	$\langle 0 \|: \bar{q}(0)[\mathrm{i}g_sG(0)\sigma]q(0) :\| 0\rangle$
u, d	−0.0464 GeV^3	−0.0875 GeV^5
s	−0.1443 GeV^3	−0.3310 GeV^5

Using the calculated results given in Table. 2, we also predict the quark virtuality $\lambda_q^2=\frac{1}{2}\langle 0 |: \bar{q}(0)[\mathrm{i}g_sG(0)\sigma]q(0) :| 0\rangle/\langle 0 |: \bar{q}(0)q(0) :| 0\rangle$ as mentioned before[3], which describes mean squared momentum of quark in QCD vacuum state. The predicted results are

$$\lambda_{\mathrm{u,d}}^2=0.9430\ \mathrm{GeV}^2\ ,\quad \lambda_{\mathrm{s}}^2=1.1469\ \mathrm{GeV}^2\ . \qquad (11)$$

All predicted values for the λ_q^2 are in reasonable agreement with the other calculations[5]. For example, the acceptable value of $\lambda_{\mathrm{u,d}}^2$ is in the range of $0.4\sim1.0$ GeV^2[5−6], and the standard QCD sum rule's estimation gives $\lambda^2_{\rm u,d} = (0.4 \pm 0.1)$ GeV^2[4-7]. The prediction of Lattice QCD gives $\lambda^2_{\rm s} = 2.50$ GeV^2[8], the instanton model calculation produces $\lambda^2_{\rm s} = 1.40$ GeV^2[9]. Our $\lambda^2_{\rm s}$ is consistent with those in Refs. [8-9].

3.2 Structure of nonlocal quark vacuum condensate in nonperturbative QCD

Substituting Eqs. (7∼8) into Eqs. (4∼6) and using of the parameters in Table 1 appearing in Eqs. (9∼10), we carry out the integration Eq. (4∼6) over p^2, ($s = p^2$), and obtain the structure of nonlocal quark vacuum condensate function. The theoretical predictions are shown in Fig. 1.

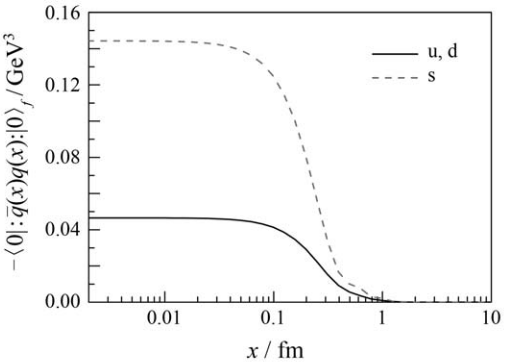

Fig. 1　The structure of the nonlocal quark vacuum condensate in non-perturbative QCD vacuum. Dashed curve denotes the strange quark structure function, while solid curve represents structure function of u and d quarks.

The dashed curve denotes the structure of strange quark nonlocal vacuum condensate, while solid line stands for u, d quark vacuum condensate structure function.

As is seen from Fig. 1, the structures of light quark u, d and s have the same behavior and simultaneously arrive at zero point, although the magnitude of s quark condensate is larger than those of u, d quark. The reason of large value of s quark is that it's mass is more larger than u, d quark mass ($m_{\rm s} \approx 117$ MeV, and $m_{\rm u,d} \approx 3$ to 5 MeV).

4 Summary and concluding remarks

Based on the DSEs for quark propagator, which determine the structure function of nonlocal quark vacuum condensate, local quark vacuum condensate and local quark-gluon mixed vacuum condensate, we predicted the structure of the nonlocal quark vacuum condensate, and calculated the local quark vacuum condensate as well as quark virtuality, using Roberts-Williams parameterized quark propagator of quark. We obtained these nonperturbative QCD properties of quarks which are very important for studying QCD vacuum dynamics. Our predicted results are in reasonable agreement with other calculations[5-9], although our predictions are a little bit larger than the others in magnitude.

The propagator of the Roberts-Williams formulism has no Lehmann representation and hence there are no quark production thresholds in any calculations of observable. The absence of such thresholds admits the interpretation that the Roberts-Williams quark propagator describes the propagator of a confined quark. The results evidently show that the Roberts-Williams propagator is useful approximation in practise, which produced a good virtuality of quark and behavior of nonlocal quark vacuum condensate in QCD vacuum state. The properties of QCD vacuum state are naturally predicted, if even it is parameterized form.

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非微扰 QCD 真空中夸克真空凝聚的结构

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摘要： 基于 Dyson-Schwinger 方程 (DSEs) 所确定的夸克传播子和算符成积展开 (OPE)，在彩虹近似下，预言了 QCD 真空中非定域夸克真空凝聚的结构。这种结构由夸克自能函数 A_f 和 B_f 决定，通过数值求解 DSEs 就可以得到这些自能函数。但是，直接数值求解 DSEs 方程非常复杂，这里采用 Roberts 和 Williams 提出的参数化方法，用参数化的夸克传播函数 $\sigma_v^f(p^2)$ 和 $\sigma_s^f(p^2)$ 计算夸克自能函数。同时，也计算了定域的夸克真空凝聚值，夸克胶子混合的真空凝聚值，以及夸克的虚度。理论预言和计算结果均与标准 QCD 求和定则、格点 QCD 和瞬子模型的理论结果大致相符。和这些模型的结果相比，参数化方法得到的轻夸克(u, d, s)的定域真空凝聚偏大，这主要是由于模型依赖导致的。与 u，d 夸克相比，s 夸克的真空凝聚比较大，这是因为 s 夸克自身质量较大的缘故。当然，Roberts-Williams 参数化的夸克传播子只是一个经验公式，只能近似描述夸克的传播。

关键词： 夸克真空凝聚；非微扰QCD；Dyson-Schwinger 方程

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