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# Excitation Energy Dependence of Level-density Parameters and Decay Properties of Heavy Nuclei

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**Abstract:** Based on the stochastic approach to fission, we have demonstrated that the ratio of level-density parameters at saddle to that at ground-state configuration, $a_{\rm f}/a_{\rm n}$, plays a significant role in the decay of an excited nucleus. It modifies not only the sensitivity of evaporation residue cross section and its spin distributions to nuclear friction, but also the dissipation properties of hot nuclei. Furthermore, we find that $a_{\rm f}/a_{\rm n}$ as a function of excitation energy appreciably affects the evolution of excitation energy at scission with the friction strength. A consequence of a change of $a_{\rm f}/a_{\rm n}$ with $E^*$ on the stability of superheavy nuclei and on modelling de-excitation processes of spallation reactions is indicated.

**Key words:** level density parameter; excitation energy; fission; stochastic model

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## 1 Introduction

The nuclear level density is an essential ingredient in calculating the statistical decay of a compound nucleus (CN) by particle evaporation and fission. So, it is generally used for studying a variety of nuclear reactions, such as fusion, multifragmentation and spallation. It is also a crucial input for modelling the decay of an excited superheavy nucleus.

For a spherical nucleus of mass number $A$ at excitation energy $E^*$ and spin $J$, the nuclear level density is often approximated by the Fermi-gas form[1] as follows:

$$\rho_{\rm FG}(E^*,J)=(2J+1)\left(\frac{\hbar^2}{2I}\right)^{3/2}\frac{\sqrt{a}}{12}\frac{\exp(2\sqrt{aU})}{U^2}\ , \qquad (1)$$

where $U=E^*-E_{\rm rot}(J)$ is the thermal excitation energy and $E_{\rm rot}(J)$ is the rotational energy. $I$ is the moment of inertia. $a$ is the nuclear level-density parameter and is parametrized as[2]

$$a=\tilde{a}\left\{1+\frac{\delta W}{U}\left[1-\exp\left(\frac{-U}{E_{\rm d}}\right)\right]\right\}\ , \qquad (2)$$

where $\tilde{a}$ is the smoothed level-density parameter. $\delta W$ is the shell correction to the liquid-drop mass, and $E_{\rm d}$ (around 20 MeV) is the rate at which the shell effect is depleted with the increase in excitation energy.

At high energy shell corrections are negligible. We focus here on the role of $\tilde{a}$, especially its excitation energy dependence, on nuclear decay properties.

It is well known that the parameters $A$, $E^*$ and $J$ have a strong influence on the decay process of heavy nuclei. The role of the parameter $a_{\rm f}/a_{\rm n}$, which is the ratio of (smoothed) level-density parameters at saddle to that at ground-state configuration, was also noticed in many previous analyses. For example, Lott et al.[3] found that to reproduce decay products of excited CNs, it is necessary to adjust the magnitude of $a_{\rm f}/a_{\rm n}$. Apart from these parameters, a great deal of work has shown that nuclear friction $\beta$ is a key to accounting for the enhancement of particle multiplicities observed in heavy-ion induced fusion-fission reactions. It has been demonstrated in numerical studies[4–10] that as $E^*$ rises a certain value, the measured neutron multiplicity deviates significantly from the calculated values using standard statistical models (SSM), and that the underlying reason for this discrepancy is due to the neglect of friction effects in nuclear fission. The friction effects evidently modify the competition between particle

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evaporation and fission. How to accurately extract $\beta$ from a multitude of fission data has been the focus of intensive researches. In recent studies[11–16] we have shown that to precisely determine $\beta$, it is important to consider the effects coming from the change in the ratio $a_f/a_n$ throughout the entire deexcitation cascade.

## 2 Model

We employ the following one-dimensional over-damped Langevin equation[17] to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{T}{M\beta}\frac{dS}{dq} + \sqrt{\frac{T}{M\beta}}\Gamma(t)\ . \tag{3}$$

Here $q$ is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, $M$ is the inertia parameter, and $\beta$ is the dissipation strength. The temperature in Eq. (3) is denoted by $T$ and $\Gamma(t)$ is a fluctuating force with $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(t')\rangle = 2\delta(t-t')$. The driving force of the Langevin equation is calculated from the entropy:

$$S(q,E^*) = 2\sqrt{a(q)[E^* - V(q)]}\ , \tag{4}$$

$E^*$ is the total internal energy of the system. Equation (4) is constructed from the Fermi-gas expression with a potential $V(q)$ in the $\{c, h, \alpha\}$ parametrization[18]. The $q$-dependent surface, Coulomb, and rotation energy terms are included in the potential $V(q)$.

In constructing the entropy, the deformation-dependent level-density parameter is used:

$$a(q) = a_1 A + a_2 A^{2/3} B_s(q)\ . \tag{5}$$

where $a_1$ and $a_2$ are constants. $B_s$ is the dimensionless surface area (for a sphere $B_s = 1$) which can be parametrized by the analytical expression[19]

$$B_s(q) = \begin{cases} 1 + 2.844(q-0.375)^2, & \text{if } q < 0.452 \\ 0.983 + 0.439(q-0.375), & \text{if } q \geqslant 0.452 \end{cases}\ . \tag{6}$$

Evaporation of prescission light particles along Langevin fission trajectories from their ground state to their scission points has been taken into account. The emission widths of light particles are calculated with Blann's formula[20].

## 3 Results

In contrast with earlier studies (e.g. Ref. [3]) where the $a_f/a_n$ parameter is adjustable, in our Langevin calculation it is completely governed by fission dynamics, not a free parameter. We first define the locations of saddle and ground state by the stationary points of entropy[17, 21], and then search these locations by comparing the magnitude of $S(q)$ at different $q$. After that, using Eq. (5) to compute the values of the level-density parameter at the saddle-point deformation ($a_f$) and at the ground state ($a_n$).

In Ref. [11] we calculated evaporation residue cross sections versus $\beta$ for systems $^{190}$Os and $^{210}$Po which have the same neutron-to-proton ratio but have a different fissility. Fig. 1 exhibits that the sensitivity to friction differs very much for the two systems. The reason is that $a_f/a_n$ is a function of fissility of the decaying system. The parameters $a_f/a_n$ and $\beta$ have an opposite role on fission rates. A greater $a_f/a_n$ increases the fission probability, which offsets the friction effects. As a result, a difference in fissility leads to a different sensitivity to $\beta$. A like effect has also been revealed[12] for the spin distributions of evaporation residue cross sections (Fig. 2). We also surveyed the sensitivity under two contrast conditions of (high $E^*$, low $\ell$)

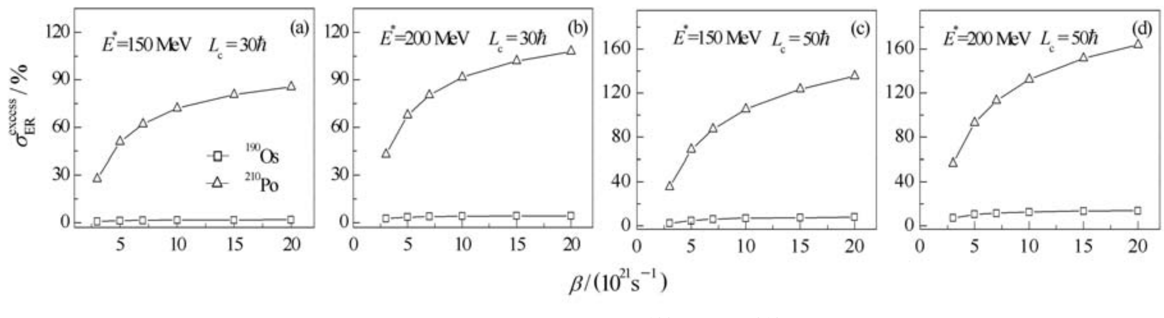

Fig. 1 Dynamical excess of the evaporation residue cross section of $^{190}$Os and $^{210}$Po relative to that of the standard statistical model prediction as a function of the dissipation strength $\beta$ at two excitation energies $E^*$ = 150 MeV and 200 MeV and two critical angular momenta $\ell_c$ = 30ħ and 50ħ. Taken from Ref. [11].

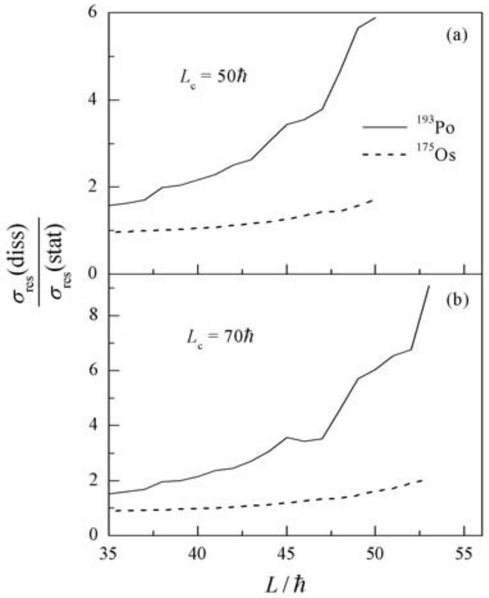

Fig. 2 Ratios of the evaporation residue spin distributions above a given spin evaluated at a dissipation strength of $5\times10^{21}$ s$^{-1}$ to that predicted by standard statistical models for systems $^{193}$Po (solid line) and $^{175}$Os (dotted line) at an excitation energy $E^*_{\mathrm{tot}}$ = 150 MeV and two critical angular momenta $\ell_c$ = 50$\hbar$ and 70$\hbar$. Taken from Ref. [12].

and (low $E^*$, high $\ell$) [13] that can be provided via peripheral heavy-ion collisions at relativistic energies and fusion mechanisms, respectively. The calculated results are displayed in Fig. 3. A significant change of the sensitivity to $\beta$ under the two different conditions is also due to an appreciable difference in the $a_f/a_n$ value.

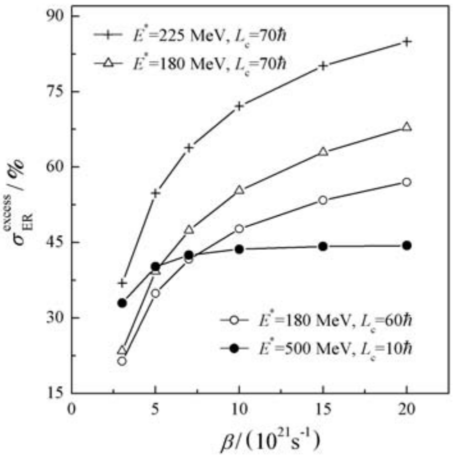

Fig. 3 Comparison of the excess of the evaporation residues over its standard-statistical values versus the dissipation strength $\beta$ for $^{200}$Hg nuclei between case (1) $E^*$ = 500 MeV, $\ell_c$ = 10$\hbar$ and case (2) $E^*$ = 180 MeV, $\ell_c$ = 60$\hbar$, 70$\hbar$; $E^*$ = 225 MeV, $\ell_c$ = 70$\hbar$. Taken from Ref. [13].

We further employed the reactions $^{16}$O+$^{181}$Ta[22] and $^{3}$He+$^{197}$Au[23] which produce similar Tl CNs that have the same excitation energy but have different angular momenta to analyze the role of $a_f/a_n$ as a function of $\ell$ in the decay mechanism of hot CNs. Rubehn et al.[23] used the statistical model of Bohr-Wheeler without friction to reproduce measured excitation function of fission cross sections of the $^{3}$He+$^{197}$Au reaction (Fig. 4).

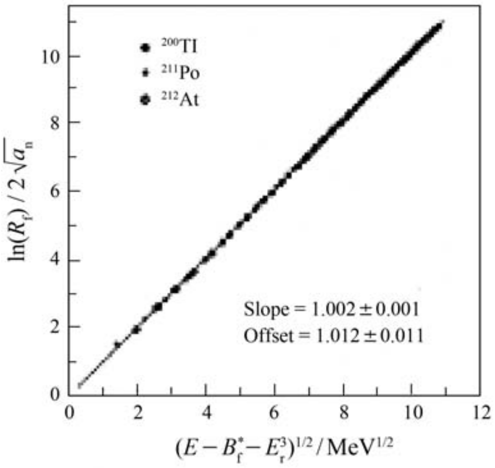

Fig. 4 The quantity $\ln R_f/2\sqrt{a_n}$ vs the square root of the intrinsic excitation energy over the saddle for fission of $^{200}$Tl, $^{211}$Po and $^{212}$At compound nuclei. The straight line represents a fit to the entire data set. Taken from Ref. [23].

However, Fig. 5 shows that SSM calculations underpredict significantly the multiplicity data of the $^{16}$O+$^{181}$Ta reaction, indicating that friction effects in fission need to be accounted for. To precisely determine $\beta$, we carried out a calculation by taking a numbers of $\beta$ values. A best fit to the measured neutrons can be obtained at $\beta = 3.5\times10^{21}$ $s^{-1}$, illustrating that fission induced in the reaction is overdamped. The reason leading to the contrast behavior of fission in the two reactions is that $\ell$ changes the magnitude of $a_f/a_n$. The comparison suggests that nuclear dissipation could have an $\ell$ dependence in addition to the known $T$ and $q$ dependence. We now discuss the excitation energy dependence of $a_f/a_n$ and its influence on the decay of hot nuclei. In previous studies, for example, Charity[24] has found that to fit energy spectra of evaporated protons or $\alpha$ particles in coincidence with evaporation residues (Fig. 6), $\tilde{a}$ is large at low $E^*$ and small at higher $E^*$, suggesting that $\tilde{a}$ must be dependent on $E^*$.

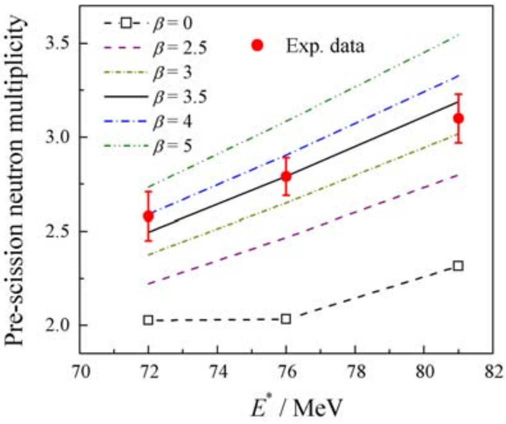

Fig. 5 (color online) Fits to measured prescission neutrons in the $^{16}$O+$^{181}$Ta system[22]. The different lines correspond to the calculation results at different friction strengths. Note that $\beta$ represents the friction strength throughout the fission process and is only adjustable parameter in our calculation, and that $\beta = 0$ denotes the standard statistical model calculation. The unit of $\beta$ is $10^{21}$ s$^{-1}$. The figure is taken from Ref. [14].

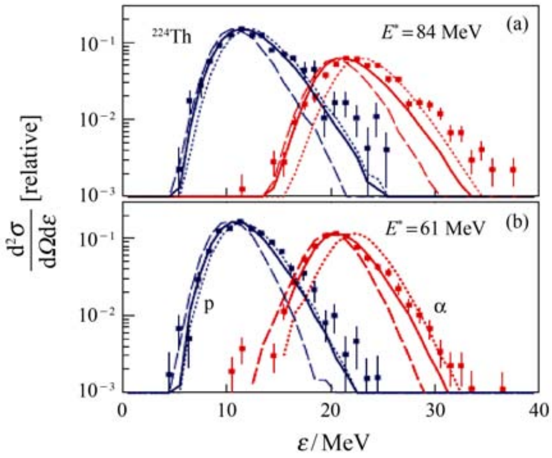

Fig. 6 (color online) Center-of-mass kinetic-energy spectra of α particles and protons detected in coincidence with evaporation residues formed in $^{16}$O+$^{208}$Pb reactions. Experimental results are shown by dotted points. The curves show spectra predicted with GEMINI model. Taken from Ref. [24].

Different from Charity's work, we chose fission processes to probe the role of $a_{\rm f}/a_{\rm n}(E^*)$. To this end, we employed the observable quantity, the excitation energy at scission $E^*_{\rm sc}$[15], which is determined by using energy conservation law

$$E^* = E^*_{\rm sc} + E_{\rm coll} + V(q) + E_{\rm evap}(t_{\rm sc}) \ . \tag{7}$$

Here $E_{\rm coll}$ is the kinetic energy of the collective degrees of freedom, and $E_{\rm evap}(t_{\rm sc})$ is the energy carried away by all evaporated particles by the scission time $t_{\rm sc}$. The Eq. (7) has been demonstrated[25] in the high energy fission case to describe excellently the experimental $E^*_{\rm sc}$ for a great number of fissioning systems which cover a wide range of fissilities and CN mass regions.

We displayed in Fig. 7 the $E^*_{\rm sc}$ vs. $\beta$ for Hg CNs. From the figure, one can easily see that with rising $E^*$ (for example, at $E^* = 350$ MeV) a little variation of $\beta$ yields a marked change in $E^*_{\rm sc}$, suggesting a significant increase in the sensitivity of $E^*_{\rm sc}$ to $\beta$ at high energy.

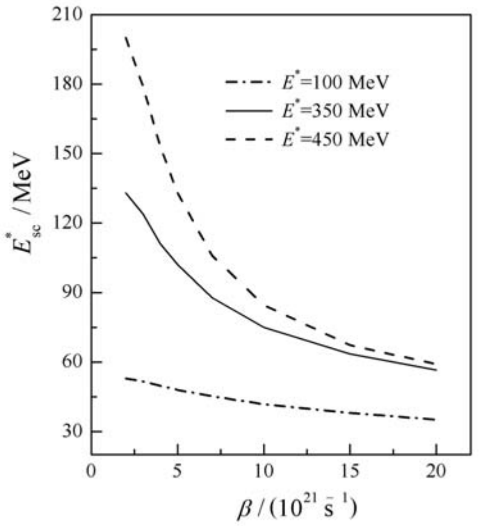

Fig. 7 Excitation energy at scission $E^*_{\rm sc}$ as a function of friction strength $\beta$ at critical angular momentum $\ell_{\rm c} = 10\hbar$ for initial excitation energies of $^{200}$Hg compound systems $E^* = 100$, 350 and 450 MeV. Taken from Ref. [15].

The mechanism responsible for the result is due to a substantial difference in the number of emitted prescission light particles with increasing $E^*$. It was observed from Fig. 8 that neutron multiplicity becomes larger at high energy, and weak decay channels like light charged particles (LCPs) have a sizeable decay probability at $E^* = 350$ MeV. In addition to shortening the particle evaporation time, high energy also favors to help LCPs overcome their Coulomb emission barriers. Furthermore, excitation energy significantly affects the magnitude of $a_{\rm f}/a_{\rm n}$, which has a consequence on the particle multiplicity at different energies. We explain it in the following way. Entropy $S$ varies with $E^*$, a change in $E^*$ shifts the position of the stationary points of the entropy. Raising $E^*$ makes the saddle-point position get closer to the ground-state configuration (see Fig. 9), i.e. the saddle-point deformation becomes smaller

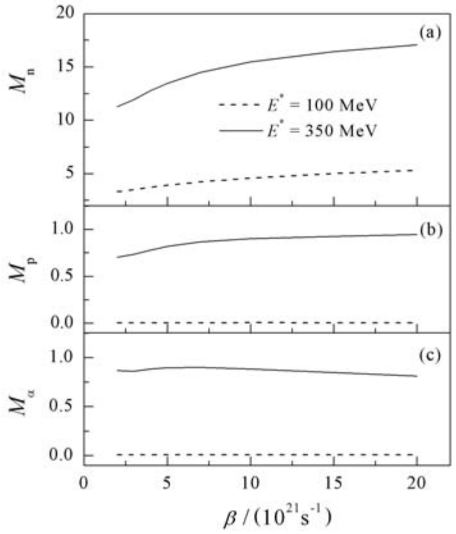

Fig. 8 Comparison of evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) for $^{200}$Hg compound nuclei between case (1) $E^* = 350$ MeV, $\ell_c = 10\hbar$ and case (2) $E^* = 100$ MeV, $\ell_c = 10\hbar$. Taken from Ref. [15].

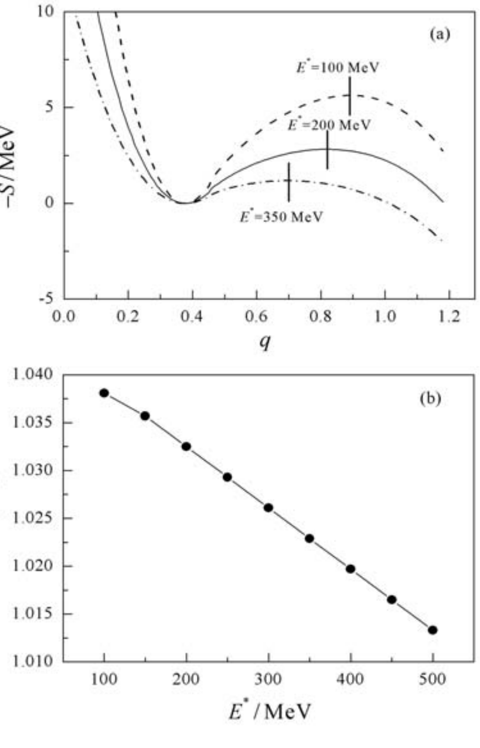

Fig. 9 (a) The negative entropies $-S(q)$ normalized to zero at the spherical shape for $^{200}$Hg at angular momentum $\ell = 10\hbar$ are plotted as a function of deformation coordinate $q$ for three excitation energies $E^* = 100$, 200, and 350 MeV. The vertical dotted lines indicate the location of saddle points. (b) Ratio of level-density parameters at saddle to that at ground-state configurations ($a_f/a_n$) as a function of excitation energy $E^*$ for $^{200}$Hg at $\ell = 10\hbar$. Taken from Ref. [15].

with respect to the location of the ground state. This decreases the values of $a_f$ and $a_f/a_n$. A smaller $a_f/a_n$ at high energy more effectively reduces the fission width, favoring the particle emission.

An enhanced neutron emission as well as the open of LCP decay channels carry away more energy from the decaying system. In addition, under a high-energy condition a longer fission time at a larger $\beta$ more strongly influences prescission particles because of a shorter particle evaporation time. These lead to a quicker cooling of the fissioning system and, correspondingly, a larger sensitivity of $E^*_{sc}$ to $\beta$ at high energy. The result has been found to be robust with respect to a change in the spin of CNs[16].

## 4 Outlook

The finding that $a_f/a_n$ depends on $E^*$ could influence the survival probability of 'hot' superhevay nuclei, because it enhances weak decay modes, i.e. neutron evaporation, with increasing energy; that is, it can increase the stability of superheavy nuclei. The conclusion has an implication for the prediction of the evaporation residue cross sections. A calculation that takes the effect into account, which is neglected in most of current calculations of neutron emission and fission in the decay of superheavy nucleus, is underway.

Experimentally, both heavy-ion reactions and energetic protons-nucleus collisions have been applied to yield highly excited decay systems. The spallation reaction is typical for proton-induced production of highly excited nuclei without major distortions due to compression, deformation and high spins which are characteristics of heavy-ion-induced reactions. These distortions complicate the description of de-excitation process of an excited nucleus. Thus, to better explore the dependence of $a_f/a_n$ on $E^*$, one can employ proton-nucleus reactions[26–27] to investigate the evolution of decay property of formed CNs with energy. Due to pre-equilibrium particle evaporation, the residual hot CNs produced in spallation have a distribution in their $E^*$. So, one can select different ranges of $E^*$ to analyze the role of $\frac{a_f}{a_n}$, facilitating a study of $\frac{a_f}{a_n}$ as a function of $E^*$.

Because spallation reactions have been used in diverse areas of applications, such as nuclear data evaluation, the finding of $\frac{a_\mathrm{f}}{a_\mathrm{n}}(E^*)$ suggests that to better describe the de-excitation stage in spallation, stochastic models to fission are needed. This actually requires a coupling of current Langevin approach to fission to intranuclear cascade models[28], quantum molecular dynamics models[29], etc. Work along this direction is in progress.

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# 激发能相关的能级密度参数和重核衰变性质

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**摘要：** 基于裂变的随机模型，研究了能级密度参数在鞍点与其基态处的比值$a_\mathrm{f}/a_\mathrm{n}$在热核衰变过程中的作用。计算表明，$a_\mathrm{f}/a_\mathrm{n}$的大小能显著影响蒸发剩余截面及其自旋分布的大小以及裂变过程中热核的耗散行为。发现，能级密度参数对激发能的依赖性$a_\mathrm{f}/a_\mathrm{n}(\mathrm{E}^*)$能改变断点处的激发能对摩擦强度的敏感性。最后指出了$a_\mathrm{f}/a_\mathrm{n}$随$E^*$的变化对超重核的稳定性和散裂反应退激阶段描述的意义。

**关键词：** 能级密度参数；激发能；裂变；随机模型

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