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# Analytical Study on Beam Steering in Low- $\beta$ Superconducting Quarter Wave Resonators<sup>\*</sup>

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**Abstract:** Superconducting(SC) cavities currently used for acceleration of ions in the velocity range from  $0.01c$  to  $0.3c$  are based frequently on quarter-wave resonators(QWR). Various types of QWR cavities across a frequency extent from 50 to 240 MHz have been constructed or are proposed for numerous applications. A drawback of this kind of cavities is the beam steering caused by transverse magnetic and electric field components, which can create emittance growth. In this paper, the analytical studies on beam steering in quarter wave resonator of frequency=80.5 and 161 MHz of the SC linac has been done in Institute of Modern Physics, Chinese Academy of Sciences, the calculated results have told us that the correction for beam deflection should be a consideration during the design of the QWR cavity, which generally involves the shaping the inner drift tube.

**Key words:** beam steering; low- $\beta$ ; quarter-wave resonator

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## 1 Introduction

The most appealing aspect of superconducting radio frequency(SRF) accelerating cavities is that they can operate in continuous wave(CW) mode or long pulse mode and provide a high accelerating gradient owing to low heat loss. The quality factor  $Q$  of SRF cavities is very high since the surface resistance of SRF cavities is 5 orders lower than that of copper cavities. SRF technology is widely used in particle accelerators around the world. Superconducting linear accelerators in the energy range from 5 MeV up to about 200 MeV are being widely studied in many laboratories for acceleration of protons and heavy ions.

The main strong points of using short, independently phased superconducting cavities, rather than large, normal conducting drift tube linacs

(DTL), are the high accelerating gradient, the high efficiency and the possibility of accelerating particles with different  $q/A$  in the same linac. Different geometries, like Spoke, Half-wave resonators(HWR), Quarter-wave resonators(QWR) and Reentrant have been proposed for beam velocities up to  $\beta=0.5$ , where multicell type superconducting cavities start to achieve a good efficiency. Among them, QWRs are preferable in comparison to other geometries due to their simplicity, accessibility and low fabrication cost<sup>[1, 2]</sup>.

One shortcoming of these superconducting QWRs is the beam steering because of the lack of symmetry about the beam axis, and then causes emittance growth. Both magnetic and transverse electric fields produce steering in the direction of the resonator axis, and a possible source of emit-

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tance increase is the change of these field components along the resonator axis.

In this paper, simple study of this beam steering has been carried out for the  $f=80.5$  MHz and  $f=161$  MHz QWRs with cylindrical inner conductors (see Fig. 1).

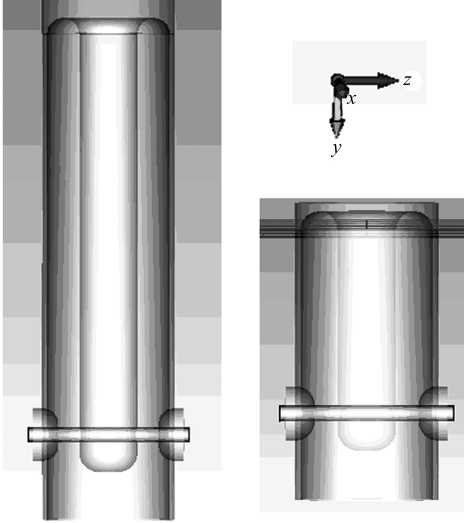


Fig. 1 The CST Microwave Studio Model (version 2009) of QWRs with  $f=80.5$  MHz (left) and  $f=161$  MHz (right).

## 2 Beam Steering in QWR

An analytical expression of the beam steering caused by transverse field components, on the basis

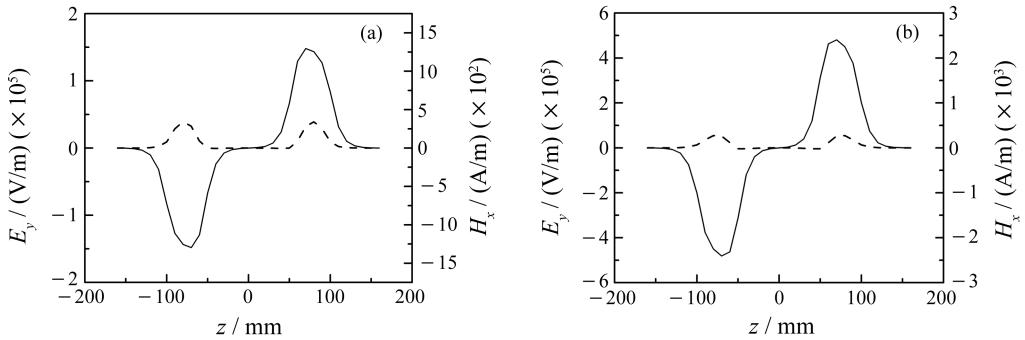


Fig. 2 Field distribution along the beam axis at the accelerating field of 1 MV/m ( $f=80.5$  MHz (a),  $f=161$  MHz (b)). The origin of the coordinates is at the center of the resonator drift tube.

The geometrical  $\beta$  value,  $\beta_G$  is defined as the velocity at which a particle crosses one cell in one-half of an RF period. For multicell elliptical cavi-

ties with  $\beta_G=1$ , the difference between optimum  $\beta$ ,  $\beta_{opt}$  and  $\beta_G$  is negligible, but for lower  $\beta$  cavities the difference becomes significant. Fig. 3 shows the

$$\Delta y' = -\frac{\Delta U}{\gamma m c^2 \beta} \text{tg} \varphi \frac{\cos\left(\frac{\pi d_y}{\beta \lambda}\right)}{\beta \sin\left(\frac{\pi d}{\beta \lambda}\right)} K_{EY}(y) - \frac{\Delta U}{\gamma m c^2 \beta} (\text{tg} \varphi) c K_{BX}(y), \quad (1)$$

where  $\Delta y' \approx \Delta p_y / p$  is the deflection angle caused by the resonator;

$$\left\{ -\frac{\Delta U}{\gamma m c^2 \beta} \text{tg} \varphi \frac{\cos\left(\frac{\pi d_y}{\beta \lambda}\right)}{\beta \sin\left(\frac{\pi d}{\beta \lambda}\right)} K_{EY}(y) \right\}$$

represents the electric deflection and

$$\left\{ -\frac{\Delta U}{\gamma m c^2 \beta} (\text{tg} \varphi) c K_{BX}(y) \right\}$$

represents the magnetic deflection;  $\Delta U$  is the particle energy gain;  $m$  is the rest mass;  $\varphi$  is the RF phase;  $d$  is the gap-to-gap (center) distance, and  $d_y$  is an effective gap-to-gap distance for the transverse electric field  $E_y$ ;  $K_{EY}(y) = \overline{E_y(y)} / \overline{E_z}$  and  $K_{BX}(y) = \overline{B_x(y)} / \overline{E_z}$  (mean values calculated in one accelerating gap and normalized to the accelerating field on the beam axis). The field distributions, calculated with the Microwave Studio code (version 2009), are shown in Fig. 2 for both the cavities.

ties with  $\beta_G=1$ , the difference between optimum  $\beta$ ,  $\beta_{opt}$  and  $\beta_G$  is negligible, but for lower  $\beta$  cavities the difference becomes significant. Fig. 3 shows the

transit time factor of both the cavities. It can be seen that for the  $f = 80.5$  MHz (a),  $\beta_G = 0.085$  QWR, the velocity acceptance is from  $0.05 c$  to

$0.21 c$  and optimum  $\beta$  is  $0.087$ ; for the  $f = 161$  MHz (b),  $\beta_G = 0.085$  QWR, the velocity acceptance is from  $0.1 c$  to  $0.5 c$  and optimum  $\beta$  is  $0.167$ .

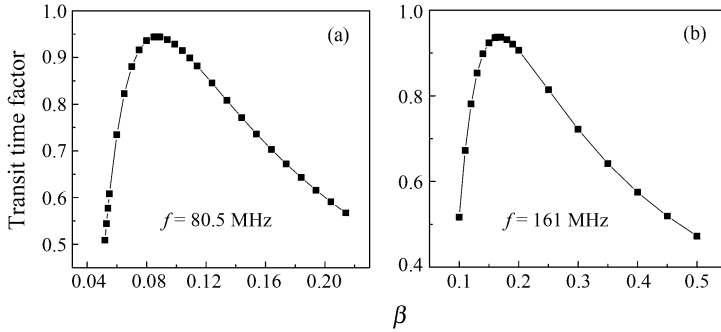


Fig. 3 Cavity transit time factor as a function of  $\beta$  ( $f=80.5$  MHz cavity (a) and  $f=161$  MHz cavity (b)).

The value of  $y$  in Eq. (1) should be the same as the beam center position of the beam center. If the beam is transported on axis,  $y$  should be 0. In the range of velocity acceptance of both the cavi-

ties, the magnetic, electric and total deflections of proton beam are calculated using Eq. (1) and presented in Fig. 4.

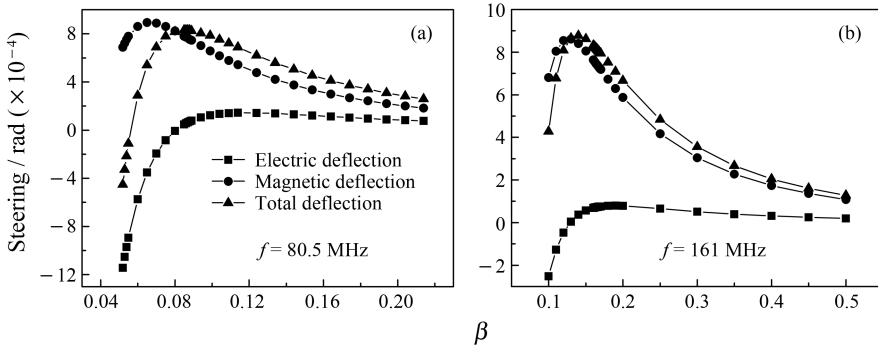


Fig. 4 Proton beam deflection calculated using Eq. (1) at  $1$  MV/m and  $\phi = -30^\circ$ .

### 3 Discussion

(1) From the deflection angle calculated with Eq. (1) shown in Fig. 4, it can be seen that the maximum steerings of the two QWRs are very close. Since normally, the magnetic deflection is the main component in the total deflection, affecting factors of the magnetic deflection will play an important role. In the Eq. (1), the value of “ $K_{BX}(0)$ ” rises with the increase of frequency, however, the item “ $1/(\gamma\beta) = \sqrt{1-\beta^2}/\beta$ ” is in a decline pattern with the mounting of  $\beta$  (see in Fig. 5). Therefore, even in the relatively low-frequency QWR with  $f=80.5$  MHz, the problem of

steering cannot be negligible.

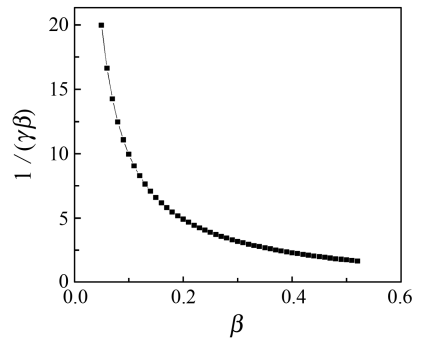


Fig. 5 The variation of  $1/(\gamma\beta)$  as a function of  $\beta$ .

(2) In addition, due to the proportional relation with the ratio  $q/A$ , the deflection will be particularly severe for high  $q/A$  particles. For in-

stance, when the two types of QWR working at 5.62 MV/m (for  $f=80.5$  MHz) and 5.20 MV/m (for  $f=161$  MHz) respectively, the deflection might be as high as 4.6 mrad for proton beam.

Two solutions can be applied for this problem, firstly, shaping of the drift-tube should be taken on the basis of cylindrical inner-conductor QWR; secondly, focusing elements should be included in the cryomodule.

In the QWR cavity, the asymmetry with respect to the horizontal plane causes the beam steering in the vertical plane. In addition, even in the low frequency cavity, we still cannot say this issue can be neglectful because the beam accelerated by the low frequency cavity usually has low velocity which will lead to large magnetic deflection. Especially in the high  $q/A$  case, the beam steering can be a big problem since the effect is proportional to

the ratio of charge to mass. An effective solution to this problem is the titling of the drift tube face, and appropriate focusing elements should be included in the linac lattice.

### 参考文献 (References):

- [1] Kakutani N, Nakayama K, Tanabe Y, *et al.* Study on Superconducting Quarter Wave Resonator for CW Intense Ion Linac. Proceedings of the 6th European Particle Accelerator conference. Stockholm; Stockholm City Congress Centre, 1998, 1858—1860.
- [2] Ceperly P H, Sokolowski J S, Ben-Zvi I, *et al.* Nucl Instr and Meth, 1976, **136**: 421.
- [3] Facco A, Zviagintsev V. Study on Beam Steering in Intermediate  $\beta$  Superconducting Quarter Wave Resonators. Proceedings of the 2001 Particle Accelerator Conference. In: Lucas P, Webber W. Chicago, Illinois US. 2001, 1095—1097.

## 低 $\beta$ 四分之一波长腔中的束流偏转的分析性研究\*

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**摘要:** 目前加速速度范围在  $0.01c$ — $0.3c$  的粒子的超导腔主要使用四分之一波长腔型。用于不同加速器上的频率范围在 50—240 MHz 的四分之一波长腔在建造或者预研中。这种腔型的一个不足是其横向电磁成分会造成束流偏转效应, 从而导致发射度的增长和束流的溢漏, 在强流重离子加速器中这种效应尤为严重。对中国科学院近代物理研究所超导直线加速器中的频率为 80.5 和 161 MHz 的四分之一波长腔的偏转效应进行了分析, 计算结果表明, 在四分之一腔体的设计时需要考虑束流偏转的修正, 这通常需要在漂移管端面上削适当大小的倾角来实现。

**关键词:** 束流偏转; 低  $\beta$ ; 四分之一波长腔

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