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Pions Enhancement in QCD Chiral Phase Transitions*

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Abstract: It is pointed out that if the QGP $SU(2)$ chiral phase transition in the LHC Pb-Pb collision process is taken place and the phase transition is in the second order, then pion strings will be formed, and decay. These phenomena lead to the pion enhancement in the low momentum region ($p \simeq 150$—400 MeV) and the number of pions produced from pion string decay can be estimated about $N_t \approx 270$, 150, 60 for different freeze temperature $T_f = 130$, 120, 110 MeV respectively.

Key words: pion string; linear sigma model; chiral phase transition

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1 Introduction

Topological defects formed during the rapid symmetry breaking phase transition[1] are very general phenomena in physics. They are studied from both theory and experiment in many fields, such as in the evolution of early universe, in condensed matter system, in superfluid ^{4}He, ^{3}He, in the second class of superconductor, in BEC system and liquid crystal.

In the early universe[2—4], topological defects, such as the cosmic string, may have been formed during the phase transitions in Big Bang and may still play an important role in the large scale structure seen in the universe. Theoretically string can be formed in phase transitions anywhere, such as the GUT scale strings which induce density perturbations in the microwave background. Light cosmic strings would only be observable since their gravitational interactions are so weak. Axion string, formed at an intermediate scale, has the exciting possibility of generating a substantial density of cold dark matter in the form of axion. The cosmological magnetic fields may be induced by global anomalous string, the π string[5, 6]. If all of the strings exist, they will provide the direct links between the observational features of our present universe and the dramatic events of the immediate aftermath of the Big Bang.

Up to now the experimental study of topological defects is only carried out in condensed matter, in which the classic example of formation of vortex lines in the nontrivial broken symmetry phase is the superfluid helium. Based on the point of view of Zurek[7], the phenomena which occur in the expanding and cooling of universe may be similar to what take place in the laboratory, such as in superfluid helium and in liquid crystals. One of the main goal in this field is the discovery of the phenomenon in superfluid ^{3}He-B phase[8]: the formation of vortices within bulk liquid in the presence of ionizing radiation. This "mini bang" allows one to study the formation of defects quantitatively in a

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time dependent second order phase transition. It is pointed out that the vortices in ^{3}He-B phase are similar to the π strings formed in high energy physics, as the two systems have the similar breaking symmetry. So it is interesting to observe the π string in nucleus-nucleus collisions.

In relativistic heavy ion collisions, as is pointed out by Rajantie[9], the experiments are so complicated that very reliable and accurate theoretical calculations are needed in order to confront the experimental results, but our present understanding of the theory is too rudimentary for that. Experimental results show that some phenomena have been observed which like what happens in the Big Bang and is called the Little Bang[10]. So the study of Little Bang from relativistic nucleus-nucleus collisions may be constructing a bridge between high energy particle physics and the cosmology. It is interesting to discuss the formation of topological defects in relativistic heavy ion collisions.

In a previous paper we proposed a model of the pion string formation and evolution in heavy ion collisions at LHC[11]. It is shown that the π strings would decay into pions which give an effective result in the low momentum region in the final state. π strings form in Kibble-Zurek mechanism, and evolute during the hot quark matter expansion, at last decay into pions at freeze out temperature. Here we will give a different picture of pion string forming and evolution. The produced picture is based on the following steps: (a) RHIC experiment shows that Bjorken hydrodynamical model works well when the temperature is above the system freeze out temperature $T_{\rm f}$. We expect that this is also right for that in LHC Pb-Pb collisions; (b) Pion string net work is formed when the system is cooling down to the critical temperature $T_{\rm c}=170$ MeV, and at Hagedorn temperature $T_{\rm H}=160$ MeV the number of the strings can be counted, according to Kibble-Zurek mechanism. It means that the Zurek temperature is equal to Hagedorn temperature $T_{\rm Z}=T_{\rm H}$; (c) Following Ref. [12] we assume that below the freeze out temperature the dynamics of shrinking of the string loop happens, the shrinking will be less dissipative. Finally the string loops will be shrinking to the small nest size, with its energy deposing in the small region, then it fragments to σ and π^0 mesons.

The Kibble-Zurek mechanism[1-7] states that in a second order phase transition as the QGP fireball cools through the temperature of phase transition where the chiral symmetry is spontaneously broken, domains of similar orientations could be formed. At the boundaries, where different causality disconnected regions meet, the order parameter does not necessarily match and a domain structure is formed leading to the formation of topological defects such as strings. In non-equilibrium transition the new low-temperature phase starts to form, due to fluctuations of the order parameter, simultaneously and independently in many parts of the system. Subsequently during further cooling, these regions grow together to form the new broken-symmetry phase. The fluctuating configuration of the order parameter is frozen out at $\varepsilon_{\rm Z}=(1-T_{\rm Z}/T_{\rm c})>0$, which means that the symmetry breaking has already occurred and topological defects are formed.

2 General Formalism

The chiral symmetry breaking has already occurred and the topological defects, the π strings are produced. The correlation length or the domain size has reached the value. The characteristic correlation length $\xi_{\rm Z}$ corresponding to $\varepsilon_{\rm Z}$ above will decide the initial density of the string. At temperature

$$T_{\rm Z}=T_{\rm c}\left(1-\sqrt{\frac{\tau_0}{\tau_{\rm Q}}}\right)\ , \tag{1}$$

the π string will be formed. If we choose QGP critical temperature $T_{\rm c}=170$ MeV, $T_{\rm H}=T_{\rm Z}=160$ MeV, then $\tau_{\rm Q}=70$ fm,

$$\xi_{\rm Z}=\xi_0\left(\frac{\tau_{\rm Q}}{\tau_0}\right)^{1/4}\simeq\left(\frac{\tau_{\rm Q}}{m_\sigma^3}\right)^{1/4}\ , \tag{2}$$

where $\xi_0 \approx \tau_0 \approx 1/m_\sigma$. $m_\sigma = 600$ MeV, $\xi_Z \approx 1.26$ fm, at the Zurek temperature. For simplicity, in the following discussion we use the parameters: $f_\pi =$ 93 MeV, $\lambda = 20$. In heavy ion collisions, the string is formed in various independent domains, each domain has the size of $\sim \xi_Z^2$, the π string energy in each domain in unit length is given by[5]

$$E_0 = \{0.75 + \log[\mu\xi_z]\}\pi f_\pi^2, \qquad (3)$$

where $\mu = \sqrt{89\lambda} f_\pi/12 \approx 327$ MeV, $E_0 \approx 205$ MeV/fm. If the system of strings is in equilibrium, the loops do not interact, it can be treated as particles in an ideal gas[13-16], and loops may be described by a Bose distribution. The distribution of the string loop is

$$\begin{aligned} \mathrm{d}n(E) &= f(E)\frac{\exp(\beta_\mathrm{H} E)}{[\exp(\beta E) - 1]}\mathrm{d}E \\ &\simeq f(E)\exp[(\beta_\mathrm{H} - \beta)E]\mathrm{d}E, \end{aligned} \qquad (4)$$

where $\beta = 1/T$, T is the temperature of the system, $\beta_\mathrm{H} = 1/T_\mathrm{H}$, $T_\mathrm{H} \approx 160$ MeV is the Hagedorn temperature, at which the distribution is given by[16]

$$f(E) = \frac{K}{\xi_Z^{3/2} E^{5/2}}, \qquad (5)$$

where $K \approx (E_0)^{3/2}$. In heavy-ion collisions only the loop string will be formed. The shortest string will be the length of $l_0 = 2\pi/\mu \approx 3.79$ fm. The number of the loop string at freeze out temperature T_f is

$$\begin{aligned} N &\approx V_\mathrm{f}\int_{E_\mathrm{min}}^{\infty} n(E)\mathrm{d}E \\ &= V_\mathrm{f}\int_{E_\mathrm{min}}^{\infty} K\frac{\exp[(\beta_\mathrm{H} - \beta_\mathrm{f})E]}{\xi_Z^{3/2} E^{5/2}}\mathrm{d}E, \end{aligned} \qquad (6)$$

where $E_\mathrm{min} = E_0 l_0 \approx 777$ MeV, R_f is the radius of the hadronic phase. In the Pb+Pb collisions, $R_\mathrm{f} \approx 10$ fm at RHIC, and $R_\mathrm{f} \approx 18$ fm at LHC[17-19], and the volume of hadronic phase is about $V_\mathrm{f} \approx 4 \times 10^3$ fm^3, 2.4×10^4 fm^3 at RHIC and LHC respectively. If the freeze out temperature is $T_\mathrm{f} = 130$, 120, 110 MeV the total number of string loops is then $N_\mathrm{t} \approx$ 90, 50, 20 in LHC and 15, 8, 3 in RHIC Pb-Pb collisions respectively.

Now we construct the picture of decay of pion string in the following. From the field of pion string, $\phi = (\sigma + \mathrm{i}\pi^0)/\sqrt{2}$, the solution of pion string is

$$\begin{aligned} (\sigma + \mathrm{i}\pi^0) &= f_\pi[1 - \exp(-\mu r)]\exp(\mathrm{i}\theta), \\ \pi^\pm &= 0. \end{aligned} \qquad (7)$$

In the evolution period ($t_Z \leqslant t \leqslant t_\mathrm{f}$) of pion string, π^0 is equivalent to σ. So after time t_f the pion string equal decays to π^0 and sigma particles, and we can approximately assume that each has half of the pion string energy. The sigma particle produced by string will soon decay to 2π. The momentum distribution of π from the processes, string→σ→2π is

$$\begin{aligned} &\mathrm{d}N_\pi(p) = 2\mathrm{d}N(p) \\ &\qquad \propto \frac{\exp[4(\beta_\mathrm{H} - \beta_\mathrm{f})\sqrt{p^2 + m_\pi^2}]}{[p^2 + m_\pi]^{7/4}}. \end{aligned} \qquad (8)$$

The mean momentum is then given by

$$\langle p \rangle = \frac{\int p\,\mathrm{d}N_\pi(p)}{\int \mathrm{d}N_\pi(p)} \simeq 154 \text{ MeV}. \qquad (9)$$

Similarly π^0 from the processes, string→π^0 is

$$\begin{aligned} &\mathrm{d}N_{\pi^0}(p) = \mathrm{d}N(p) \\ &\qquad \propto \frac{\exp[2(\beta_\mathrm{H} - \beta_\mathrm{f})\sqrt{p^2 + m_\pi^2}]}{[p^2 + m_\pi]^{7/4}}. \end{aligned} \qquad (10)$$

The mean momentum is then given by

$$\langle p_{\pi^0} \rangle = \frac{\int p\,\mathrm{d}N_{\pi^0}(p)}{\int \mathrm{d}N_{\pi^0}(p)} \simeq 394 \text{ MeV}. \qquad (11)$$

From the above discussion, it gives out that each string loop decays to three pions. So in the process Pb+Pb, with $\sqrt{s} = 5500$ AGeV at LHC, if the number of pion string loops is $N_\mathrm{t} \approx 90$, 50, 20 for different freeze temperature $T_\mathrm{f} = 130$, 120, 110 MeV respectively, the number of pions produced by string loops will be 270, 150 and 60. This pion enhancement effect may be observed.

3 Conclusion

At the LHC Pb-Pb collision energies, the QGP $SU(2)$ chiral phase transition is taken place. If the chiral phase transition is in a second order, then pion strings will be formed, and decay into pions at freeze out temperature. We find that the main effect of this phenomenon will induce a pion enhancement in the low momentum region ($p \simeq$ 150—400 MeV), and the number of pions produced from pion strings decay is about $N_t \approx 270$, 150, 60 for different freeze temperature $T_f = 130$, 120, 110 MeV, respectively.

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QCD 手征相变中的 π 介子增强 *

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摘　要：讨论了在 LHC 的 Pb-Pb 碰撞过程中，如果 QGP $SU(2)$手征相变出现二级相变，那么系统将会有 π 弦产生，并且 π 弦最终将衰变为 π 介子。于是以上效应将导致在低动量区域($p \simeq$150—400 MeV)内的 π 介子增强。对应于不同的冷却温度 $T_f = 130$，120，110 MeV，产生于 π 弦衰变的 π 介子的数量分别是 $N_t \approx$ 270，150，60.

关 键 词：π 弦；线性 σ 模型；手征相变

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