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# Dynamic Aperture Studies for CSRe Storage Ring<sup>\*</sup>

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**Abstract:** Dynamic aperture(DA) is playing a more and more important role in circular accelerators, especially in the modern storage rings. In this paper, the DA of CSRe is analyzed by MAD program. Comparing the DA under various assumptions, we find that the multipole errors in dipoles or quadrupoles, and the sextupoles which bring strong non-linearities, and limit the DA of CSRe. Fortunately, the DA is larger than the physical aperture in all the cases, and that is large enough to satisfy the high precision physical experimental request.

**Key words:** storage ring; multipole error; dynamic aperture

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## 1 Introduction

In order to satisfy the high precision physical experimental request, the modern accelerators develop with the trend to higher energy, higher current and longer life. In the new generation of accelerators, for example, the storage ring of the new generation synchrotron radiation source, high focusing are used to achieve low emittances and high brilliances with very strong sextupoles to correct the large chromaticities<sup>[1]</sup>. Consequently, sextupoles with high strengths introduce various kinds of geometric and chromatic aberrations, which limit the dynamic aperture (DA). In the error dominated machines, for example, the higher-order field errors of magnets in the large hadron collider(LHC) are the main source of non-linearities, due to the poor field quality of superconducting magnets. Errors are from of the systematic field imperfections, which are generated by persistent current field distortions at low excitation, and the random errors from manufacturing tolerances. So

the DA is limited by these errors and needs to be improved by compensating these errors. But normally, in above kinds of machines, a large DA is necessary to accommodate the oscillations of scattered particles to obtain a long beam lifetime.

## 2 Introduction of the CSRe

CSRe is the experimental ring of HIRFL-CSR complex<sup>[2]</sup>. The heavy ion beams from the HIRFL will be accumulated, cooled and accelerated in CSRm, and then extracted to produce radioactive ion beams(RIB) or highly charged heavy ions. The secondary beams can be accepted by the CSRe and stored in it for internal-target experiments or high precision spectroscopy with e-cooling.

CSRe is a racetrack shape and consists of two quasi-symmetric parts<sup>[2]</sup>. One is the internal target part and the other is the e-cooler part. Each part consists of two identical arc sections. Each arc section consists of four dipoles, two triplets or one triplet and one doublet. 11 independent variables

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for quadrupole are used in CSRe. CSRe is also a high-resolution spectrometer. In order to meet the different experimental needs, three lattice modes are adopted. The first one is the internal-target mode with small  $\beta$ -function in the target point and large transverse acceptance. The second one is the high-resolution mode also called Normal mode used for mass spectrometer with large momentum acceptance. The third mode is the isochronous mode for the mass measurements of short lifetime nuclei using Time-of-Flight method. Table 1 is the lattice parameters of CSRe for the Normal mode, and Fig. 1 is the distribution of  $\beta$ -functions and dispersion for this mode.

**Table 1 Lattice parameters of the Normal mode**

Quantity	Value
Circumference/m	128.80
$B\rho_{\max}/\text{Tm}$	8.4
$\gamma_{\text{tr}}$	2.629
Betatron tune values	$Q_x/Q_y=2.53/2.57$
Natural chromaticity	$Q'_x/Q'_y=-3.10/-3.74$
Max. $\beta$ -amplitude/m	$\beta_x/\beta_y=17.6/8.2$ (Dipole) $\beta_x/\beta_y=30.9/22.3$ (Quad.)
Max. Dispersion/m	$D_{\max}=6.5$ (Dipole, $\beta_x=13$ ) $D_{\max}=7.8$ (Quad., $\beta_y=16$ )
Injection section/m	$\beta_x=30.4, D_x=0$ (Septum) $\beta_x=30.9, D_x=0$ (Quad.)
E-cooler section/m	$\beta_x/\beta_y=12.5/16.0, D_x=0$

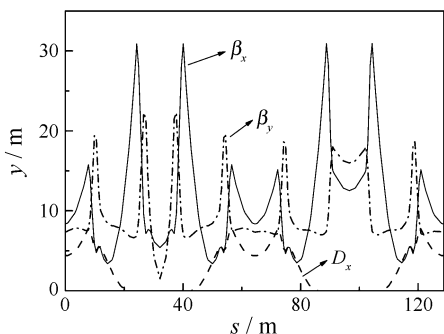


Fig. 1 The  $\beta$ -functions and dispersion for the Normal mode of the CSRe.

### 3 Discussion on Dynamic Aperture

In the design of accelerator, it is very impor-

tant to ensure that beam loss due to the nonlinear multipole error is as small as possible. When the initial amplitude and phase of particle motion,  $(A_0, \varphi_0)$ , are given, it is required that the amplitude at any time  $t$  should satisfy  $|A(t)| < B$ , where  $B$  is a finite boundary. If we can find a maximum  $A_0$  for any phase  $\varphi_0$  such that  $A(t)$  keeps finite for a long enough time, and the betatron motion with an initial amplitude smaller than  $A_0$  is stable, such an  $A_0$  is called DA<sup>[3]</sup>.

Here we use the E. Todesco theory<sup>[4]</sup>, and analyze a 4D symplectic mapping, which can be written as

$$X' = F(X), \quad X = (x, p_x, y, p_y), \quad (1)$$

where  $X$  is a vector in the 4D Euclidean phase space. The linear motion is given by the direct product of two constant rotations in the planes  $(X, P_x)$  and  $(Y, P_y)$  by the linear tunes  $\nu_1$  and  $\nu_2$ . Let us consider the phase space volume of the initial conditions which are bounded after  $N$  iterations:

$$\iiint \chi(x, p_x, y, p_y) dx dp_x dy dp_y, \quad (2)$$

where  $\chi(x, p_x, y, p_y)$  is the generalization of the characteristic function to the 4D case. If the particles still running after  $N$ -turns,  $\chi(x, p_x, y, p_y) = 1$  otherwise  $\chi(x, p_x, y, p_y) = 0$ .

In order to exclude the disconnected part of the stability domain, we have to choose a suitable coordinate transformation:

$$\begin{aligned} x &= r \cos \alpha \cos \theta_1, \\ p_x &= r \cos \alpha \sin \theta_1, \\ y &= r \sin \alpha \cos \theta_2, \\ p_y &= r \sin \alpha \sin \theta_2, \end{aligned}$$

where

$$\begin{aligned} r &\in [0, +\infty] \\ \theta_1, \theta_2 &\in [0, 2\pi] \\ \alpha &\in [0, \pi/2] \end{aligned} \quad (3)$$

Substituting Eq. (3) in Eq. (2) we obtain

$$\int \chi(r, \alpha, \theta_1, \theta_2) r^3 \sin(\alpha) \cos(\alpha) dr d\alpha d\theta_1 d\theta_2. \quad (4)$$

Fixed  $\alpha, \theta_1$ , and  $\theta_2$  and let  $r(\alpha, \theta_1, \theta_2)$  be the first value of  $r$  whose orbit is not bounded after  $N$

iterations, then, the area of a connected stability domain is

$$A_{\alpha, \theta_1, \theta_2} = \frac{1}{8} \iiint [r(\alpha, \theta_1, \theta_2)]^4 \times \sin(\alpha) d\alpha d\theta_1 d\theta_2. \quad (5)$$

We define the DA as the radius of the hypersphere that has the same volume as the stability domain:

$$D_{\alpha, \theta_1, \theta_2} = \left[ \frac{2A_{\alpha, \theta_1, \theta_2}}{\pi^2} \right]^{\frac{1}{4}}. \quad (6)$$

## 4 Simulation of CSRe DA

The DA of CSRe storage ring has been investigated using the MAD program<sup>[5]</sup>. In order to estimate the effects of the chromaticity sextupoles, multipole errors in the dipoles and quadrupoles and the momentum spread, several cases are simulated and compared with each other.

### 4.1 The DA of Ideal Lattice and with sextupoles

First, we simulate the Ideal Lattice of CSRe with different momentum spread at energy 0.6 GeV. Then, we study the case with sextupoles. From Fig. 2, we can see that the impact of the chromaticity sextupoles on the DA is very strong,

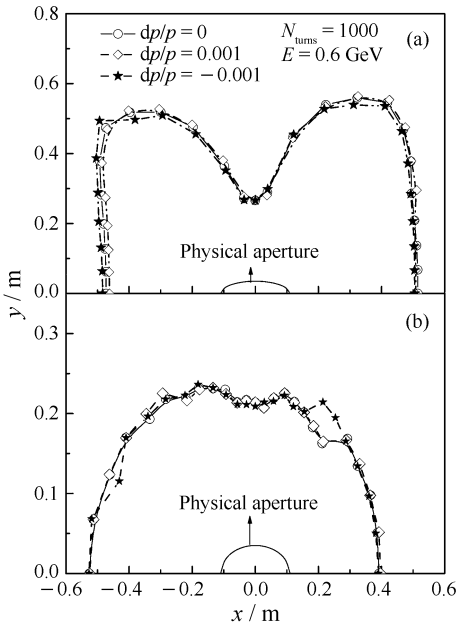


图 2 DA of Ideal Lattice and with sextupoles with different  $dp/p$   
(a) DA without sextupoles, (b) DA with sextupoles.

but the effect of the momentum spread on the DA is small.

### 4.2 The DA with multipoles in dipoles and quadrupoles

Considering the high-order component errors in dipoles and quadrupoles, the DA of CSRe should be investigated. Fig. 3 shows the DA with the chromaticity sextupoles and multiple errors in dipoles and quadrupoles. From these results, we can find that multiple errors in the quadrupoles play a little effect on the DA, and the effects of the multiple errors in the dipoles play a more important role in the DA of CSRe.

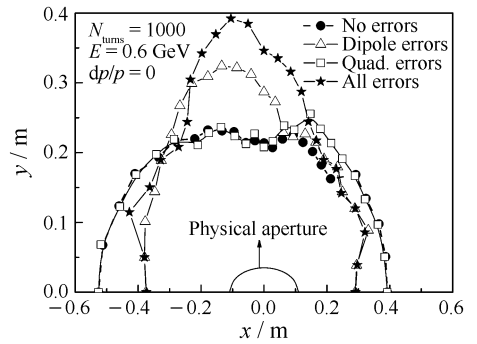


Fig. 3 DA with sextupoles and multipoles errors.

### 4.3 The DA of different turns

In order to estimate the negative effects of these higher order multipoles on the long-term stability of the CSRe, the DA of  $10^4$  and  $10^5$  turns are also simulated. Fig. 4 shows the DA of the three different turns, the results in these cases are exactly the same.

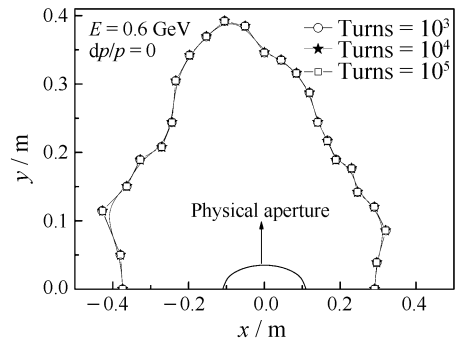


Fig. 4 The compare of different turns.

### 4.4 The experiment result of CSRe

From the following formula we can calculate

the beam life.

$$N_t = N_0 e^{-t/\tau}, \quad \tau = \frac{t}{\ln(N_0/N_t)}. \quad (7)$$

The calculated beam life in CSRe is about  $1 \times 10^3$  s, and the particles will run about  $2 \times 10^9$  turns during this time. So in Fig. 4, we get the same result in different turns, for the beam in the ring is very stable.

Through the above simulation, we find that the beam in CSRe is very stable in theory. Actually, we find the same case in experiment. Fig. 5 shows the result of DCCT for the 660 MeV  $C^{6+}$  beam in CSRe in Oct 2007, and we can see that the beam current of CSRe is about 1.2 mA, that is enough for physical experiment.

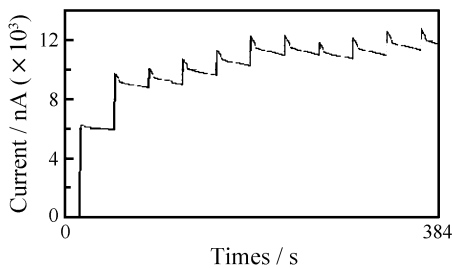


Fig. 5 The DCCT of the CSRe with  $C^{6+}$ .

## 5 Conclusion

In this paper we simulate the DA of CSRe in

several cases by MAD program, and we find that the DA reduces rapidly with the chromaticity sextupoles, and the multipoles errors in dipoles take a much more negative effect than that in quadrupoles. The DA is very large if only the multiple errors in the dipoles and quadrupoles are included. However it will reduce rapidly when the chromaticity sextupoles are switched on. From the more turns study and the actually result, we can conclusion that the DA of CSRe is much larger than the physical aperture, and that is large enough to satisfy the high precision physical experimentals request.

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## CSRe 储存环动力学孔径研究\*

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**摘要:** 动力学孔径对环形加速器, 尤其是现代的储存环, 起着越来越重要的作用。采用 MAD 程序研究了兰州重离子加速器实验环(CSRe)的动力学孔径。通过对比几种情况下的模拟结果, 发现六极铁和二极铁的高阶场对束流的动力学孔径影响较大, 使 CSRe 的动力学孔径减小, 但减小后的动力学孔径也远大于该环的物理孔径。因此, 束流可以长期、稳定的存在。

**关键词:** 储存环; 多极场误差; 动力学孔径

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