

Article ID: 1007-4627(2009)Suppl.-0158-07

# Signature Inversion in Odd-odd Nuclei<sup>\*</sup>

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**Abstract:** Signature inversion in odd-odd nuclei is investigated by using a proton and a neutron coupling to the coherent state of the core. Two parameters are employed in the Hamiltonian to set the energy scales of rotation, neutron-proton coupling and their competition. Typical level staggering is extracted from the calculated level energies. The calculation can approximately reproduce experimental signature inversion. Signature inversion is attributed to the rotational motion and neutron-proton residual interaction having reversed signature splitting rules. It is found signature inversion can appear at axially symmetric shape and high- $K$  band.

**Key words:** signature; odd-odd nuclei; neutron-proton interaction

**CLC number:** O571.2      **Document code:** A

## 1 Introduction

An interesting phenomenon in deformed odd-odd nuclei is signature inversion. The total angular momentum  $I$  of two-particle band in odd-odd nuclei can be classified as two  $\Delta I=2$  branches characterized by signature quantum number  $\alpha=0, 1$ . The branch with  $I-j_n-j_p$  being even ( $\alpha=1/2$  [ $(-1)^{j_n-1/2} + (-1)^{j_p-1/2}$ ]) is favored, i. e. lower in energy, while the other  $I-j_n-j_p$  being odd is disfavored, where  $j_n$  and  $j_p$  is the respective angular momentum of valence neutron and proton. When the rule is broken at low spin, it is the so-called low-spin signature inversion. Several explanations for signature inversion have been made including Coriolis effects<sup>[1]</sup>, triaxial deformation<sup>[2]</sup>, neutron-proton residual interaction<sup>[3-5]</sup>, band crossing<sup>[6, 7]</sup>, band mixing<sup>[8]</sup>, quadrupole pairing<sup>[9]</sup>, and the interaction boson-fermion model<sup>[10]</sup>. Usually, it is difficult for the theory to explain signature inver-

sion in high- $K$  bands without configuration mixing, particularly when the  $z$  component of the total angular momentum  $K=K_n+K_p$ , like  $\pi h_{11/2} \otimes \nu i_{13/2}$  bands of  $A \sim 170$  nuclei<sup>[3, 11, 12]</sup>. In this paper we will employ a very simple theory of coherent state plus angular momentum projection to reduce and describe the very complex inversion mechanics in odd-odd nuclei, which have always proven difficult to study for high level density. We will pay special attention to signature inversion in  $\pi h_{11/2} (9/2^- [514]) \otimes \nu i_{13/2} (7/2^+ [633])$  band ( $K^\pi = 8^-$ )<sup>[3, 11, 12]</sup>.

## 2 Outline of the Theoretical Approach

When one neutron and one proton is added into the even-even core to form a two-particle band, we construct an intrinsic state for the system by filling the lowest possible Nilsson orbits. Because of the exclusion principle each level can accommo-

\* **Received date:** 12 Sep. 2008; **Received date:** 16 Sep. 2008;

\* **Foundation item:** National Natural Sciences Foundation of China(10505025, 10775158); Major State Basic Research Development Program of China(2007CB815005); Chinese Academy of Sciences

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date at most two identical nucleons with adverse  $z$  component of angular momentum; their wave function with  $K=0$  is not angular-momentum eigenfunction, but the coherent state of different angular-momentum. On the other hand, the basic idea of the Interacting Boson Approximation is to assume that the identical nucleons couple in pairs only to  $s$  and  $d$  bosons and the low-lying collective excitations of even-even nuclei can be described in terms of the energies and interactions of such pairs. This leads that the two identical nucleon pair can be approximately described by the coherent state of boson  $\{s^\dagger + \beta [\cos\gamma d_0^\dagger + (1/2)\sin\gamma (d_2^\dagger + d_{-2}^\dagger)]\}^{[13]}$ . Here parameters  $\beta$  and  $\gamma$  describe intrinsic shape and we set  $\gamma=0$  to study only the axially symmetric case. For the  $2N$ -nucleon core, the coherent state has the form

$$\frac{\{s^\dagger + \beta \cos\gamma d_0^\dagger\}^N}{\sqrt{N! (1 + \beta^2)^N}} |0\rangle^{[13]}.$$

Since a decoupled state has a weak  $j$ -admixture, it is close to a spherical one, of angular momentum  $j$  and its  $z$  component  $m^{[7]}$ . The Nilsson orbit of valence neutron and proton may be written approximately as  $|j_n K_n\rangle$  and  $|j_p K_p\rangle$  respectively with  $K = |m|^{[7]}$ . Finally, we assume a many-body wave function for the system to have the form of product of the wave functions of two single particles and the boson coherent state of the core

$$\begin{aligned} & |N\beta; j_p K_p; j_n K_n\rangle \\ &= \frac{a_{j_n K_n}^\dagger a_{j_p K_p}^\dagger \{s^\dagger + \beta \cos\gamma d_0^\dagger\}^N}{\sqrt{N! (1 + \beta^2)^N}} |0\rangle. \end{aligned} \quad (1)$$

However, this many-body wave function is

not appropriate for the description of nuclear states with given angular momentum. In order to infer the level energies of the odd-odd nuclei within the framework of angular momentum projection, we resort to the expansion of the coherent state in a series of angular momentum eigenfunction

$$\begin{aligned} & |N\beta; j_p K_p; j_n K_n\rangle \\ &= |j_n K_n\rangle |j_p K_p\rangle \sum_R C_R |R0\rangle \\ &= \sum_I C_{IK} |IK\rangle, \end{aligned} \quad (2)$$

where  $R(I)$  and  $0(K=K_p+K_n)$  are the core (total) angular momentum and  $z$  component, respectively. The coefficient  $C_R$  is determined by the angular momentum coupling of boson. The component  $C_I$  of  $|IK\rangle$  in this many-body wave function can be deduced by using operator  $\hat{R}(\theta) = e^{-i\theta \hat{J}_y}$

$$\begin{aligned} & \langle N\beta; j_p K_p; j_n K_n | \hat{R}(\theta) | N\beta; j_p K_p; j_n K_n \rangle \\ &= \sum_I C_I^2 \langle IK | e^{-i\theta \hat{J}_y} | IK \rangle \\ &= \sum_I \sum_M C_I^2 \langle IK | d_{MK}^I | IM \rangle \\ &= \sum_I C_I^2 d_{KK}^I(\theta), \end{aligned} \quad (3)$$

where  $\hat{J}_y$  is the rotation about the  $y$  axis through an angle  $\theta$ . Using Eq. (3) and the orthogonality of the  $d$ -functions  $d_{MK}^I(\theta)^{[14]}$

$$\begin{aligned} & \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{MK}^I(\theta) d_{M'K'}^I(\theta) \\ &= \delta_{II'} \delta_{MM'} \delta_{KK'}, \end{aligned} \quad (4)$$

we obtain the norm for two-particle angular momentum  $J$  described by Clebsch-Gordan coefficients  $(j_p K_p j_n K_n | JK)$  and  $(JKR0 | IK)$ ,

$$\begin{aligned} N_{IK} &= C_I^2 \\ &= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{KK}^I \langle N\beta; j_p K_p; j_n K_n | \hat{R}(\theta) | N\beta; j_p K_p; j_n K_n \rangle \\ &= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{KK}^I d_{K_n K_n}^{j_n} d_{K_p K_p}^{j_p} \left\{ \sum_{R'} C_{R'} \sum_R C_R d_{00}^R \delta_{RR'} \right\} \\ &= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{KK}^I \sum_{J=|j_p-j_n|}^{J=j_p+j_n} (j_p K_p j_n K_n | JK)^2 d_{KK}^I \sum_R C_{R0}^2 d_{00}^R \end{aligned}$$

$$\begin{aligned}
&= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta_{KK}^I \sum_J (j_p K_p j_n K_n | JK)^2 \sum_R C_R^2 \sum_{I'=|R-J}^{I'=R+J} (JKR0 | I'K)^2 d_{KK}^{I'} \\
&= \sum_R \sum_{J=|j_p-j_n|}^{J=j_p+j_n} C_R^2 (j_p K_p j_n K_n | JK)^2 (JKR0 | IK)^2, \tag{5}
\end{aligned}$$

where the overlap

$$\begin{aligned}
&\langle N\beta; j_p K_p; j_n K_n | \hat{R}(\theta) | N\beta; j_p K_p; j_n K_n \rangle \\
&= \langle j_n K_n | e^{-i\theta \hat{J}_y} | j_n K_n \rangle \langle j_p K_p | e^{-i\theta \hat{J}_y} | j_p K_p \rangle \left\{ \sum_{R'} C_{R'} \sum_R C_R \langle R'0 | e^{-i\theta \hat{J}_y} | R0 \rangle \right\} \tag{6}
\end{aligned}$$

can be got by using the following relationship<sup>[14]</sup>:

$$\begin{aligned}
&\langle jK | e^{-i\theta \hat{J}_y} | jK \rangle = d_{kK}^I \\
&d_{M_1 K_1}^{I_1} d_{M_2 K_2}^{I_2} = \sum_{I=|I_1-I_2|}^{I=I_1+I_2} (I_1 M_1 I_2 M_2 | IM) (I_1 K_1 I_2 K_2 | IK) d_{MK}^I. \tag{6}
\end{aligned}$$

The Hamiltonian reads  $H = H^c + H^{cn} + H^{cp} + H^{np}$  and the core rotational Hamiltonian  $H^c = \lambda R^2$  with the energy scale  $\lambda$  satisfies

$$H^c | R0 \rangle = \lambda R(R+1) | R0 \rangle. \tag{7}$$

The other interactions are of multipole-multipole types<sup>[14]</sup>. The energy of the level  $|IK\rangle$  is given by angular momentum projection<sup>[6, 7, 14]</sup>:

$$E_{IK} = \frac{H_{IK}}{N_{IK}}. \tag{8}$$

The core-nucleon interplay can be approximately deduced by the difference between the rotational energy of odd- $A$  nucleus and its even-even core,  $\tau$  representing proton or neutron,

$$H^{\tau\tau} = \lambda [I^2 - R^2] = \lambda [I^2 - R^2 - j_\tau^2] + \lambda j_\tau^2 = 2\lambda j_\tau R + \lambda j_\tau^2. \tag{9}$$

Here we omit the single-particle energy and assume the odd- $A$  nucleus has the same rotation constant  $\lambda$  as the even-even core for simplicity. We neglect the proton-neutron quadrupole-quadrupole interaction  $\mathbf{Q}_{j_p K_p} \cdot \mathbf{Q}_{j_n K_n}$ ,

$$\begin{aligned}
H_{IK} &= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta_{KK}^I \langle N\beta; j_p K_p; j_n K_n | \mathbf{Q}_{j_p K_p} \cdot \mathbf{Q}_{j_n K_n} \hat{R}(\theta) | N\beta; j_p K_p; j_n K_n \rangle \\
&= \left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta_{KK}^I \langle j_p K_p; j_n K_n | \mathbf{Q}_{j_p K_p} \cdot \mathbf{Q}_{j_n K_n} \hat{R}(\theta) | j_p K_p; j_n K_n \rangle \sum_R C_R^2 d_{00}^R \\
&= (j_p K_p 20 | j_p K_p) (j_n K_n 20 | j_n K_n) \langle j_p || \mathbf{Q}_{j_p} || j_p \rangle \langle j_n || \mathbf{Q}_{j_n} || j_n \rangle N_{IK}, \tag{10}
\end{aligned}$$

which contributes the level energy a term  $\propto \langle j_p K_p 20 | j_p K_p \rangle \langle j_n K_n 20 | j_n K_n \rangle \langle j_p || \mathbf{Q}_{j_p} || j_p \rangle \langle j_n || \mathbf{Q}_{j_n} || j_n \rangle$  and represent just a renormalization of the core, where

$$\begin{aligned}
&\langle j_p K_p; j_n K_n | \mathbf{Q}_{j_p K_p} \cdot \mathbf{Q}_{j_n K_n} \hat{R}(\theta) | j_p K_p; j_n K_n \rangle \\
&= \sum_{K'_n, K'_p} \langle j_p K_p; j_n K_n | \mathbf{Q}_{j_p K_p} \cdot \mathbf{Q}_{j_n K_n} d_{K'_n K'_n}^{j_p} d_{K'_p K'_p}^{j_n} | j_p K'_p; j_n K'_n \rangle \\
&= \sum_{K'_n, K'_p, \mu} d_{K'_n K'_n}^{j_p} d_{K'_p K'_p}^{j_n} (j_p K_p 2\mu | j_p K'_p) (j_n K_n 2\mu | j_n K'_n) \langle j_p || \mathbf{Q}_{j_p} || j_p \rangle \langle j_n || \mathbf{Q}_{j_n} || j_n \rangle \\
&= \sum_{K'_n, K'_p} d_{K'_n K'_n}^{j_p} d_{K'_p K'_p}^{j_n} (j_p K_p 20 | j_p K'_p) (j_n K_n 20 | j_n K'_n) \langle j_p || \mathbf{Q}_{j_p} || j_p \rangle \langle j_n || \mathbf{Q}_{j_n} || j_n \rangle \\
&= d_{K'_n K'_n}^{j_p} d_{K'_p K'_p}^{j_n} (j_p K_p 20 | j_p K_p) (j_n K_n 20 | j_n K_n) \langle j_p || \mathbf{Q}_{j_p} || j_p \rangle \langle j_n || \mathbf{Q}_{j_n} || j_n \rangle. \tag{11}
\end{aligned}$$

Here we use the condition  $K = K_p + K_n = K'_p + K'_n = K_p + \mu + K_n + \mu$ , i. e.  $\mu = 0$ . The other higher-rank multipole-multipole interactions give the similar results and only the dipole-dipole interaction  $j_n \cdot j_p$ <sup>[14]</sup> affects the level spacing, which has been used in Ref. [5] to account for signature inversion. The Hamiltonian is hence simplified as

$$H = \lambda R^2 + 2\lambda j_n R + 2\lambda j_p R + 2\xi j_n \cdot j_p, \quad (12)$$

where  $\xi$  represents the energy scale of the neutron-proton residual interaction. The energy contributed by the core can be expressed in the following form:

$$\begin{aligned} E_{IK}^c &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d^l_{KK} \langle N\beta; j_p K_p; j_n K_n \mid \lambda R^2 \hat{R}(\theta) \mid N\beta; j_p K_p; j_n K_n \rangle}{N_{IK}} \\ &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d^l_{KK} d^l_{K_n K_n} d^l_{K_p K_p} \sum_R C_R^2 d_{00}^R \langle R0 \mid \lambda R^2 \mid R0 \rangle}{N_{IK}} \\ &= \frac{\sum_R \sum_{J=|j_p-j_n|}^{J=j_p+j_n} C_R^2 \langle j_p K_p j_n K_n \mid JK \rangle^2 \langle JKR0 \mid IK \rangle^2 \lambda R(R+1)}{N_{IK}}. \end{aligned} \quad (13)$$

The energies related to the interactions between the core and the particle outside the core is

$$\begin{aligned} E_{IK}^{cp} &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d^l_{KK} \langle N\beta; j_p K_p; j_n K_n \mid 2\lambda j_p \cdot R \hat{R}(\theta) \mid N\beta; j_p K_p; j_n K_n \rangle}{N_{IK}} \\ &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d^l_{KK} d^l_{K_n K_n} \langle N\beta; j_p K_p \mid 2\lambda j_p \cdot R \hat{R}(\theta) \mid N\beta; j_p K_p \rangle}{N_{IK}} \\ &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d^l_{KK} d^l_{K_n K_n} \sum_R C_R^2 \langle R0; j_p K_p \mid 2\lambda j_p \cdot R \hat{R}(\theta) \mid R0; j_p K_p \rangle}{N_{IK}} \\ &= \frac{\sum_R \sum_{I_p=|I-j_n|}^{I_p=I+j_n} C_R^2 \langle I_p K_p j_n K_n \mid IK \rangle^2 \langle j_p K_p R0 \mid I_p K_p \rangle^2 \lambda [I_p(I_p+1) - j_p(j_p+1) - R(R+1)]}{N_{IK}}, \\ E_{IK}^{cn} &= \frac{\sum_R \sum_{I_n=|I-j_p|}^{I_n=I+j_p} C_R^2 \langle I_n K_n j_p K_p \mid IK \rangle^2 \langle j_n K_n R0 \mid I_n K_n \rangle^2 \lambda [I_n(I_n+1) - j_n(j_n+1) - R(R+1)]}{N_{IK}}. \end{aligned} \quad (14)$$

where

$$\begin{aligned} &\langle R0; j_p K \mid 2\lambda j_p \cdot R \hat{R}(\theta) \mid R0; j_p K \rangle \\ &= \sum_{M, K'} \langle R0; j_p K \mid 2\lambda j_p \cdot R d^R_{M0} d^l_{K'K} \mid RM; j_p K' \rangle \\ &= \sum_{M, K'} d^R_{M0} d^l_{K'K} \langle R0; j_p K \mid \lambda(I^2 - j_p^2 - R^2) \mid RM; j_p K' \rangle \\ &= \lambda d^R_{00} d^l_{K_p K_p} \{ \langle R0; j_p K \mid I^2 \mid R0; j_p K \rangle - [j_p(j_p+1) + R(R+1)] \} \\ &= \lambda d^R_{00} d^l_{K_p K_p} \{ \langle R0 \otimes j_p K; I_p K \mid I^2 \mid R0 \otimes j_p K; I_p K \rangle - [j_p(j_p+1) + R(R+1)] \} \\ &= \lambda d^l_{K_p K_p} \langle j_p K_p R0 \mid I_p K_p \rangle^2 \{ I_p(I_p+1) - j_p(j_p+1) - R(R+1) \} \end{aligned} \quad (15)$$

The energy of neutron-proton residual interaction reads

$$\begin{aligned}
E_{IK}^{\text{np}} &= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{KK}^I \langle N\beta; j_p K_p; j_n K_n | 2\xi j_n \cdot j_p \hat{R}(\theta) | N\beta; j_p K_p; j_n K_n \rangle}{N_{IK}} \\
&= \frac{\left(I + \frac{1}{2}\right) \int_0^\pi \sin(\theta) d\theta d_{KK}^I \sum_R C_R^2 d_{00}^R \langle j_p K_p; j_n K_n | \xi (J^2 - j_n^2 - j_p^2) \hat{R}(\theta) | j_p K_p; j_n K_n \rangle}{N_{IK}} \\
&= \frac{\sum_R \sum_{J=|j_p-j_n|}^{J=j_p+j_n} C_R^2 \langle j_p K_p; j_n K_n | JK \rangle^2 \langle JKR0 | IK \rangle^2 \xi [J(J+1) - j_n(j_n+1) - j_p(j_p+1)]}{N_{IK}}.
\end{aligned} \tag{16}$$

The vertex of  $J(J+1) - j_p(j_p+1) - j_n(j_n+1)$  lies at the position  $\sqrt{j_n(j_n+1) + j_p(j_p+1) - (1/4) - (1/2)}$  and therefore the neutron-proton residual interaction is somewhat similar to the parabolic rule<sup>[15]</sup>. Signature splitting and inversion can be discussed by the level staggering  $S(I)$ ,

$$\begin{aligned}
S(I) &= S^c(I) + S^{\text{cn}}(I) + S^{\text{cp}}(I) + S^{\text{np}}(I), \\
S^\chi(I) &= E_{IK}^\chi - E_{I-1K}^\chi - \frac{1}{2} [E_{I+1K}^\chi - E_{IK}^\chi + E_{I-1K}^\chi - E_{I-2K}^\chi],
\end{aligned} \tag{17}$$

where  $\chi$  represents c, cn, cp and np (see Eqs. (13), (14) and (16)).

### 3 Discussions

Fig. 1 displays the dependence of  $C_R^2$  on  $R$  with  $\beta=0.25$ ,  $N=30$  and  $C_0^2=1$ ; the trend of the curve indicates  $C_R$  drops rapidly as  $R$  increases. Once it is true, the  $|I-j_n-j_p\rangle$  or  $|I-j_n-j_p+1\rangle$  is the largest component among the coherent states, signed by  $R$  located in the respective interval of  $|I-j_n-j_p|$  to  $I+j_n+j_p$  as  $I-j_n-j_p$  being even or  $|I-j_n-j_p+1|$  to  $I+j_n+j_p-1$  as  $I-j_n-j_p$  being odd with increment 2. The level staggering  $S^c(I) + S^{\text{cn}}(I) + S^{\text{cp}}(I)$  is shown in Fig. 2 with a rotational constant  $\lambda \simeq 7$  keV for ground band in  $^{174}\text{W}$ <sup>[16]</sup>; normal signature splitting can be seen above  $I=12$  below which the total angular momentum is smaller than  $j_n+j_p$  and the numbers of coherent states  $|R0\rangle$  decrease abruptly. The splitting at higher spins can be understood as follows. Indeed, the  $|IK\rangle$  and  $|I+1K\rangle$  states have the same coherent states if  $I-j_n-j_p$  being odd, where  $R$  ranges from  $I-j_n-j_p+1$  to  $I+j_n+j_p-1$  with step 2; this means that the rotation of the corresponding core for  $I-j_n-j_p$  state is relatively faster than the  $I-j_n-j_p+1$  state. This leads the branch with  $I-j_n-j_p$  being even is favored, i. e. low in energies,

while the other  $I-j_n-j_p$  being odd is unfavored. It is obvious that the split for given total angular momentum  $I$  is caused by the interference lower or upper limit and numbers of the core angular momentum eigenfunctions  $|R0\rangle$ , which differs from the split induced by Coriolis effects in the particle-rotor model<sup>[5]</sup>.

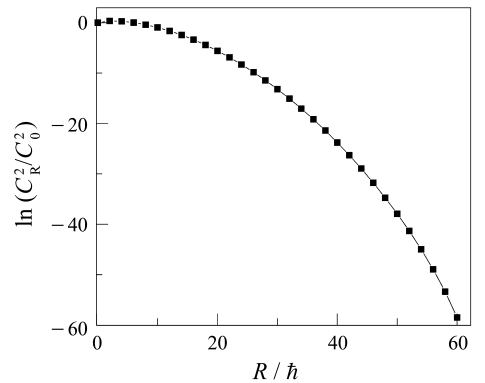


Fig. 1 Relative coherent-state probability  $\ln(C_R^2/C_0^2)$  against angular momentum  $R$  of the core.

In Ref. [3] where the particle-rotor model is used to discuss signature inversion, the member  $J_{\text{max}}=j_n+j_p$  of two particle multiplet is dominated in the favored states, while the member  $J_{\text{max}}-1$  is dominated in the unfavored states. The similar re-

sult can be obtained in this work. It is because the largest component of  $R$  is  $I - j_n - j_p$  from the above discussion and therefore the main components of  $J$

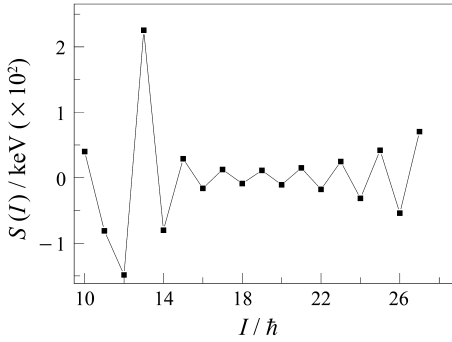


Fig. 2 Signature splittings arising from the core rotation and particle-core couplings vs total angular momentum  $I$ .

is  $j_n + j_p$  due to  $j_n + j_p \geq J \geq I - R$  when  $I - j_n - j_p$  being even. On the other hand,  $j_n + j_p - 1$  and  $j_n + j_p$  are the main components when  $I - j_n - j_p$  being odd. If the value of neutron-proton interaction among  $j_n + j_p$  is larger than the value of neutron-proton interaction among  $j_n + j_p - 1$ , the neutron-proton residual interaction makes the band having the contrary splitting rule against the one resulting from the core rotation and particle-core couplings. Fig. 3 exhibits the neutron-proton residual interactions  $\xi(j_p K_p j_n K_n | JK)^2 [J(J+1) - j_p(j_p+1) - j_n(j_n+1)]$  for  $\pi h_{11/2} (9/2^- [514]) \otimes \nu i_{13/2} (7/2^+ [633])$  two-particle multiplet with  $K = 9/2 + 7/2 = 8$  for a tentative interaction strength  $\xi = 50$  keV. The signature splitting  $S^{np}(I)$  is shown in Fig. 4. It can be seen from Figs. 2 and 4, the splitting

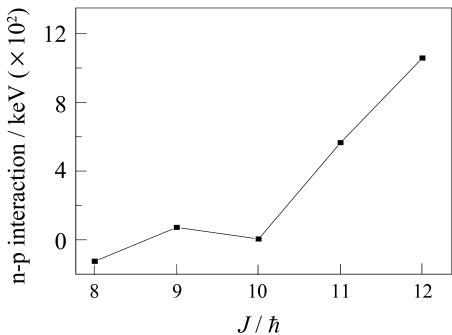


Fig. 3 Neutron-proton residual interaction for the two-particle multiplet  $\pi h_{11/2} \otimes \nu i_{13/2}$  with  $K=8$  for a tentative interaction strength  $\xi=50$  keV.

amplitudes are also contrary at higher spins. The former increases gradually with increasing total angular momentum, whereas the latter decreases gradually with increasing total angular momentum.

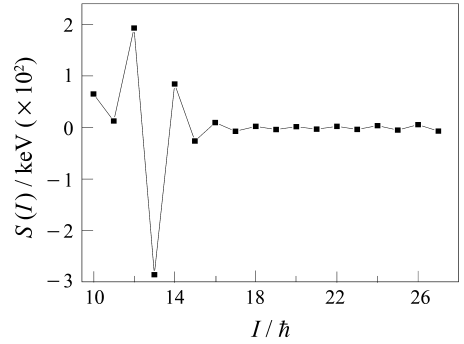


Fig. 4 Signature splittings arising from neutron-proton residual interaction for the two-particle multiplet  $\pi h_{11/2} \otimes \nu i_{13/2}$  vs total angular momentum  $I$ .

Due to the splitting arising from neutron-proton interaction dominated at low spin, the competition of two mechanics  $S^c(I) + S^{cn}(I) + S^{cp}(I)$  and  $S^{np}(I)$  results in low-spin signature inversion, showed in Figs. 5 and 6 indicates the experimental

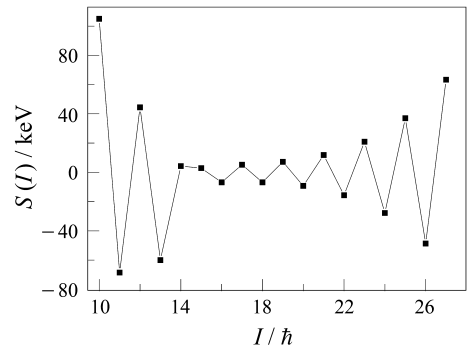


Fig. 5 Plot of calculated signature splittings vs total angular momentum  $I$ .

signature inversion for  $\pi h_{11/2} (9/2^- [514]) \otimes \nu i_{13/2} (7/2^+ [633])$  band ( $K^\pi = 8^-$ ) in  $^{176}\text{Re}^{[3]}$ . By Comparing Figs. 5 and 6, it is obvious that the calculation can approximately reproduce the experimental signature inversion point. Because of high single-particle level density in odd-odd nuclei which results in a multitude of rotation bands, further theoretical investigation to reproduce the smaller

splitting magnitude in Fig. 6 by using configuration mixing is needed. This is, however, beyond the scope of this work.

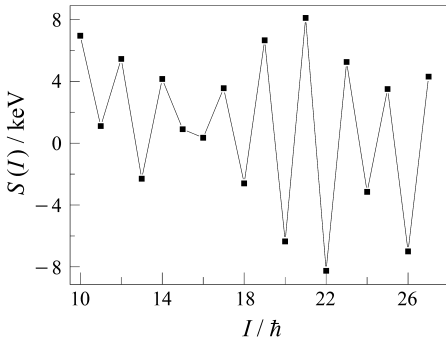


Fig. 6 Plot of signature splittings vs total angular momentum  $I$  for the two-particle bands observed in  $^{176}\text{Re}$ .

## 4 Conclusions

Signature inversion is studied by using a proton and a neutron coupling to the coherent state of the core and angular momentum projection theory. The level energy is simply scaled by two parameters representing the rotational motion, neutron-proton residual interaction and their competition. This provides a relatively simple and yet straightforward way to reduce the very complex odd-odd nuclear systems. The calculated level staggering indicates that signature inversion arises from the rotational motion and neutron-proton residual interaction having reversed signature splitting rules. Signature inversion occurs at low spin where the

splitting originated from neutron-proton residual interaction is dominated. The calculation can approximately reproduce signature inversion at axially symmetric shape and high- $K$  ( $K_n + K_p$ ) band without configuration mixing.

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