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Shell-model Study of Neutron-rich Λ-hypernucleus $^{10}_{\Lambda}$Li *

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Abstract: We investigate a Σ-mixing probability of a neutron-rich Λ-hypernucleus $^{10}_{\Lambda}$Li by using microscopic shell-model calculations considering a Λ-Σ coupling in the first order perturbation. The theoretical Σ-mixing probability in $^{10}_{\Lambda}$Li is found to be about 0.48%, due to the appearance of multi-configuration Σ Nuclear excited states which can be strongly coupled with the Λ ground state in $^{10}_{\Lambda}$Li.

Key words: hypernuclei; neutron-rich; shell model

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1 Introduction

One of the most important subjects in strange nuclear physics is a study of neutron-rich Λ-hypernuclei[1]. It is expected that a Λ hyperon has a glue-like role in nuclei beyond the neutron-drip line, together with an additional attraction of a three-body ΛNN force caused by a strong ΛN-ΣN coupling[2, 3], which might induce a Σ-mixing in nuclei. The knowledge of behavior of hyperons in a neutron-excess environment affects significantly our understanding of neutron stars, because it makes the Equation of State(EOS) soften[4]. The purpose of our study is to clarify theoretically the structure of the neutron-rich Λ-hypernuclei by a nuclear shell model, which can succeed a description of the neutron-excess nuclei.

Recently, Saha and his collaborators have performed the first successful measurement of a neutron-rich Λ-hypernucleus $^{10}_{\Lambda}$Li by the double-charge exchange reaction (π^-, K^+) on a ^{10}B target[5]. However, the magnitude and incident-momentum dependence of the experimental production cross sections cannot be reproduced by a theoretical calculation by Tretyakova and Lanskoy[6], who predicted that the cross section for $^{10}_{\Lambda}$Li is mainly explained by a two-step process, $\pi^- p \to K^0 \Lambda$ followed by $K^0 p \to K^+ n$, or $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+ \Lambda$ with the distorted-wave impulse approximation, rather than by a one-step process, $\pi^- p \to K^+ \Sigma^-$ via $\Sigma^- p$ doorways due to the $\Sigma^- p \leftrightarrow \Lambda N$ coupling. This problem might suggest the importance of Σ-mixing in the Λ-hypernucleus. We have shown that the analysis of the (π^-, K^+) reaction provides to examine precisely a wave function involving the Σ-mixing in $^{10}_{\Lambda}$Li, as well as a mechanism of this reaction[7].

In this paper, we investigate the Σ-mixing probability of the neutron-rich hypernucleus $^{10}_{\Lambda}$Li, in microscopic shell-model calculations considering the Λ-Σ coupling effect. We find that the Σ-mixing probability is about 0.48%, due to the appearance of multi-configuration Σ Nuclear excited states which can be strongly coupled with the Λ ground state in $^{10}_{\Lambda}$Li.

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2 Formalism

We consider a Λ Nuclear state involving a Σ-mixing in a Λ-hypernucleus ${}^{A}_{\Lambda}Z$ in a microscopic nuclear shell model. The state of the Λ-hypernu- cleus is represented by $|({}^{A}_{\Lambda}Z)\nu TT_zJM\rangle$, where A is the mass number, T and J are the isospin and the angular momentum, respectively, and T_z and M are their z-components. The atomic number denotes $Z=A/2+T_z$; the index ν is introduced to distinguish states with the same T and J.

In the configuration space for the Λ-hypernucleus involving a Λ-Σ coupling, the Hamiltonian is given as

$$H=H_\Lambda+H_\Sigma+V_{\Lambda\Sigma}+V_{\Sigma\Lambda}, \tag{1}$$

where H_Λ is the Hamiltonian in the Λ configuration space, H_Σ is that in the Σ configuration space, and $V_{\Lambda\Sigma}$ and its Hermitian conjugate $V_{\Sigma\Lambda}$ denote the two-body Λ-Σ coupling interaction, $\Lambda N\leftrightarrow\Sigma N$. Then, we can write the ΛNuclear state as

$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle=\sum_\mu C_{\nu,\mu}\,|\psi^\Lambda_\mu,\ TJ\rangle+\sum_{\mu'}D_{\nu,\mu'}\,|\psi^\Sigma_{\mu'},\ TJ\rangle\ , \tag{2}$$

where $|\psi^\Lambda_\mu,\ TJ\rangle$ and $|\psi^\Sigma_{\mu'},\ TJ\rangle$ are eigenstates for the Λ configuration and the Σ configuration, respectively, which are given by

$$H_\Lambda\,|\psi^\Lambda_\mu;\ TJ\rangle=E^\Lambda_\mu\,|\psi^\Lambda_\mu;\ TJ\rangle\ , \tag{3}$$

$$H_\Sigma\,|\psi^\Sigma_{\mu'};\ TJ\rangle=E^\Sigma_{\mu'}\,|\psi^\Sigma_{\mu'};\ TJ\rangle\ . \tag{4}$$

Although the coefficients $C_{\nu,\mu}$ and $D_{\nu,\mu'}$ are determined by diagonalization of the full Hamiltonian H, we treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ as perturbation because a Σ hyperon has about 80 MeV higher mass than a Λ hyperon. When taking into account up to the first-order terms, the coefficients can be written as

$$C_{\nu,\mu}=\delta_{\nu\mu},$$

$$D_{\nu,\mu'}=-\frac{\langle\psi^\Lambda_\nu,\ TJ\ |\ V_{\Lambda\Sigma}\ |\ \psi^\Sigma_{\mu'},\ TJ\rangle}{E^\Sigma_{\mu'}-E^\Lambda_\nu}\ . \tag{6}$$

Then, the Σ-mixing probability in the Λ-nuclear state $|({}^{A}_{\Lambda}Z)\nu TJ\rangle$ is given as

$$P_\Sigma=\sum_{\mu'}P_{\Sigma,\mu'}, \tag{7}$$

where

$$P_{\Sigma,\mu'}=|\ D_{\nu,\mu'}\ |^2 \tag{8}$$

is a Λ-Σ coupling strength for each Σ eigenstate $|\psi^\Sigma_{\mu'},TJ\rangle$.

It has been well-known that a Λ hyperon in a Λ-hypernucleus is described by the single-particle picture very well because a ΛN interaction is weak. On the other hand, in terms of a Σ hyperon, the nuclear configuration would change due to the strong spin-isospin dependence in a ΣN interaction. In order to evaluate the single-particle picture for a hyperon, we consider a spectroscopic factor for a hyperon-pickup reaction from $|\psi^{Y}_\mu,\ TJ\rangle$,

$$S_\mu(\nu_N T_N J_N,j_Y)=\frac{|\ \langle\psi^{Y}_\mu,\ TJ\ \|\ a^\dagger_{j_Y}\ \|\ ({}^{A-1}Z)\nu_N\ T_N\ J_N\rangle\ |^2}{(2T+1)(2J+1)}\ , \tag{9}$$

where $|({}^{A-1}Z)\nu_N T_N J_N\rangle$ is a state of a core nucleus and $a^\dagger_{j_Y}$ is a creation operator of a single-particle state of the hyperon with an angular momentum j_Y. The matrix element $\langle\cdot\ \|\ \cdot\ \|\ \cdot\rangle$ is reduced with respect to both isospin and angular momentum. The spectroscopic factor satisfies the sum rule

$$\sum_{\nu_N T_N J_N}S_\mu(\nu_N T_N J_N,j_Y)=n_{j_Y}, \tag{10}$$

where n_{j_Y} is the number of the hyperon in the orbit j_Y. If a hyperon in the hypernucleus provides the single-particle nature, the state $|\psi^{Y}_\mu,\ TJ\rangle$ is represented as a tensor product of a nuclear core state $|({}^{A-1}Z)\nu_c T_c J_c\rangle$ and a hyperon state $|j_Y\rangle$; we obtain $S_\mu(\nu_N T_N J_N,j_Y)=\delta_{\nu_N\nu_c}$, where $\nu_N=\nu_c$ means the core state is equivalent to the ${}^{A-1}Z$ state with the weak coupling limit.

In the present shell-model calculations, we construct wave functions of ${}^{A}_{\Lambda}Z$ as follows. Four nucleons are inert in the ^{4}He core and $(A-5)$ valence nucleons move in the p-shell orbits. The Λ or Σ hyperon is assumed to be in the lowest $0s_{1/2}$ orbit. For the NN effective interaction, we adopt the Cohen-Kurath (8—16) 2BME[8], which is a traditional and empirical interaction for ordinary *p*-shell nuclei, and is one of reliable effective interactions for stable and semi-stable nuclei. The YN effective interaction is written as

$$V_Y = V_0(r) + V_\sigma(r)\boldsymbol{s}_N\cdot\boldsymbol{s}_Y + V_{LS}(r)\,\ell\cdot(\boldsymbol{s}_N+\boldsymbol{s}_Y) + V_{ALS}(r)\ell\cdot(\boldsymbol{s}_N+\boldsymbol{s}_Y) + V_T(r)S_{12}, \quad (11)$$

where $V(r)$'s are radial functions of the relative coordinate $r=|\boldsymbol{r}_N-\boldsymbol{r}_Y|$ between the nucleon and the hyperon. $\boldsymbol{s}_N$ and $\boldsymbol{s}_Y$ are spin operators for the nucleon and the hyperon, respectively, and ℓ is the angular momentum operator of the relative motion. The tensor operator S_{12} is defined by

$$S_{12} = 3(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_N)(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_Y) - (\boldsymbol{\sigma}_N\cdot\boldsymbol{\sigma}_Y) \quad (12)$$

with $\boldsymbol{\sigma}=2\boldsymbol{s}$ and $\hat{\boldsymbol{r}}=(\boldsymbol{r}_N-\boldsymbol{r}_Y)/r$. In Table 1, we list the parameters of radial integrals $\overline{V}$, Δ, S_+, S_- and T, which correspond to V_0, V_σ, V_{LS}, V_{ALS} and V_T, respectively. We adopt the values of the ΛN interaction V_Λ and the Λ-Σ coupling interaction $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ which are given in Ref. [9], and the ΣN interaction V_Σ given in Ref. [10].

Table 1 Radial integrals for a YN effective interaction in unit of MeV. The values are listed in Ref. [9] for the ΛN interaction and the Λ-Σ coupling interaction, and Ref. [10] for the ΣN interaction

	Isospin	$\overline{V}$	Δ	S_+	S_-	T
V_Λ	$T=1/2$	−1.220 0	0.430 0	−0.202 5	0.187 5	0.030 0
V_Σ	$T=1/2$	−3.160 0	−2.330 0	−0.079 0	−0.010 0	−0.483 0
V_Σ	$T=3/2$	−2.040 0	4.960 0	−0.167 0	0.018 0	0.226 0
$V_{\Lambda\Sigma}$, $V_{\Sigma\Lambda}$	$T=1/2$	1.450 0	3.040 0	−0.085 0	0.000 0	0.157 0

3 Results and Discussion

We calculate the states with $T=3/2$ and $J^\pi=1^-$ of the neutron-rich Λ-hypernucleus $^{10}_{\Lambda}$Li, including the ground and excited states. We assume that the difference between Λ and Σ threshold energies is $E(^9\text{Li}_{gs}+\Sigma)-E(^9\text{Li}_{gs}+\Lambda)=80$ MeV. The dimension of the Hamiltonian matrix elements is 47 (12 Λ Nuclear states and 35 Σ Nuclear states). The calculated spectrum of Λ eigenstates $|\psi^\Lambda_\mu, TJ\rangle$ is shown in the left panel of Fig. 1. Here, we set the energy of the ground state to 0 MeV. This spectrum is very similar to the spectrum of ^{9}Li with $T=3/2$ and $J^\pi=1/2^-$, $3/2^-$, which is shown in the second panel from the left in Fig. 1. The gaps between the energy levels of $^{10}_{\Lambda}$Li slightly change from those of ^{9}Li because the ΛN interaction is weak. We confirm that the ^{9}Li core state is hardly changed by the addition of the Λ hyperon, and that the Λ hyperon behaves as the single-particle motion in the nucleus[10]. The results are also supported by the Λ-pickup spectroscopic factors. In Fig. 1, we also show the spectroscopic factors S_Λ for the Λ ground and two excited states; (a) $(T,J^\pi)=(3/2,1^-)_{gs}$ at 0.0 MeV, (b) $(3/2,1^-)_5$ at 9.5 MeV, and (c) $(3/2,1^-)_{10}$ at 16.0 MeV. In the case (a), we obtain $S_\Lambda\approx1$ for the ground

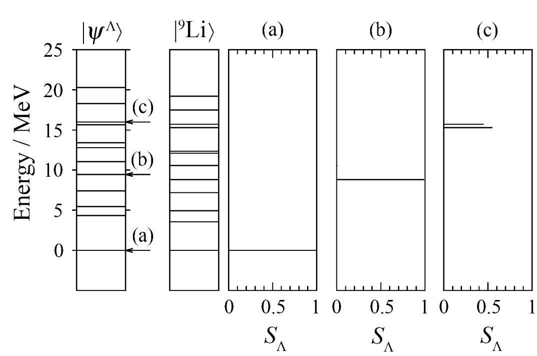

Fig. 1 Calculated energy spectra for Λ eigenstates of $^{10}_{\Lambda}$Li and eigenstates of ^{9}Li. Λ-pickup spectroscopic factors for three eigenstates, labelled by (a), (b)and (c) in each panel.

state of ^{9}Li and $S_\Lambda\approx0$ for other eigenstates. Similarly, in the case (b), $S_\Lambda\approx1$ for the corresponding eigenstate. In the case (c), S_Λ has a large value for the two eigenstates, because these states are close each other in the levels.

In Fig. 2 we display that the calculated spectra of Σ eigenstates in ${}^{10}_{\Lambda}$Li and eigenstates in ^{9}Be with $T=1/2,\ 3/2,\ 5/2$ and $J^{\pi}=1/2^{-},3/2^{-}$, together with the Σ-pickup spectroscopic factors S_{Σ^-} for three states; (a) $(3/2,\ 1^{-})_{gs}$ at 0.0 MeV, (b) $(3/2,\ 1^{-})_{10}$ at 17.3 MeV, and (c) $(3/2,\ 1^{-})_{20}$ at 24.7 MeV, where the energy of the Σ ground state $|\psi^{\Sigma}_{gs}\rangle$ is 58.4 MeV higher than that of the Λ ground state $|\psi^{\Lambda}_{gs}\rangle$. The distributions of S_{Σ^-} for excited states, (b) and (c), spread widely with the multi-configuration of ^{9}Be*, as seen in Fig. 2. This implies that the Σ hyperon has the ability of changing the nuclear configuration largely.

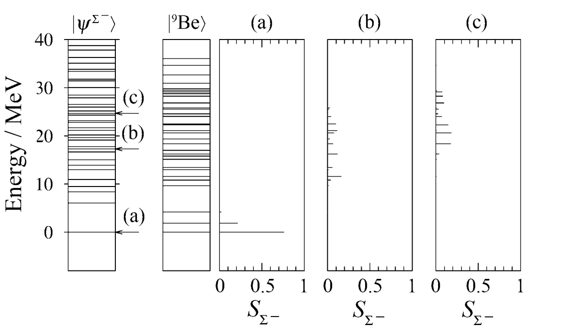

Fig. 2 Calculated energy spectra for Σ eigenstates of ${}^{10}_{\Lambda}$Li and eigenstates of ^{9}Be. Σ^{-}-pickup spectroscopic factors for three eigenstates, labelled by (a), (b) and (c) in each panel.

We find that the theoretical Σ-mixing probability P_{Σ} in the ground state of ${}^{10}_{\Lambda}$Li, which is calculated by the first-order perturbation, is 0.48%. The Λ-Σ coupling strengths $P_{\Sigma,\mu'}$ of the Σ eigenstates, in the ground state of ${}^{10}_{\Lambda}$Li are shown in Fig. 3. A contribution of the ground state $|\psi^{\Sigma}_{gs}\rangle$ ($E^{\Sigma}_{gs}-E^{\Lambda}_{gs}=58.4$) to the Σ−mixing of the ground state of ${}^{10}_{\Lambda}$Li is reduced $P_{\Sigma,gs}=0.001\%$, whereas the several Σ excited states in the $E^{\Sigma}_{\mu'}-E^{\Lambda}_{gs}=65$—70 MeV region considerably contribute to the Σ-mixing. Those contributions are enhanced by the configuration mixing due to the ΣN interaction V_{Σ}.

We used the values of $\overline{V}$ in the ΣN effective interaction, $\overline{V}_{\Sigma}(T=1/2)=-3.16$ and $\overline{V}_{\Sigma}(T=3/2)=-2.04$ MeV in Table 1, which mean the ΣN interaction is attractive. Since recent studies suggest that the ΣN interaction may be repulsive[11, 12], we demonstrate the Σ-mixing probability P_{Σ} as a function of the energy difference, $E^{\Sigma}_{gs}-E^{\Lambda}_{gs}$, between the Λ and Σ ground states. As shown in Fig. 4, we find the probability is 0.2%—0.35%, if the ΣN interaction is more repulsive than that used in the present calculation and the energy difference $E^{\Sigma}_{gs}-E^{\Lambda}_{gs}=70$—90 MeV.

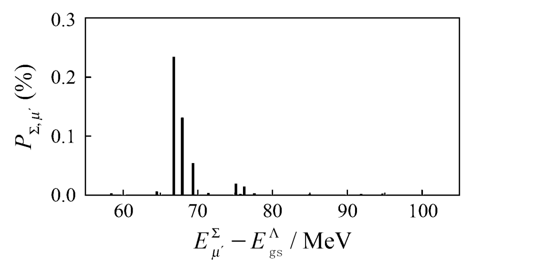

Fig. 3 Λ-Σ coupling strengths $P_{\Sigma,\mu'}$ of the Σ eigenstates in the ground state of ${}^{10}_{\Lambda}$Li.

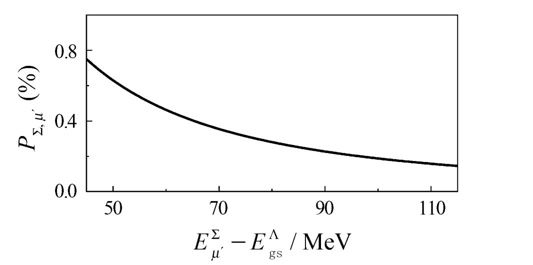

Fig. 4 The Σ-mixing probability P_{Σ} in the ground state of ${}^{10}_{\Lambda}$Li. P_{Σ} is a function of the energy difference, $E^{\Sigma}_{gs}-E^{\Lambda}_{gs}$, between the Λ and Σ ground states.

4 Conclusion

The purpose of the present study has been to investigate the Σ-mixing probabilities of the neutron-rich ${}^{10}_{\Lambda}$Li hypernucleus, in shell-model calculations considering the Λ-Σ coupling in the first-order perturbation. The present shell-model calculations have shown that the addition of the Σ hyperon changes the nuclear configuration mixing largely while the addition of the Λ hyperon does not change that. We have found that the Σ-mixing probability is about 0.48%, due to the appearance of multi-configuration Σ excited states which can be strongly coupled with the Λ ground state in $^{10}_{\Lambda}$Li.

For distribution of the Λ-Σ coupling strengths, a detailed discussion of the ΛN ↔ ΣN interaction is necessary. The analysis of the ΛN↔ΣN interaction will be published elsewhere, together with further numerical analysis of neutron-rich nuclei with/without a hyperon.

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