

# Three- and Four-body Structure of Light Hypernuclei\*

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**Abstract:** Hypernuclear physics has become very exciting owing to new epoch-making experimental data. Recent progress in theoretical and experimental studies of hypernuclei and future developments in this field are discussed.

**Key words:** hypernuclear physics; hyperon-nucleon interaction; many-body structure

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## 1 Introduction

One of the main goals in hypernuclear physics is to understand the baryon-baryon interaction. The baryon-baryon interaction is fundamental and important for the study of nuclear physics. In order to understand the baryon-baryon interaction, two-body scattering experiments are most useful. For this purpose, many NN scattering experiments have been performed and the total number of NN data are more than 4000. However, due to the difficulty of performing two-body hyperon(Y)-nucleon(N) and hyperon(Y)-hyperon(Y) scattering experiments, the total number of YN scattering data are very limited. Namely, the number of differential cross section are only about 40 and there are no YY scattering data. Therefore, the YN and YY potential models so far proposed have large ambiguities.

As a substitute for the limited two-body YN and non-existent YY scattering data, the systematic investigation of light hypernuclear structure is essential. The strategy to extract useful information about YN and YY interactions from the study of light hypernuclear structure follows(cf. Fig. 1).

(1) Firstly, we begin with candidate YN and

YY interactions which are based on meson theory and the constituent quark model. (2) Secondly, we utilize hypernuclear spectroscopy experiments performed in order to provide information about the YN and YY interactions. However, these experiments do not directly give any information about the interactions. (3) Using the interactions in (1),

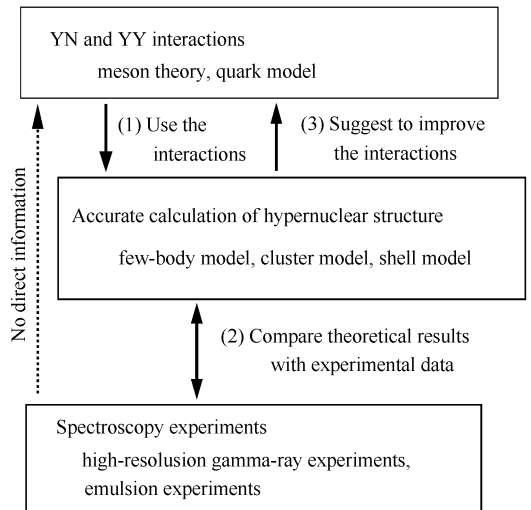


Fig. 1 Strategy for extracting information about YN and YY interactions from the study of the structure of light hypernuclei.

accurate calculations of hypernuclear structure are

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performed. The calculated results are compared with the experimental data. (4) From this comparison, improvements in the interactions are proposed.

Within the elements of the program outlined in (1) to (4), the author's role is to contribute to (3) and (4) using an accurate three- and four-body calculational method developed by the author and her collaborators (cf. the next section).

## 2 Method

In order to solve three- and four-body problems accurately, we employ the Gaussian Expansion Method (GEM) which is a variational method using Gaussian basis functions generated on all possible sets of Jacobi coordinates. The method was proposed in Ref. [1] and have been developed by the Kyushu group including the present author. The method has been successfully applied to various types of three- and four-body systems, which is summarized in Ref. [2].

Especially, regarding four-body systems, recently in Ref. [3], seven different few-body research groups (including the present author) performed a benchmark-test calculation for the four-nucleon bound state, namely, the ground state of  ${}^4\text{He}$  using a realistic force AV8'. Good agreement was obtained among the seven different methods for the binding energy, the rms radius, and the two-body correlation function.

## 3 $S = -1$ Hypernuclei and the YN Interaction

Following the strategy mentioned in Sec. 1, we have obtained information about the spin-spin, spin-orbit and tensor term of the YN interaction from the study of  $S = -1$  hypernuclei. As an example, the investigation of the YN spin-orbit force is explained.

In the YN interaction, there are two kinds of LS forces, a symmetric LS force ( SLS ) and an antisymmetric LS ( ALS ) force, defined by

$$\begin{aligned} V_{\text{SLS}} &= \mathbf{L} \cdot (\mathbf{s}_\Lambda + \mathbf{s}_N) v_{\text{SLS}}(r) , \\ V_{\text{ALS}} &= \mathbf{L} \cdot (\mathbf{s}_\Lambda - \mathbf{s}_N) v_{\text{ALS}}(r) , \end{aligned} \quad (1)$$

where  $\mathbf{s}_\Lambda$  and  $\mathbf{s}_N$  are spins of the  $\Lambda$  and  $N$ , respectively. The ALS force vanishes in conventional nuclei because of the Pauli principle. On the other hand, the ALS force is present in hypernuclei since no Pauli principle works between the  $\Lambda$  and  $N$ .

Historically, it is well known that the ALS force differs between meson theory and the constituent quark model<sup>[4]</sup>. For instance, the quark model of the Kyoto-Niigata group<sup>[5]</sup> predicts that the strength of the ALS amounts to approximately 85% of that of the SLS but with the opposite sign. On the other hand, the meson based interaction of the Nijmegen group<sup>[6, 7]</sup> generates much smaller strength in the ALS, some 20%—40% of that of the SLS, also with the opposite sign. Therefore, it is important to extract information about these LS forces from the study of the structure of  $\Lambda$  hypernuclei.

For the study of the spin-orbit force,  ${}^9_\Lambda\text{Be}$  and  ${}^{13}_\Lambda\text{C}$  are very useful. Recently, in high-resolution  $\gamma$ -ray experiments, BNL-E930<sup>[8]</sup> and BNL-E929<sup>[9]</sup>, the spin-orbit splitting energies of  ${}^9_\Lambda\text{Be}$  and  ${}^{13}_\Lambda\text{C}$  were measured. Namely, the first (E930) observed  $\gamma$  rays from the decay of the  $5/2_1^+$  and the  $3/2_1^+$  states to the  $1/2_1^+$  ground state in  ${}^9_\Lambda\text{Be}$ , while the second (E929) measured those from the  $3/2_1^-$  and  $1/2_1^-$  states to the  $1/2_1^+$  ground state in  ${}^{13}_\Lambda\text{C}$ .

Before the measurements were made, we predicted those energy splittings in Ref. [10]. We took an  $\alpha + \alpha + \Lambda$  three-body model for  ${}^9_\Lambda\text{Be}$  and an  $\alpha + \alpha + \alpha + \Lambda$  four-body model for  ${}^{13}_\Lambda\text{C}$ . We employed two types of the YN spin-orbit force, namely, the Nijmegen meson-theory based YN interaction and the Kyoto-Niigata group's quark based YN interaction mentioned above. The predicted energy splittings of  ${}^9_\Lambda\text{Be}$  and  ${}^{13}_\Lambda\text{C}$  are listed in the second and third columns of Table 1. In both nuclei, the splittings given by using the quark based LS force are significantly smaller than those

using the meson based LS force.

**Table 1 Spin-orbit splitting energy in  ${}^9_{\Lambda}\text{Be}$  and  ${}^{13}_{\Lambda}\text{C}$**

Splitting	Cal. (meson theory)	Cal. (quark model)	Exp.
	/ keV	/ keV	/ keV
${}^9_{\Lambda}\text{Be } E(5/2^+ - 3/2^+)$	80 – 200	35 – 40	$43 \pm 5$
${}^{13}_{\Lambda}\text{C } E(3/2^- - 1/2^-)$	390 – 960	150 – 200	$150 \pm 54 \pm 36$

Recently, experimental data for these energy splittings of  ${}^9_{\Lambda}\text{Be}$ <sup>[8]</sup> and  ${}^{13}_{\Lambda}\text{C}$ <sup>[9]</sup> have been reported to be  $43 \pm 5$  and  $152 \pm 54 \pm 36$  keV, respectively, as shown in Table 1. We see that the predicted energy splitting using the quark-model based spin-orbit force can explain both data. On the other hand, the predictions using the meson-theory based force are much larger than the data.

The reason why the meson-theory based YN interaction produces a large spin-orbit splitting in the case of  ${}^9_{\Lambda}\text{Be}$  is as follows. Using the SLS force only, the splitting energy is 140–250 keV depending on the five models in the YN interaction, it is not a small value. When the ALS force is included, the ALS with the opposite sign of the SLS reduces this splitting. But, the strength of the ALS in the case of Nijmegen model, 20%–40% of the SLS as mentioned before, is not enough to reproduce the observed data. On the other hand, in the quark model, the ALS is strong enough to reproduce the data. Therefore, we suggested that there are two paths to improve the meson based model; one is to reduce the SLS strength and the other is to enhance the ALS strength so as to reproduce the observed spin-orbit splittings in  ${}^9_{\Lambda}\text{Be}$  and  ${}^{13}_{\Lambda}\text{C}$ .

Recently, a new YN interaction based on meson theory was proposed by the Nijmegen group (ESC06)<sup>[11]</sup>, they proposed a reduced strength of the SLS. Using this potential, we obtained the energy splitting in  ${}^9_{\Lambda}\text{Be}$  to be 98 keV in the case of the SLS only and 39 keV with including the ALS, which is in good agreement with the data.

To summarize in reference to the numbers in parentheses in the strategy diagram of Fig. 1, (1)

we used two types of the YN spin-orbit models, the Nijmegen model and the Kyoto-Niigata model and calculated the energy splittings of  ${}^9_{\Lambda}\text{Be}$  and  ${}^{13}_{\Lambda}\text{C}$ ; (2) We then compared our results with the experimental data; (3) We suggested improving the strength of the LS force. After that, the Nijmegen group proposed a new potential version ESC06. Using this potential, we calculated the energy splitting, and we compared with the experimental data. Then, the calculated results were in good agreements with the experimental data. Since 1998, we have many  $\gamma$ -ray spectroscopic data<sup>[12, 13]</sup>. By combined analysis of experiments and theoretical calculations, we are succeeding in extracting information about the spin-spin, spin-orbit and tensor terms of  $\Lambda\text{N}$  interaction.

## 4 $S = -2$ Hypernuclei and the YY Interaction

It is interesting to investigate the structure of multi-strangeness systems, when one or more  $\Lambda$ s are added to an  $S = -1$  nucleus. It is conjectured that the extreme limit, which includes many  $\Lambda$ s (and other hyperons) in nuclear matter, is the core of a neutron star. In this scenario, the sector of  $S = -2$  nuclei, double  $\Lambda$  hypernuclei and  $\Xi$  hypernuclei, is just the entrance into the multi-strangeness world. However, we have hardly any knowledge of the YY interaction, because there exist no YY scattering data. Therefore, in order to understand the YY interaction, it is crucial to study the structure of double  $\Lambda$  hypernuclei and  $\Xi$  hypernuclei. The equation of state with the strangeness degree of freedom includes a crucial component in under-

standing neutron stars.

Recently, an epoch-making datum has been reported by the KEK-E373 experiment. Namely, the double  $\Lambda$  hypernucleus  ${}_{\Lambda\Lambda}^6\text{He}$  was observed<sup>[14]</sup>. This observation was called the NAGARA event. The formation of  ${}_{\Lambda\Lambda}^6\text{He}$  was uniquely identified by the observation of sequential weak decays, and a precise experimental value of the  $2\Lambda$  binding (separation) energy,  $B_{\Lambda\Lambda} = (7.25 \pm 0.19^{+0.18}_{-0.11})\text{MeV}$ , was obtained.

Following the strategy mentioned in Sec. 1, we studied double  $\Lambda$  hypernuclei with  $A = 6 - 10$ <sup>[17]</sup>. Firstly, (1) we employed the  $\Lambda\Lambda$  interaction of Nijmegen model D and performed an  $\alpha + \Lambda + \Lambda$  three-body calculation for  ${}_{\Lambda\Lambda}^6\text{He}$ ; (2) by comparing the theoretical result with the experimental data of the binding energy of  ${}_{\Lambda\Lambda}^6\text{He}$ ; (3) we suggested reducing the strength of the  ${}^1S_0$  term in the  $\Lambda\Lambda$  interaction by half to reproduce the data. Again, (2) using the improved potential, we predicted energy spectra of new double  $\Lambda$  hypernuclei with  $A = 7 - 10$ <sup>[15]</sup>, which is discussed below.

In fact, it is planned at J-PARC to produce many double  $\Lambda$  hypernuclei by emulsion techniques<sup>[16]</sup>. However, it will be difficult to determine the spin-parities and to know whether the observed state is the ground state or an excited state. Therefore, it will be necessary to compare the data with theoretical studies for the identification of the state. The author's role is to contribute to the theoretical analysis by using few-body techniques.

A successful example of determining the spin-parity of double  $\Lambda$  hypernuclei is the case of  ${}_{\Lambda\Lambda}^{10}\text{Be}$ . There was one more event found in the E373 experiment named the “Demachi-Yanagi” event<sup>[17, 18]</sup>. The most probable interpretation of this event is the production of a bound state of  ${}_{\Lambda\Lambda}^{10}\text{Be}$  having  $B_{\Lambda\Lambda}^{\text{exp}} = 12.33^{+0.35}_{-0.21}\text{MeV}$ . But the experiment could not determine whether this state was the ground state or an excited state. In order to determine this, our calculation<sup>[5]</sup> mentioned above was useful as following: we studied  ${}_{\Lambda\Lambda}^{10}\text{Be}$  by em-

ploying an  $\alpha + \alpha + \Lambda + \Lambda$  four-body model. The  $\Lambda\Lambda$  interaction is the one improved from the Nijmegen Model D as mentioned above. The  $\Lambda\Lambda$ ,  $\alpha\Lambda$  and  $\alpha\alpha$  interactions were chosen so as to reproduce the binding energies of all the subsystems,  ${}_{\Lambda\Lambda}^6\text{He}$ ,  ${}_{\Lambda}^5\text{He}$ ,  ${}^8\text{Be}$  and  ${}^9_{\Lambda}\text{Be}$ . As shown in Fig. 2, it is striking that our calculated value of  $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{10}\text{Be}(2^+))$  is 12.28 MeV that agrees with the experimental data. Therefore, the Demachi-Yanagi event can be interpreted most probably as the observation of the  $2^+$  excited state in  ${}_{\Lambda\Lambda}^{10}\text{Be}$ .

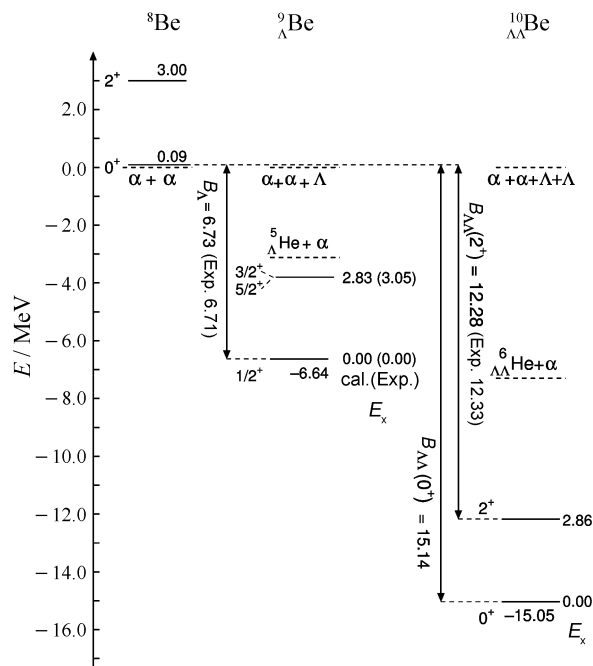


Fig. 2 Calculated energy levels of  ${}^8\text{Be}$ ,  ${}^9_{\Lambda}\text{Be}$  and  ${}_{\Lambda\Lambda}^{10}\text{Be}$  on the basis of the  $\alpha + \alpha$ ,  $\alpha + \alpha + \Lambda$ , and  $\alpha + \alpha + \Lambda + \Lambda$  models, respectively. The level energies are measured from the particle breakup thresholds or are given by the excitation energies  $E_x$ . The calculated  $2^+$  state of  ${}_{\Lambda\Lambda}^{10}\text{Be}$  explains the Demachi-Yanagi event.

In this way, we succeeded in interpreting the spin-parity of  ${}_{\Lambda\Lambda}^{10}\text{Be}$  by comparing the experimental data and our theoretical calculation. Therefore, our four-body calculation is considered to have predictive power. Hoping to observe new double  $\Lambda$  hypernuclei in future experiments, we have predicted, as shown in Fig. 3, the level structure of double  $\Lambda$  hypernuclei with  $A = 7 - 9$  taking as the framework the  $\alpha + x + \Lambda + \Lambda$  models with  $x = n, p$ ,

d, t and  $^3\text{He}$ <sup>[15]</sup>.

By comparing our theoretical predictions with future experimental data, we can interpret the

spectroscopy of the double  $\Lambda$  hypernuclei. I hope that many double  $\Lambda$  hypernuclei will be produced at the J-PARC facility.

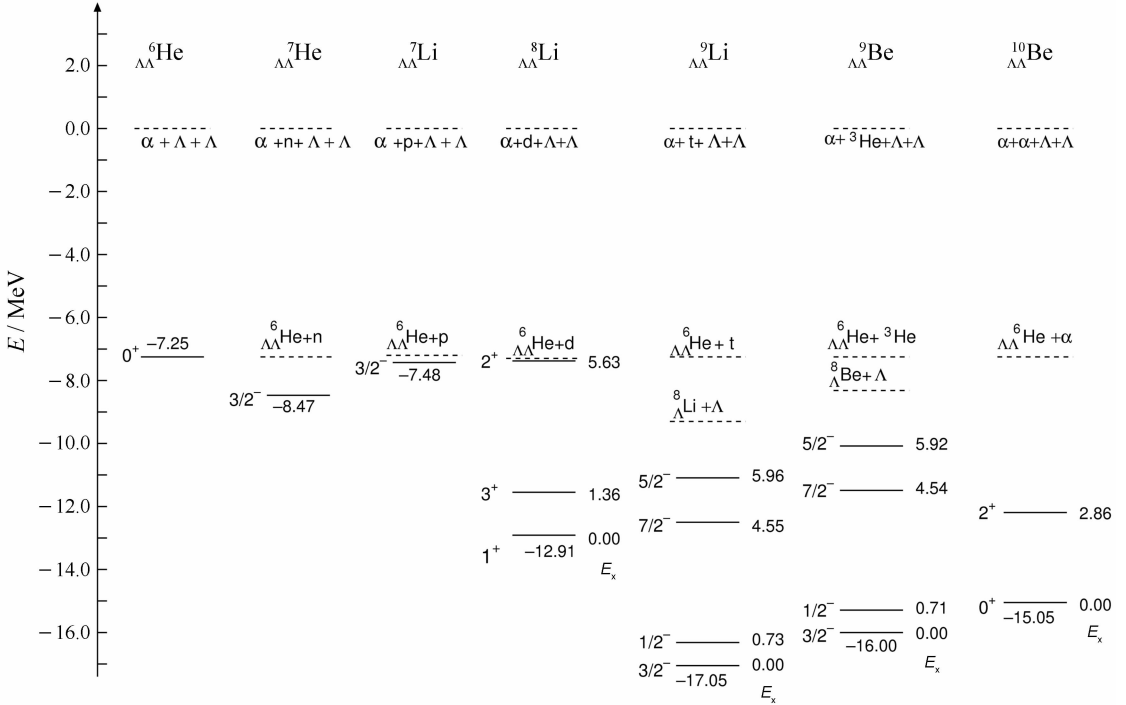


Fig. 3 Energy levels of double- $\Lambda$  hypernuclei,  ${}_{\Lambda\Lambda}^6\text{He}$ ,  ${}_{\Lambda\Lambda}^7\text{He}$ ,  ${}_{\Lambda\Lambda}^7\text{Li}$ ,  ${}_{\Lambda\Lambda}^8\text{Li}$ ,  ${}_{\Lambda\Lambda}^9\text{Li}$ ,  ${}_{\Lambda\Lambda}^9\text{Be}$  and  ${}_{\Lambda\Lambda}^{10}\text{Be}$  calculated using the  $\alpha+x+\Lambda+\Lambda$  model with  $x=0, n, p, d, t, {}^3\text{He}$  and  $\alpha$ , respectively.

### 5 Future Subjects

In the  $S=-1$  sector, the following two subjects are still open questions: 1) charge symmetry breaking and 2)  $\Lambda\text{N}-\Sigma\text{N}$  coupling. For these studies, it is planned in J-PARC to do experiments on  ${}_{\Lambda}^1\text{B}$  and  ${}_{\Lambda}^4\text{He}$  in Ref. [19], and on  ${}_{\Lambda}^9\text{He}$  and  ${}_{\Lambda}^6\text{H}$  in Ref. [20].

The  $S=-2$  sector is the entrance to the multi-strangeness world, the study of which is one of the major goals of the hypernuclear physics. In this sector, the most important subjects still to be studied are the  $\Lambda\Lambda-\Xi\text{N}$  coupling and the  $\Xi\text{N}-\Xi\text{N}$  interaction.

Firstly, we discuss  $\Lambda\Lambda-\Xi\text{N}$  coupling. Because the difference between the threshold energy of  $\Xi\text{N}$  and that of  $\Lambda\Lambda$  is very small ( $\sim 25$  MeV), the  $\Lambda\Lambda-\Xi\text{N}$  particle conversion is considered to be strong in multi-strangeness systems. However, the effect of  $\Lambda-\Lambda-\Xi\text{N}$  coupling is small in  ${}_{\Lambda\Lambda}^6\text{He}$ .

The reason why is as follows: in the shell model picture, in the lowest s-shell, two neutrons, two protons and two  $\Lambda$ s occupy the available space and the s-shell is closed. When the two  $\Lambda$ s occupying the s-shell are converted into  $\Xi\text{N}$  by  $\Lambda\Lambda-\Xi\text{N}$  coupling, the valence nucleon is forbidden to occupy the s-shell due to Pauli Principle effects<sup>[21, 22]</sup>. On the other hand, for the study of  $\Lambda\Lambda-\Xi\text{N}$  coupling, s-shell double  $\Lambda$  hypernuclei such as  ${}_{\Lambda\Lambda}^4\text{H}$ ,  ${}_{\Lambda\Lambda}^5\text{H}$  and  ${}_{\Lambda\Lambda}^5\text{He}$  are very suitable<sup>[23-25]</sup>. Because for example, in the case of  ${}_{\Lambda\Lambda}^5\text{H}$ , in the s-shell, two neutrons, one proton and two  $\Lambda$ s occupy the space. When two  $\Lambda$ s are converted into  $\Xi^-p$  by  $\Lambda\Lambda-\Xi\text{N}$  coupling, the valence proton can occupy the s-shell because it is not Pauli blocked<sup>[6]</sup>. Therefore, it is thought that the  $\Lambda\Lambda-\Xi\text{N}$  coupling can be large in s-shell double  $\Lambda$  hypernuclei such as  $A=4$  and 5 systems.

Next, let us discuss the  $\Xi\text{N}-\Xi\text{N}$  interaction.

The study of the structure of  $\Xi$  hypernuclei will be useful for investigating the  $\Xi N$  interaction. Since there has been no observation of  $\Xi$  hypernuclei, it is necessary to produce in the near future  $\Xi$  hypernuclei as bound states. For this purpose, it is planned at J-PARC<sup>[27]</sup> to produce the  ${}_{\Xi}^{12}\text{Be}$  ( $= {}^{11}\text{B} + \Xi^-$ ) hypernucleus by the  $(K^-, K^+)$  reaction using a  ${}^{12}\text{C}$  target. In this experiment, we would have the first observation of a  $\Xi$  hypernucleus, which should contribute to extracting information about the  $\Xi N$  interaction. After this experiment, we hope that many experiments searching for  $\Xi$  hypernuclei will be performed successfully at J-PARC.

In conclusion, we have discussed a strategy to study the  $YN$  and  $YY$  interactions in connection with the structure of hypernuclei having  $S = -1$  and  $S = -2$ . At J-PARC, JLAB, DAΦNE and GSI facilities, production of many hypernuclei in the  $S = -1$  and  $S = -2$  sectors is planned. We expect that hypernuclear physics will be much enhanced by the experimental data from these facilities and the related theoretical studies.

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