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High Order Isospin-dependent Nucleon-ω-ρ Coupling Correction in Relativistic Mean Field Theory*

LIANG Jun
(*China Institute of Atomic Energy*, *Beijing* 102413, *China*;
Department of Physics, *University of Technology of Taiyuan*, *Taiyuan* 030024, *China*)

Abstract: The sensitivity of the neutron skin thickness S in ^{208}Pb to the new addition of the high order isospin-dependent nucleon-ω-ρ coupling term in the relativistic mean field model is studied. Calculations show that the high order isospin-dependent nucleon-ω-ρ coupling term can further soften the symmetry energy, and thus further decrease the neutron radius of ^{208}Pb without affecting other ground-state observables.

Key words: relativistic mean field; nucleon-ω-ρ coupling; neutron skin thickness
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1 Introduction

Much attention[1—10] has been devoted to studying the neutron skin thickness S in ^{208}Pb, a quantity defined as the difference between the root-mean-square(rms) radius of neutrons, $\sqrt{\langle r_n^2 \rangle}$, and that of protons, $\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$. It is of significance, indeed, a precise measurement of the neutron radius of ^{208}Pb, together with existing high-precision measurements of the proton radius of ^{208}Pb, will yield a precise value for the neutron skin thickness, thus providing one of most stringent tests for current models of nuclear structure. As the neutron radius of ^{208}Pb is not tightly constrained by existent ground-state observables, some authors[10—15] by introducing a nonlinear isoscalar-isovector coupling or higher order coupling modify the poorly known dependence of the symmetry energy, and thus change the neutron radius of ^{208}Pb. In this article, the author also make a study of that problem by introducing

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Biography: Liang Jun (1968—), male(Han Nationality), Shanxi Datong, Lecturer, working on nuclear many body theory; E-mail: liangjun@iris.ciae.ac.cn

a new addition of high order nucleon-ω-ρ coupling term to Relativistic Mean Field (RMF) model.

2 Formalism

The basic physics underlying RMF models and their application in nuclear physics can be found in Refs. [16—18]. The following derivations are standard. The Lagrangian of isodoublet nucleon field(ψ) interacting via the exchange of one scalar (σ for the sigma) and three vector (ω^μ for the omega, $\boldsymbol{b}^\mu$ for the rho, and A^μ for the photon) fields. That is,

$$\mathscr{L}=\bar{\psi}[\gamma^\mu(\mathrm{i}\partial_\mu-g_\omega\omega_\mu-g_\rho\boldsymbol{\tau}\cdot\boldsymbol{b}_\mu-\frac{1}{2}e(1+\tau_3)A_\mu)-(M+g_\sigma\sigma)\psi+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma-\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}+\frac{1}{2}m_\omega^2\omega^\mu\omega_\mu-\frac{1}{4}\boldsymbol{b}^{\mu\nu}\cdot\boldsymbol{b}_{\mu\nu}+\frac{1}{2}m_\rho^2\boldsymbol{b}^\mu\cdot\boldsymbol{b}_\mu-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-U_{\mathrm{eff}}(\sigma,\omega^\mu,\boldsymbol{b}^\mu)\ , \tag{1}$$

where M, m_σ, m_ω, m_ρ are the nucleon-, the σ-, the ω- and the ρ-masses, respectively, while g_σ, g_ω, g_ρ and $e^2/4\pi=1/137$ are the corresponding constants for the mesons and the photon; the various field tensors have been defined as follow:

$$\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu\ , \tag{2}$$

$$\boldsymbol{b}_{\mu\nu}=\partial_\mu\boldsymbol{b}_\nu-\partial_\nu\boldsymbol{b}_\mu\ , \tag{3}$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ , \tag{4}$$

$$U_{\mathrm{eff}}(\sigma,\omega^\mu,\boldsymbol{b}^\mu)=\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4-\frac{1}{4}c_3(\omega^\mu\omega_\mu)^2-4\Lambda_{\mathrm{v}}g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu g_\omega^2\omega_\mu\omega^\mu\ , \tag{5}$$

where g_2, g_3 and c_3 are the nonlinear parameters for the self-interactions of the scalar and vector fields, Λ_{v} is nonlinear mixed isoscalar-isovector coupling coefficient. The author tries introducing the following new coupling of the nucleon current to ω- and ρ-meson fields:

$$\mathscr{L}_{\mathrm{new}}=-\Lambda_\omega\bar{\psi}g_\rho\gamma^\mu\frac{g_\omega^2\omega^\mu\omega_\mu}{M^2}\boldsymbol{\tau}\cdot\boldsymbol{b}_\mu\psi\ , \tag{6}$$

where Λ_ω is coupling coefficient. Thus the total Lagrangian density is:

$$\mathscr{L}'=\mathscr{L}+\mathscr{L}_{\mathrm{new}}\ . \tag{7}$$

Within the framwork of RMF models, substituting the Lagrangian density into the Euler-Lagrangian equations of motion leads to Dirac equations of motion for the nucleon spinors and Klein-Gordon equatinos for the mesons and photons. For a static, spherically symmetric system ^{208}Pb, the single-particle solutions of the Dirac equation may be written as: (using $|\boldsymbol{x}\equiv\boldsymbol{r}$)

$$\Psi_\alpha=\mathscr{U}_{n\kappa mt}=\frac{1}{r}\begin{pmatrix}g_{n\kappa t}(r)\mathscr{Y}_{\kappa m}(\hat{\boldsymbol{x}})\\ \mathrm{i}f_{n\kappa t}(r)\mathscr{Y}_{-\kappa m}(\hat{\boldsymbol{x}})\end{pmatrix}\zeta_t\ , \tag{8}$$

where $\{\alpha\}\equiv\{a=n\kappa t;\ m\}$ denotes the collection of all quantum numbers required to describe the single-particle Dirac spinor. ζ_t denotes a two-component spinor in issopin space (with $t=\pm\frac{1}{2}$ for protons and neutrons, respectively), n and m are the principal and magnetic quantum numbers, respectively, and the spin-spherical harmonics are defined as:

$$\mathscr{Y}_{\kappa m}(\hat{\boldsymbol{x}})\equiv\langle\hat{\boldsymbol{x}}\mid l\frac{1}{2}jm\rangle;\quad j=|\kappa|-\frac{1}{2}\ ; \tag{9}$$

$$l=\begin{cases}\kappa, & \text{if}\quad \kappa>0\\ -1-\kappa, & \text{if}\quad \kappa<0\end{cases}\ . \tag{10}$$

Further, the following spinor normalization has been adopted:

$$\int\mathrm{d}^3x\,\mathscr{U}_\alpha^\dagger(\boldsymbol{x})\mathscr{U}_\alpha(\boldsymbol{x})=\int_0^\infty\mathrm{d}r(g_a^2(r)+f_a^2(r))=1\ . \tag{11}$$

It then follows that the coupled differential equations satisfied by the radial components of the Dirac spinor are given by:

$$E_a g_a(r)=\left(-\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)f_a(r)+(M+S(r)+V(r)g_a(r)\ , \tag{12}$$

$$E_a f_a(r)=\left(-\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)g_a(r)-(M+S(r)-V(r))f_a(r)\ , \tag{13}$$

where

$$V(r)=g_\omega\omega_0+g_\rho\left(1+\Lambda_\omega\left(\frac{g_\omega\omega_0}{M}\right)^2\right)\tau_3 b_0+$$

$$\frac{1}{2}e(1+\tau_3)A_0 \quad , \tag{14}$$

$$S(r)=g_\sigma\sigma \quad . \tag{15}$$

The meson field equations reduce to the following set of equations:

$$\left(-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+m_\sigma^2\right)\sigma = -g_\sigma\rho_s - g_2\sigma^2 - g_3\sigma^3 \quad , \tag{16}$$

$$\left(-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+m_\omega^2\right)\omega_0 = g_\omega\rho_v - c_3\omega_0^3 - 8\Lambda_v g_\rho^2 g_\omega^2 b_0^2\omega_0 + \frac{2\Lambda_\omega g_\rho b_0\rho_3 g_\omega^2\omega_0}{M^2} \quad , \tag{17}$$

$$\left(-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+m_\rho^2\right)b_0 = -g_\rho\rho_3\left(1+\Lambda_\omega\left(\frac{g_\omega\omega_0}{M}\right)^2\right) - 8\Lambda_v g_\rho^2 g_\omega^2\omega_0^2 b_0 \quad , \tag{18}$$

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)A_0(r) = -e\rho_p \quad . \tag{19}$$

The corresponding densities are given by:

$$\rho_s(r)=\rho_{s,p}(r)+\rho_{s,n}(r) \quad , \tag{20}$$

$$\rho_v(r)=\rho_{v,p}(r)+\rho_{v,n}(r) \quad , \tag{21}$$

$$\rho_3(r)=\rho_{v,p}(r)-\rho_{v,n}(r) \quad , \tag{22}$$

$$\rho_p(r)=\rho_{v,p}(r) \quad , \tag{23}$$

where scalar and vector densities have been defined as:

$$\begin{pmatrix}\rho_{s,t}(r)\\ \rho_{v,t}(r)\end{pmatrix} = \sum_{n\kappa}^{occ}\left(\frac{2j_\kappa+1}{4\pi r^2}\right)\left(g_{n\kappa t}^2(r) \mp f_{n\kappa t}^2(r)\right) \quad , \tag{24}$$

where t=n,p for neutron or proton, respectively. The neutron and proton rms radii are directly related to the neutron and proton density distributions via the following relationship:

$$R_t=\sqrt{\langle r_t^2\rangle}=\sqrt{\frac{\int d\boldsymbol{x}\rho_t(r)r^2}{\int d\boldsymbol{x}\rho_t(r)}} \quad . \tag{25}$$

The total binding energy of the system is:

$$E=\sum_\alpha^{occ}E_\alpha(2j_\alpha+1)-\frac{1}{2}\int d\boldsymbol{x}\left[g_\sigma\sigma(r)\rho_s(r)+\frac{1}{3}g_2\sigma^3(r)+\frac{1}{2}g_3\sigma^4(r)\right]-\frac{1}{2}\int d\boldsymbol{x}\left\{g_\rho b_0(r)\cdot\left[1+3\Lambda_\omega\left(\frac{g_\omega\omega_0(r)}{M}\right)^2\right]\rho_3(r)-8\Lambda_v(g_\rho g_\omega\omega_0(r)b_0(r))^2\right\}-\frac{1}{2}\int d\boldsymbol{x}\left[g_\omega\omega(r)\rho_v(r)-\frac{1}{2}c_3\omega_0^4(r)\right]-\frac{1}{2}\int d\boldsymbol{x}A_0(r)\rho_c(r)-MA \quad . \tag{26}$$

The coupled, ordinary differential equations (9)—(10) and (13)—(16) must be self-consistently solved, see Refs.[10,19], for details.

3 Result and Discussion

Three models are considered in this article, the very successful NL3[12,20] along with the S271 and Z271[13—15]. These three models are constrained to the following properties of symmetric nuclear matter: nuclear saturation at a fermi momentum of $\kappa_F=1.30\ \mathrm{fm}^{-1}$; a binding energy per nucleon of 16.24 MeV; and a incompressibility of $K=271$ MeV. The various parameter sets are listed in Table 1.

The symmetry energy[12,13,15] is given by

$$a_{sym}(\rho)=\frac{k_F^2}{6E_F^*}+\frac{g_\rho^2}{3\pi^2}\frac{k_F^3}{m_\rho^{*2}} \quad , \tag{27}$$

where k_F is the Fermi momentum, $\sqrt{k_F^2+M^{*2}}$, $M^*=M+g_\sigma\sigma$ is the effective nucleon mass.

Table 1　Model parameters used in the calculations*

Model	m_s	g_σ	g_ω	g_2	g_3	c_3
NL3	508.194	10.271	12.868	−10.431	−28.885	0.00
S271	505.000	9.006	10.806	−12.370	−17.323	0.00
Z271	465.000	7.031	8.406	−5.434	63.691	49.941

* The parameter g_2 is given in fm^{-1}; the scalar mass m_σ is given in MeV; the nucleon, rho, and omega masses are kept fixed at $M=939$, $m_\rho=763$ and $m_\omega=783$ MeV, respectively, except in the case of the NL3 model where it is fixed at $m_\omega=782.5$ MeV.

Further, the effective rho-meson mass has been difined as follows:

$$m_\rho^{*2}=\frac{m_\rho^2+8\Lambda_v g_\rho^2 g_\omega^2\omega_0^2}{\Gamma^2} \quad , \tag{28}$$

where $\Gamma=1+\Lambda_\omega(g_\omega\omega/M)^2$.

The symmetry energy at saturation density corresponding to $k_F = 1.30\ \text{fm}^{-1}$, is not well constrained experimentally. Rather, an average of the symmetry energy is constrained by the nuclear binding energy. The following prescription is adopted: the value of the NN_ρ coupling constant is adjusted so that all parameter sets have a symmetry energy of $a_{sym} = 25.68$ MeV at $k_F = 1.15\ \text{fm}^{-1}$ ($\rho = 0.10\ \text{fm}^{-1}$)[13]. That is

$$g_\rho^2 = \frac{m_\rho^2 \Delta a_{sym}}{\frac{k_F^3}{3\pi^2}\Gamma^2 - 8\Lambda_v g_\omega^2 \omega_0^2 \Delta a_{sym}}\ , \tag{29}$$

where $\Delta a_{sym} = (a_{sym} - k_F^2/6E_F^*)$.

The results are listed in Table 2. From the Table, we see that the combination of Λ_v and Λ_ω can further decrease the neutron skin thickness S in ^{208}Pb while still maintaining the binding energy and the proton radius nearly constant. The binding energy and proton radius are accurately determined experimentally. The experimental binding energy per nucleon $(E/A)_{exp}$ in ^{208}Pb is -7.868 MeV[21], considering the correction for center-of-mass motion is neglected in the self-consistent RMF calculation, an equivalent experimental $(E/A)_{exp}$ is -7.843 MeV for ^{208}Pb, which is estimated via the harmonic oscillator phenomenological formula $E_{cm} = 0.75\ \hbar\omega_0$, with $\hbar\omega_0 = 41A^{-1/3}$[22]; and the fact that the accurately measured value of the proton rms radius R_p of ^{208}Pb is (5.45 ± 0.02) fm[23]. Moveover for Z271 parameter set, which sacrifices good fits to the experimental binding energy per nucleon, the absolute value of the binding energy of ^{208}Pb using Z271 effective interacion is smaller than the experimental one; by introducing the isospin-dependent high order nucleon-ω-ρ coupling correction to RMF model, the combination of Λ_v and Λ_ω can make the absolute value of the binding energy go near the experimental one.

Table 2 Results for the NL3, S271 and Z271 effective interaction, the binding energy per nucleon, E/A, the proton rms radius R_p and the neutron skin thickness S in ^{208}Pb

Model	Λ_v	Λ_ω	g_ρ	E/A	R_p	S
NL3	0.0	0.0	4.461	−7.854	5.460	0.280
	0.005	0.0	4.606	−7.864	5.460	0.265
		−1.6	5.054	−7.898	5.461	0.218
	0.01	0.0	4.766	−7.872	5.461	0.251
		−1.2	5.133	−7.897	5.463	0.215
	0.015	0.0	4.946	−7.879	5.463	0.237
		−0.9	5.249	−7.897	5.465	0.209
	0.02	0.0	5.146	−7.885	5.465	0.233
		−0.6	5.371	−7.897	5.467	0.204
	0.025	0.0	5.374	−7.890	5.468	0.209
		−0.4	5.544	−7.897	5.470	0.196
S271	0.0	0.0	4.622	−7.940	5.459	0.254
	0.01	0.0	4.785	−7.952	5.460	0.238
		−2.5	5.138	−7.984	5.459	0.199
	0.02	0.0	4.963	−7.964	5.461	0.221
		−1.6	5.212	−7.984	5.461	0.196
	0.03	0.0	5.164	−7.974	5.462	0.205
		−0.9	5.320	−7.985	5.463	0.191
Z271	0.0	0.0	4.749	−7.774	5.459	0.241
	0.01	0.0	4.811	−7.777	5.460	0.235
		−11.0	5.371	−7.840	5.457	0.170
	0.02	0.0	4.874	−7.783	5.460	0.228
		−10.0	5.401	−7.840	5.458	0.169
	0.03	0.0	4.940	−7.788	5.460	0.222
		−9.0	5.430	−7.839	5.459	0.168
	0.04	0.0	5.009	−7.793	5.460	0.215
		−8.0	5.459	−7.839	5.460	0.167
	0.05	0.0	5.080	−7.801	5.460	0.201
		−7.0	5.489	−7.838	5.460	0.166
	0.06	0.0	5.155	−7.803	5.461	0.203
		−6.0	5.518	−7.837	5.461	0.166

The results can be explained as follows: the neutron radius of ^{208}Pb thus neutron skin thickness S relies on the density dependence of symmetry. Stiff density dependence (i. e., pressure) for neutron matter pushes neutrons out against surface tension, leading to a larger radius. Fig. 1 clearly shows that the new introducing coupling term can further soften the density dependence of the symmetry energy (i. e., cause the symmetry energy to grow more slowly with density) and thus leads to a smaller radius.

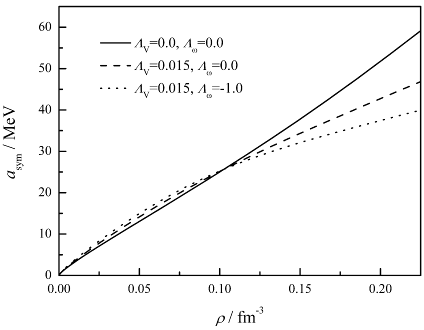

Fig. 1　The symmetry energy a_{sym} (in MeV) in symmetric nuclear matter as a function of density (in fm^{-3}) in the NL3 model.

4　Summary and Conclusion

Present model-dependent analyses of experimental data with hadronic probes yield values of neutron skin thickness S of ^{208}Pb varying between 0.0 and 0.2 fm[1−6]. According to Table 2, it is clearly seen that the results of neutron skin thickness S of ^{208}Pb, after adding the new isospin-dependent high order nucleon-ω-ρ coupling term

$$\mathscr{L}_{new} = -\Lambda_\omega \bar{\psi} g_\rho \gamma^\mu \frac{g_\omega^2 \omega^\mu \omega_\mu}{M^2} \boldsymbol{\tau} \cdot \boldsymbol{b}_\mu \psi \quad , \qquad (30)$$

seem to be more consistent with the present model-dependent analyses. However, hadronic measurements suffer from controversial uncertainties in the nuclear reaction mechanism. The Parity Radius Experiment (PREX) at Jefferson Laboratory aims to measure the neutron radius in ^{208}Pb via parity violating electron scattering to an unprecedented accuracy of 1% (±0.05 fm)[10, 24], which should provide a unique observational constraint on the thickness of the neutron skin of a heavy nucleus.

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在相对论平均场框架下研究同位旋依的高阶核子-ω-ρ介子耦合*

梁　钧

(中国原子能科学研究，北京 102413；
太原理工大学物理系，山西 太原 030024)

摘　要：在相对论平均场的框架下，对^{208}Pb中子皮对新的同位旋依赖的高阶核子-ω-ρ介子耦

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