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Unified Description of Signature Inversion, Chiral and Wobbling Bands*

CHEN Yong-shou[1,2], GAO Zao-chun[1]
(1 *China Institute of Atomic Energy*, *Beijing* 102413, *China*;
2 *Institute of Theoretical Physics*, *Chinese Academia of Sciences*, *Beijing* 100080, *China*)

Abstract: The Reflection Asymmetric Shell Model has been generated to include the axial symmetry break. In this new development, the shell model space is spanned by the triaxially-deformed multi-quasi-particle basis. A unified shell model description of the signature inversion, chiral and wobbling bands can be achieved. In the frame work of shell model, the inclusion of the triaxiality, the exact three dimensional projection and up to six quasiparticle configuration mixing make the theory powerful to study the phenomena in highly rotating triaxial nuclei at a new level.

Key words: triaxiality; signature inversion; wobbling mode; chiral band
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1 Introduction

There have been many unusual and interesting features discovered in the nuclear high-spin rotational spectra. Among them, the signature inversion, the chiral and wobbling bands, observed in highly rotating heavy nuclei, are currently most interests. To describe them, it is impossible to use the conventional shell models which employ a spherical basis. The Reflection Asymmetric Shell Model (RASM) has been successful for a description of the octupole bands in heavy nuclei[1], where the projections of angular momentum and parity treat the states fully quantum-mechanically by collecting all the energy-degenerate mean-field states associated with the rotational and reflection symmetries. Recently the model has been generated to include the axial symmetry break. In this new development, the shell model space is spanned by the triaxially-deformed multi-quasi-particle basis, and thus the exotic nuclear shapes can be described. In the frame work of shell model, the inclusion of the triaxiality, the exact three dimensional projection and up to six quasiparticle configuration mixing make the theory powerful to study the phenomena in highly rotating triaxial nuclei at a new level. As the applications of the present model, we first address the long-standing question on signature inversion. Secondly, we apply the model for descriptions of the chiral and wobbling bands. We briefly introduce the theory and report the new but preliminary results of calculations for these phenomena.

2 Brief Description of the Theoretical Model

The shell model Hamiltonian considered involves a large number of nucleons moving in a

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Biography: Chen Yong-shou(1939—), male(Han Nationality), Sichuan Luzhou, Professor, working on Nuclear Structure and Nuclear Astrophysics; E-mail: yschen@iris.ciae.ac.cn

spherical Nilsson potential and an interaction of separable multipole $Q \cdot Q$ plus monopole pairing plus quardrupole pairing,

$$H = H_0 - \frac{1}{2}\sum_{\lambda=2}^{4}\chi_\lambda \sum_{\mu=-\lambda}^{\lambda} Q^\dagger_{\lambda\mu} Q_{\lambda\mu} - G_0 P^\dagger_{00} P_{00} - G_2 \sum_{\mu=-2}^{2} P^\dagger_{2\mu} P_{2\mu} . \qquad (1)$$

Where H_0 is the spherical modified harmonic-oscillator single particle Hamiltonian, and the operators Q and P are expressed as

$$Q_{\lambda\mu} = \sum_{\alpha,\beta} \langle \alpha \mid \rho^2 Y_{\lambda\mu} \mid \beta \rangle \, c^\dagger_\alpha c_\beta , \qquad (2)$$

$$P^\dagger_{00} = \frac{1}{2}\sum_\alpha c^\dagger_\alpha c^\dagger_{\bar\alpha} , \qquad (3)$$

$$P^\dagger_{2\mu} = \frac{1}{2}\sum_{\alpha,\beta} \langle \alpha \mid \rho^2 Y_{2\mu} \mid \beta \rangle \, c^\dagger_\alpha c^\dagger_{\bar\beta} . \qquad (4)$$

Where, the eigenstates of a single particle moving in the spherical Nilsson potential can be labelled by the quantum numbers $\alpha = nljm$, and $\bar\alpha$ denotes the time-reversed state of α. The QQ interaction strength χ is determined in such a way that it has a self-consistent relation with the quadrupole deformation[1,2]. The monopole pairing strength G_M is of the standard form G/A. The quadrupole pairing strength G_Q is proportional to G_M. For the present calculation, $G_M = 19.6$ for neutrons and 17.2 for protons in the $A=130$ region, and $G_M = 17.93$ for both protons and neutrons in the $A=160$ region. These pairing parameters approximately reproduce the observed odd-even mass differences in the considered mass regions. The used quadrupole pairing strength is $G_Q = 0.16 G_M$.

For the present purpose, we do not need the parity projection, the wave-function can be written as

$$\mid \Psi^\sigma_{IM} \rangle = \sum_{K\kappa} f^\sigma_{IK_\kappa} \hat{P}^I_{MK} \mid \Phi_\kappa \rangle , \qquad (5)$$

where $\hat{P}^I_{MK}$ is the three-dimensional angular-momentum-projection operator,

$$\hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega D^I_{MK}(\Omega)\, \hat{R}(\Omega) , \qquad (6)$$

and σ in Eq. (5) specifies the states with the same angular momentum I. The dimension of the summation in Eq. (5) is $K \times \kappa$, where $|K| \leqslant I$ and κ is usually in the order of 10^2.

Where $|\Phi_\kappa\rangle$ represents a set of multi-qp states associated with the triaxially deformed qp vacuum $|0\rangle$. For odd-odd nuclei one has

$$\{ \alpha^\dagger_{\nu_1} \alpha^\dagger_{\pi_1} \mid 0\rangle, \alpha^\dagger_{\nu_1} \alpha^\dagger_{\nu_2} \alpha^\dagger_{\nu_3} \alpha^\dagger_{\pi_1} \mid 0\rangle, \alpha^\dagger_{\nu_1} \alpha^\dagger_{\pi_1} \alpha^\dagger_{\pi_2} \alpha^\dagger_{\pi_3} \mid 0\rangle, \alpha^\dagger_{\nu_1} \alpha^\dagger_{\nu_2} \alpha^\dagger_{\nu_3} \alpha^\dagger_{\pi_1} \alpha^\dagger_{\pi_2} \alpha^\dagger_{\pi_3} \mid 0\rangle \} , \qquad (7)$$

and for odd proton nuclei one has

$$\{ \alpha^\dagger_{\pi_1} \mid 0\rangle, \alpha^\dagger_{\nu_1} \alpha^\dagger_{\nu_2} \alpha^\dagger_{\pi_1} \mid 0\rangle, \alpha^\dagger_{\nu_1} \alpha^\dagger_{\nu_2} \alpha^\dagger_{\pi_1} \alpha^\dagger_{\pi_2} \alpha^\dagger_{\pi_3} \mid 0\rangle \} . \qquad (8)$$

The triaxially deformed single particle states are generated by the Nilsson Hamiltonian

$$H_N = H_0 - \frac{2}{3}\hbar\omega\varepsilon_2 \left(\cos\gamma\, Q_0 + \sin\gamma \frac{Q_{+2} + Q_{-2}}{\sqrt{2}} \right) , \qquad (9)$$

where H_0 is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force[3]. The parameters ε_2 and γ describe quadrupole deformation and triaxial deformation, respectively.

3 Signature Inversion

The signature inversion phenomenon has been widely observed in nuclear rotational spectrum but never been convincingly explained. We will show that the phenomenon can be naturally described by shell-model-type calculations without invoking any unusual assumptions.

The signature is a quantum number, associated with the invariance of a system with intrinsic quadrupole deformation under a rotation of 180° around a principal axis[4]. Due to this symmetry, rotational energies $E(I)$ of a high-j band can be split into two branches with $\Delta I = 2$, classified by the signature quantum number α. As a rule, the energetically favored sequence in a doubly-odd nucleus has signature $\alpha_f = |(-1)^{j_p-1/2} + (-1)^{j_n-1/2}|/$

2, where j_p and j_n are the angular momenta of the last proton and the last neutron carries respectively. Therefore, the $\pi h_{11/2}\, \nu h_{11/2}$ bands are expected to have the $\alpha=1$ signature (odd spins) favored over the $\alpha=0$ sequence (even spins). The signature splitting exhibits then a zigzag phase in an $E(I)-E(I-1)$ plot. As example, the signature inversion phenomena were found in odd-odd nuclei $^{118-130}Cs$ where the energetically favored states have even-integer spin states denoted by $\alpha=0$, presenting a reversed zigzag phase, at low spins, but have odd-integer spin states denoted by $\alpha=1$, presenting a normal zigzag phase, at spins higher than a critical spin I_{rcv}[5,6].

Calculations are carried out for doubly-odd nuclei $^{118-130}Cs$ with the deformation parameters listed in Table 1. The quadrupole deformation parameters ε_2 are consistent with those obtained from the TRS calculations. The γ values are adjusted to best describe the bands. The results are compared with available data in Fig. 1. As one can see, an excellent agreement has been achieved. With an increasing neutron number, the trend of decreasing signature splitting, i. e. decreasing zigzag amplitude, has been reproduced. What is also correctly described is the increasing trend of I_{rcv} as neutron number increases.

To investigate how signature inversion occurs in our theory we have carried out calculation of the three components of angular-momentum operator, $\hat{I}_i^2$ ($i=x$, y and z). It is found that for a irrotational triaxial odd-odd nuclear system, the rotational axis is along the shortest axis at low spins and becomes along the intermediate axis after about I_{rcv}. According to these calculations we have interpreted the phenomenon of signature inversion as a manifestation of the dynamical drift of the rotational axis from the shortest principal axis to the intermediate one as an odd-odd nucleus is rotating at the presence of the triaxiality[7]. The calculations to examine the role of the neutron-proton force and quadrupole-pairing have also been carried out. Our results show that neither the neutron-proton force as a residual interaction nor the quadrupole-pairing is a decisive factor for causing signature inversion.

Table 1　Quadrupole deformation ε_2 and triaxial deformation γ employed in the calculation

Nucleus	118	120	122	124	126	128	130
ε_2	0.30	0.29	0.26	0.26	0.21	0.20	0.19
γ/(°)	30	30	31	31	35	37	39

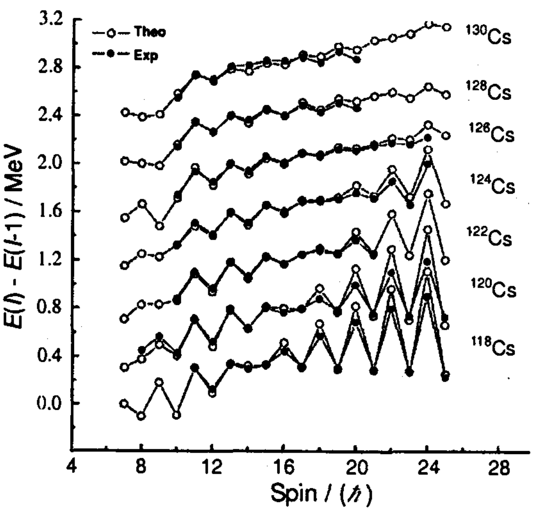

Fig. 1　Comparison of calculated energies with data for the $\pi h_{11/2}\, \nu h_{11/2}$ bands in $^{118-130}Cs$. Note the increasing trend in the reversion spin: 14.5 for ^{118}Cs, 16.5 for ^{120}Cs, 17.5 for ^{122}Cs, 18.5 for ^{124}Cs, 20.5 for ^{126}Cs, 21.5 for ^{128}Cs (prediction), and 22.5 for ^{130}Cs (prediction).

4　Chiral Band

The concept of chirality was first given by Lord Kelvin in 1904[8], "I call any geometrical figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot brought to coincide with itself." The chirality is common and has important consequence in science. Many biological and pharmaceutical molecules have static chirality when they are composed of four different atoms. In particle physics, the chirality exists in massless particles having a parallel or antiparallel orientation of spin and momentum. The chirality may exist in nuclei when a particular angular momentum coupling scheme appears, where three angular momenta are mutually perpendicular so that a left- and a right-handed system can be formed. It was suggested to occur in odd-odd triaxial nuclei[9] when the Fermi level is located in the lower part of the valence proton high-j subshell resulting in its angular momentum oriented along the short axis of the triaxial core, and in the upper part of the valence neutron subshell leading to its angular momentum aligned with the long axis, while the angular momentum of the core is along the intermediate axis due to its largest moment of inertia for irrotational-like flow[4]. The necessary restoration of the spontaneously broken intrinsic chiral symmetry in laboratory frame will manifest as degenerate doublet $\Delta I=1$ bands in the rotational spectrum. The nearly degeneracy of the chiral sister bands has been found experimentally, for example, in odd-odd $N=75$ isotones, ^{130}Cs, ^{132}La, ^{134}Pr and ^{136}Pm[10].

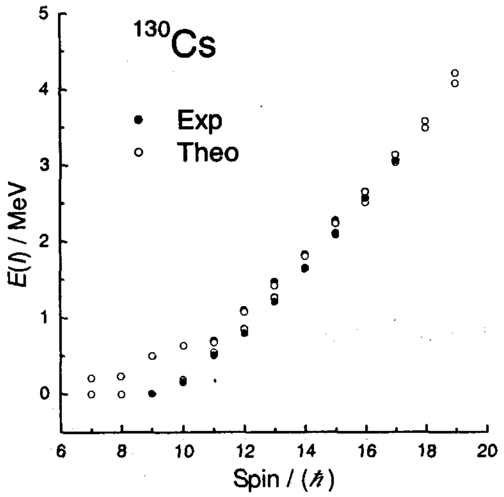

Fig. 2 Calculated rotational bands, energy as function of spin, in ^{130}Cs. Note the calculated lowest two bands well reproduce the experimental chiral sister bands, and the predicted states with no experimental data are also shown.

In Fig. 2 the calculated rotational bands, energy as function of spin, for ^{130}Cs with deformation parameters as the list in Table I. The calculated lowest two $\pi h_{11/2}\nu h_{11/2}$ bands are in good agreement with the measured so called chiral sister bands, which are nearly degenerate at high spins. To investigate the coupling structure of angular momentum, we have carried out calculations of components of angular momenta of the lowest q p states and the vacuum. It was found that the angular momenta of the last $h_{11/2}$ neutron, proton and the vacuum are oriented along the short axis, the long axis and the intermediate axis respectively, providing support for the interpretation of chiral bands in this nucleus. However, the chirality in nuclear physics is still a question open for further research, especially the measurements of the electromagnetic transitions need to be down.

5 Wobbling Band

The wobbling motion is a direct consequence of rotation of a triaxial body with moments of inertia $J_x \gg J_y \neq J_z$. In nuclear high spin spectra, the wobbling excitation leads to sequences of bands with an increasing number of wobbling quanta, $n_W = 0, 1, 2, \cdots$. The wobbling phonon energy is given by $\hbar\omega_W = \hbar\omega_{rot}\sqrt{(J_x - J_y)(J_x - J_z)/(J_y J_z)}$ with $\hbar\omega_{rot} = I/J_x$[4]. The energy of wobbling bands is $E(I, n_W) = I(I+1)/2J_x + \hbar\omega_W(n_W + 1/2)$. Although the wobbling degree of freedom is expected to be general for triaxial rotational nuclei, it had never been realized in experimental high spin spectra until a clear evidence for wobbling mode was found in ^{163}Lu in 2001[11]. A year later, the more firm evidences were found experimentally in the same nucleus by the same research group[12]. In this experiment, three triaxial strongly deformed bands called TSD1, TSD2 and TSD3 together with the connecting transitions between them were measured in ^{163}Lu. The firm interpretation of TSD2 band as a one-phonon wobbling excitation built on the TSD1 band was given by the observed strong E2 transitions feeding from the TSD2 into the TSD1, competing with the collective along-band E2 transitions, characterized with large values of $B(E2)_{out}/B(E2)_{in}$. The possible two-phonon wobbling excitation built on the TSD1 was found as the TSD3 band. A similar evidence for the wobbling mode was also found in strongly deformed triaxial nuclei ^{165}Lu[13] and ^{167}Lu[14]. The TSD bands were found in the Hf isotopes, for example, in ^{168}Hf[15], however, there is no evidence found for the wobbling mode.

Calculation was performed for typical wobbler nucleus ^{163}Lu with the quadrupole deformation parameters $\varepsilon_2 = 0.38$ and $\gamma = 20°$, which are consistent with those obtained from the TRS calculations. Our results are compared with experimental data in Fig. 3. As one can see, an excellent agreement has been achieved up to highest spins for the three measured TSD bands. This means that the present calculation reproduces not only the each rotational band but also the relative excitation energies between these TSD bands, namely the wobbling phonon energies. Furthermore, the preliminary calculated results of E2 transitions, not given here, show large B(E2) values of interband transitions, further confirming the wobbling excitation nature. In Fig. 3 the calculated fourth TSD band is also shown as a prediction for the possible three-phonon wobbling excitation in this nucleus, no available experimental data for a comparison.

To examine the effect of triaxiality on the wobbling motion we have carried out a calculation with the same parameters but without triaxial deformation, namely setting $\gamma = 0$. In Fig. 4 the theoretical wobbling bands (the yrast TSD1 band and the one-phonon excitation TSD2 band) calculated with $\gamma = 20°$, which well reproduce the data as shown in Fig. 3, are compared with the lowest two bands, SD1 and SD2, calculated with $\gamma = 0°$. One can see that the SD2 band is more than 1 MeV higher than the SD1 band, which is a typical cranking rotational excitation and clearly no wobbling nature. The comparison in Fig. 4 indicates that the triaxiality is decisive for the appearance of wobbling motion.

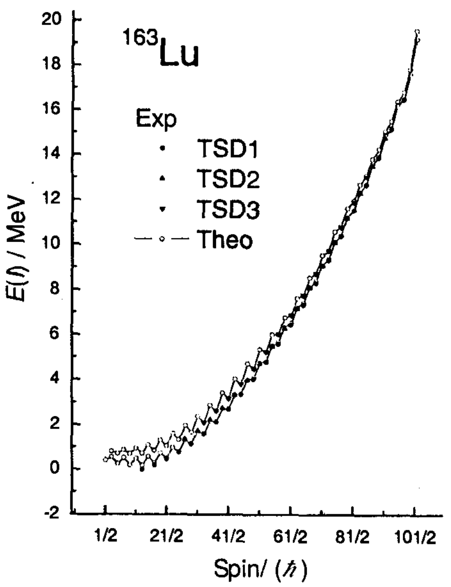

Fig. 3 Calculated lowest four rotational TSD bands compared with data[12]. Superdeformed experimental bands TSD1, TSD2 and TSD3 which form wobbling excitations are well reproduced by the calculation. Note the calculated TSD4 band may be a 3 phonon wobbling one.

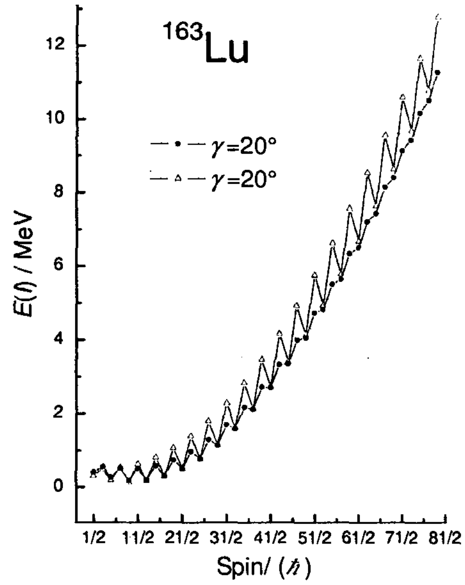

Fig. 4 Calculated lowest two TSD bands with $\gamma = 20°$ and lowest two SD bands with $\gamma = 0°$. Note, no wobbling but the cranking picture occurs for the case of $\gamma = 0°$.

The experimental data for signature inversion are reproduced systematically with high accuracy, and the phenomenon of signature inversion has been interpreted as a manifestation of the dynamical drift of the rotational axis in a triaxial rotational nucleus. The calculated results of wobbling and chiral sister bands are also in good agreement with experimental data. The signature inversion, wobbling mode and chirality are deeply related to the axial symmetry break in nuclei. The RASM has been generated to include the triaxiality and therefore can provide an unified description for these phenomena in a natural way. The key to the success is the shell-model nature of the method. In the frame work of shell model, the inclusion of the triaxiality, the exact three dimensional projection and up to six quasiparticle configuration mixing make the theory powerful to study the phenomena in highly rotating triaxial nuclei at a new level.

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旋称反转手征带和摇摆带的统一描述*

陈永寿[1,2]，高早春[1]

(1 中国原子能科学研究院，北京 102413；
2 中国科学院理论物理研究所，北京 100080)

摘　要：反射不对称壳模型推广到包括轴对称破缺。在这一发展中，壳模型空间由三轴形变的多准粒子基所构建，实现了对旋称反转、手征带和摇摆带的统一的壳模型描述。在壳模型框架下，三轴性的引入，严格的三维投影和直到六准粒子的组态混合，使该理论可以在一个新的水平上研究高速转动三轴形变核中的物理现象。

关 键 词：三轴性；旋称反转；摇摆带；手征带

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