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Anomalous Magnetic Moment of Muon in Composite Models of Leptons*

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Abstract: Based on the Bethe-Salpeter formalism and the general relativistic covariant quantum field theory, we illustrate with a simple composite model that the observed deviation of $(g-2)_\mu$ can be an effect of the substructure of muon and give the constraints on the radius of muon in different cases of light constituents and heavy constituents.

Key words: composite model; lepton; anomalous magnetic moment

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1 Introduction

Recently, the E821 group[1] reported their measurement result for the anomalous magnetic moment of the muon as

$$a_\mu = \frac{g-2}{2} = 11\ 659\ 202(14)(6) \times 10^{-10}. \quad (1)$$

In the Standard Model (SM), the anomalous magnetic moment, $a_\mu^{\rm SM}$, is estimated to be[2]

$$a_\mu = 116\ 591\ 597(67) \times 10^{-11}, \quad (2)$$

where the error is mainly from the hadronic contribution the magnitude of which is still under debates. From these results one obtains present deviation from SM

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (4.3 \pm 1.6) \times 10^{-9}. \quad (3)$$

If this discrepancy is confirmed it will give a hint of new physics. There have been many works for explaining this discrepancy along different directions. In particular, supersymmetric loop effects and radiative mass mechanism have been reviewed in Ref. [2]. The reason of the deviation has also been probed in technicolor models[3], preonic models[4] and other new physics. In Ref. [4], the deviation is attributed to the presence of heavy exotic colored lepton and extra Z-boson states arising in a preonic model and essentially the bound state description of lepton is not touched in their calculations. The constraint from the anomalous magnetic moments of leptons on possible substructure of leptons has been analyzed in terms of general formalism for describing a bound state in early 80's[5,6]. In the non-relativistic theory of bound state, where the binding energy and the inverse radius R^{-1} are much smaller than the mass of the bound state, the magnetic moment of the bound state is the vectorial sum of the magnetic moments of the constituents. This would contradicts the precise experimental data for a_μ if μ is such a bound state[7]. It has been shown that this is not the case for the relativistic bound state[5,6]. In this paper, we will consider the theoretical implication of the E821 new result in a composite model of lepton, based on the bound state description of muon and the general relativistic covariant quantum field theory. We generalize the previous investigations to higher orders in α and including both the heavy and light constituent cases. We shall repeat some previous derivations in

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order to make the paper self-contained. For the sake of simplicity we calculate the anomalous magnetic moment of a muon in a simple composite model in which the lepton is assumed to be a bound state composed of a fermion and a scalar boson. We show that the deviation, δa_μ, can be a signal for compositeness of muon and poses a constraint on the charge radius of muon (and the masses of constituents if constituents are heavy).

2 Matrix Element of Electric Current of Composite Particle at the Lowest Order in α

For simplicity, we assume that the lepton is the bound state composed of a charged fermion and a neutral scalar boson which we shall also call as preons for convenience. We shall confine us in this article to the relativistic tightly bounded states with $R^{-1} \gg m_l$ and $R^{-1} \geq m_F$, where R is the charge radius of the composite lepton and m_F is the mass of the charged fermionic preon. We shall consider two cases. In the case A, the mass m_F is of the same order as R^{-1} and much heavier than m_l. In the case B, m_F and m_l are both much smaller than R^{-1}. In this case, if the force between the preons is some confining gauge interaction, there may be an approximate global chiral symmetry which naturally results in the small lepton mass m_l. Essentially R^{-1} is also the confinement scale of the gauge interaction.

The general Lorentz and space inversion invariant Bethe-Salpeter (B-S) wave function for a composite lepton composed of a fermionic and a scalar preon is

$$\chi_P^s(p) = \int d^4x e^{ip\cdot x} \langle 0|T^*\left(\psi\left(\frac{x}{2}\right), \Phi\left(\frac{-x}{2}\right)\right)|P,s\rangle = \left(f_1 + \frac{i\hat{p}}{M}f_2\right)u^s(P) , \tag{4}$$

where ψ and Φ are fields of the fermion and bosonic preon respectively, $\hat{p} = p \cdot \gamma$, $P = p_1 + p_2$ is the momentum of lepton, $p = (m_B p_1 - m_F p_2)/(m_B + m_F)$ is the relative momentum between preons, m_B and m_F are the masses of the bosonic preon and fermionic preon respectively. The two cases mentioned above are: A. $m_F = O(R^{-1}) \gg m_l$. B. $m_F m_l \ll R^{-1}$. In Eq. (4) the constant M with mass dimension is introduced for convenience, $f_i = f_i(P \cdot p, p^2) \equiv f_i(P, p)$, $(i = 1,2)$, are real functions corresponding to the S wave state, and $u^s(P)$ is the Dirac spinor with spin component $s_z = s$. It is straightforward to derive from invariance under space-time inversion that the B-S wave function for a lepton final state is

$$\bar{\chi}_P^s(p) = \bar{u}^s(P)\left(f_1 + \frac{i\hat{p}}{M}f_2\right) . \tag{5}$$

The simplest diagram for the lowest order electro-magnetic interaction of composite lepton is illustrated in Fig. 1. The corresponding bound state matrix element is [5,5]

$$\Gamma_\mu^{(0)}(P,q) = \langle P + \frac{q}{2}, s|J_\mu(0)|P - \frac{q}{2}, s\rangle = -\frac{1}{(2\pi)^4}\int d^4k \bar{\chi}_{P+\frac{q}{2}}\left(k + \frac{q}{4}\right)\gamma_\mu \chi_{P-\frac{q}{2}}\left(k - \frac{q}{4}\right) i\Delta_F'^{-1}\left(\left(\frac{P}{2} - k\right)^2\right)$$
$$= -\bar{u}\left(P + \frac{q}{2}\right)\frac{1}{(2\pi)^4}\int d^4k\left[f'_1 + \frac{i}{M}\left(\hat{k} + \frac{\hat{q}}{4}\right)f'_2\right]\gamma_\mu\left[f_1 + \frac{i}{M}\left(\hat{k} - \frac{\hat{q}}{4}\right)f_2\right] i\Delta_F'^{-1}\left(\left(\frac{P}{2} - k\right)^2\right)u\left(P - \frac{q}{2}\right) , \tag{6}$$

where q is the momentum of photon,

$$f_i = f_i\left(P - \frac{q}{2}, k - \frac{q}{4}\right), \qquad f'_i = f_i\left(P + \frac{q}{2}, k + \frac{q}{4}\right) \text{ for } i = 1,2,$$

and Δ_F' is the propagator of the scalar preon.

Using the Dirac equation, it is easy to obtain by a straightforward calculation

$$\Gamma_\mu^{(0)} = \bar{u}\left(P + \frac{q}{2}\right)\{\gamma_\mu F_1^{(0)}(q^2) - \frac{1}{2}[\gamma_\mu, \hat{q}]F_2^{(0)}(q^2)\}u\left(P - \frac{q}{2}\right) , \tag{7}$$

where
$$F_1^{(0)}(q^2) = S_1 + \frac{2T}{M^2} - \frac{2m_l}{M}V_{1P} + \frac{1}{M^2}\left(m_l^2 - \frac{q^2}{4}\right)T_{PP} + \frac{q^2}{M^2}T_{qq} + \frac{1}{4}\frac{q^2}{M^2}V_{2P} - \frac{1}{16}\frac{q^2}{M^2}S_2 , \tag{8}$$

$$F_2^{(0)}(q^2) = \frac{m_l}{M^2}T_{PP} + \frac{1}{2M}S_3 - \frac{1}{M}V_{1P} + \frac{2}{M}V_{1q} - \frac{m_l}{2M^2}V_{2P} . \tag{9}$$

In Eqs. (8) and (9) the Lorentz invariant functions S, etc. are defined by and

$$S_1 = \frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_1 f_1 \Delta'^{-1}_F\left(\left(\frac{P}{2} - k\right)^2\right) .$$

$$S_2 = \frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2 f_2 \Delta'^{-1}_F ,$$

$$S_3 = \frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2 f_1 \Delta'^{-1}_F .$$

$$\frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2 f_1 \Delta'^{-1}_F k_\mu = V_{1P}P_\mu + V_{1q}q_\mu .$$

$$\frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2 f_2 \Delta'^{-1}_F k_\mu = V_{2P}P_\mu ,$$

$$\frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2 f_2 \Delta'^{-1}_F k_\mu k_\nu$$
$$= T_{PP}P_\mu P_\nu + T_{qq}q_\mu q_\nu + T\delta_{\mu\nu} . \tag{10}$$

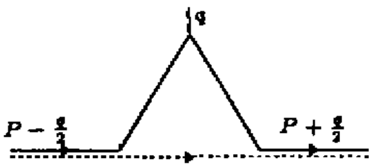

Fig. 1 The lowest order diagram of the electro-magnetic interaction of a composite lepton in the composite model of leptons. The solid (dashed) line represents the fermionic (bosonic) preon and the wave line represents the photon.

The left sides of the last two formulae in Eq. (10) is even in q, hence there is no the terms linear in q_μ in the right sides of them.

It follows from Eq. (8) that the normalization condition of electric charge is

$$S_1(0) + \frac{2}{M^2}T(0) - \frac{2m_l}{M}V_{1P}(0) - \frac{m_l^2}{M^2}T_{PP}(0) = 1 , \tag{11}$$

which can approximately be written as

$$S_1(0) + \frac{2}{M^2}T(0) = 1 \tag{12}$$

if $m_l/M \ll 1$.

$$T(0) = \frac{1}{4}\frac{-\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k f_2^2 \Delta'^{-1}_F k^2 . \tag{13}$$

Let us choose M to make the integral of $f_i f_j \Delta'^{-1}_F$ to be of the same order for $i, j = 1, 2$. From (13) we have

$$T(0) = O(R^{-2})\, S_1(0) . \tag{14}$$

Since $V_{1P}(0)$ and $V_{1q}(0)$ are of the same order of $S_1(0)$, we obtain from (8), (9), (12) and (14)

$$\frac{F_2(0)}{F_1(0)} = \frac{C_2 M}{M^2 + C_1 R^{-2}} , \tag{15}$$

where C_1 and C_2 are constants of the order 1.

Eq. (15) is in agreement with the result in Eq. (25) of Ref. [6] derived with a different approach. Authors of Ref. [6] asserted that M should be equal to m_F. In our opinion, $M = m_F$ is likely to hold in theories with vector or pseudo-vector interactions as a chirality flip along the fermion line is required for the F_2 term. However, it may not be true for scalar or pseudo-scalar interaction, where a natural guess is $M = O(R^{-1})$. In case A, m_F and R^{-1} are of the same order, these two possibilities coincide and from (15) the anomalous magnetic moment of μ from compositeness at the leading order of a is

$$a_\mu = O\left(\frac{m_l}{R^{-1}}\right) . \tag{16}$$

In case B, $m_F \ll R^{-1}$, from (15) we obtain:

$$a_\mu = O\left(\frac{m_l}{R^{-1}}\right) , \tag{17}$$

for $M = O(R^{-1})$ and

$$a_\mu = O\left(\frac{m_l m_F}{R^{-2}}\right) , \tag{18}$$

for $M = O(m_F)$.

Therefore, a_μ can be suppressed linearly or quadratically by $O(R^{-1})$ depending on whether $m_F = O(R^{-1})$ or $m_F \ll R^{-1}$ and also on different internal dynamics.

From the above results, the magnetic moment of the bound state is found to approximately be the Dirac magnetic moment $e/2m_l$ at the zeroth order of

α provided that the charge radius of lepton is small enough.

3 Radiative Corrections at α order

It is necessary to examine the radiative corrections in higher orders of α in the composite model in view of the agreement between the experimental data and the standard model prediction for a_μ up to the order 10^{-8} which is of the order $(\alpha/\pi)^3$. In this section, we will consider radiative corrections to the anomalous magnetic moment of composite lepton at the α order.

To illustrate how the corrections from compositeness at the α order is suppressed let us consider first the case A, $m_F = O(R^{-1}) \gg m_l$. For simplicity we set $m_F = m_B = m$ in this section. The simplest diagram at this order is shown as Fig. 2 and the corresponding matrix element is given by

$$\Gamma_\mu^{(1)} = -\frac{i}{(2\pi)^4}\int d^4p\,\bar{\chi}_{P-\frac{q}{2}}\left(\bar{p}+\frac{q}{4}\right) \Lambda_\mu \chi_{P-\frac{q}{2}}\left(\bar{p}-\frac{q}{4}\right)\cdot \Delta'^{-1}_F\left(\left(\frac{P}{2}-\bar{p}\right)^2\right), \tag{19}$$

where

$$\Lambda_\mu = \frac{ie^2}{(2\pi)^4}\int d^4k\,\frac{1}{k^2}\gamma_\nu\frac{-i(\not{p}'_1-\not{k})+m}{(p'_1-k)^2+m^2}\gamma_\mu\frac{-i(\not{p}_1-\not{k})+m}{(p_1-k)^2+m^2}\gamma_\nu,$$

$$p_1 = \frac{P}{2}+p-\frac{q}{2}, \qquad p'_1 = \frac{P}{2}+p+\frac{q}{2}. \tag{20}$$

Firstly, let us consider the contribution of the f_1 term in the B-S wave function (4). Using both the Dirac equation and the symmetry of integrand under permuting Feynman parameters, it is easy to show that (19) can be transformed into the same form as Eq. (7), which is expected from the conservation of electric current.

In calculating the anomalous magnetic moment q^2 can be put to zero. The correction to $F_1(0)$ from Eq. (19) is absorbed by the normalization condition $F_1(0)=1$. We know by examining Eq. (19) that the contribution to $F_2(0)$ comes from the term

$$\frac{\alpha}{\pi}m\bar{u}\left(P+\frac{q}{2}\right)u\left(P-\frac{q}{2}\right)\frac{-i}{(2\pi)^4}\int d^4p\,f_1^2(P,p,q=0)\,\Delta'^{-1}_F\left(\left(\frac{P}{2}-p\right)^2\right)\cdot$$
$$\int_{x_1+x_2\le 1}dx_1dx_2\frac{2(1-x_1-x_2)(P/2+p)_\mu}{(x_1+x_2)[(1-x_1-x_2)(P/2+p)^2+m^2]}. \tag{21}$$

From the normalization condition (12) of electric charge and (14), we have

$$S_1(0) = O(1) \tag{22}$$

in the present case $M=O(R^{-1})=O(m_F)$.

$m \gtrsim R^{-1} \gg m_l$, so $O(m_l/m)$ terms can be neglected. When the momentum of the internal preon $P/2+p$ is in the Euclidean region, the term in the square brackets in the denominator of the integrand in the second line of (21) is larger than m^2. Therefore, once the wave function can be continued into the Euclidean region, as verified by Wick[9], the contribution of (21) to the anomalous magnetic moment is

$$O\left(\alpha\frac{e}{2m}\right) = O\left(\alpha\frac{e}{2R^{-1}}\right). \tag{23}$$

The conclusions will not change when the contribution from the f_2 term of the B-S wave function (4) is included.

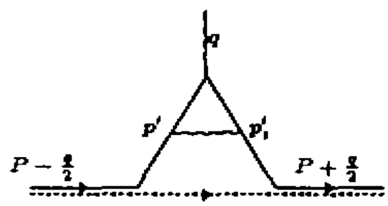

Fig. 2 The α order diagram of the electro-magnetic interaction of a composite lepton in the composite model of leptons.

Similar analysis can be carried out for the case B $m \ll R^{-1}$ where m^2 in the denominator of (21) can be neglected. A difference arises if in addition $M=$

$O(m_F)$. In that case the $(f_2/M)^2$ term dominates and instead of (22) we have

$$\frac{T(0)}{M^2} = O(1) \; . \qquad (24)$$

The contribution of the f_2^2 term can be obtained by replacing f_1^2 by $(f_2/M)^2 p^2$ in (21). Using (24) we find that the contribution to the anomalous magnetic moment of Fig. 2 is $O(\alpha(em/2R^{-2}))$ if $m \ll R^{-1}$ and $M = O(m)$.

When the "very strong" interaction binding preons into a lepton is considered, it will bring corrections to the electro-magnetic vertex of the fermionic preon. Accordingly, there will appear much more complex diagrams. If we assume that the constituent fermion interacts with the constituent boson through exchanging bosons, we get diagrams in Fig. 3 in lowest orders. Such diagrams can be calculated with the spectra representation of the B-S wave function. The details of the analysis can be found in Ref. [5]. From such analysis we found that the contribution to $F_2(0)$ from any individual diagram, which contains B-S wave functions and additional finite number of propagators of the preons or the particles mediating the "very strong" interaction, is suppressed by $O(\alpha R)$ or $O(\alpha mR^{-2})$ at the order α. However, can the QED result for a_μ at the α order approximately be obtained in the composite model and how large is the deviation from the QED result? This question is answered as follows.

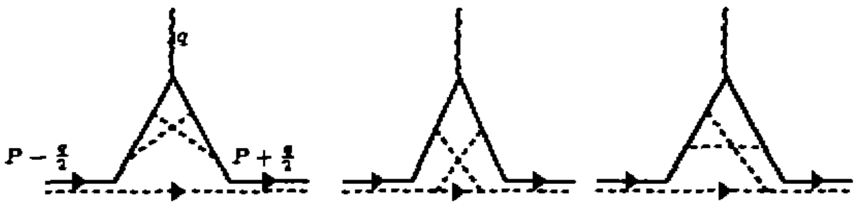

Fig. 3 Some examples of complex diagrams. The dashed line represents the boson which mediates the "very strong" interaction.

Adding infinite diagrams, we will get the pole term contribution of the bound state (see Fig. 3). The four point Green function $\mathcal{K}$ in Fig. 4 contains the pole of the lepton bound state. It was shown in (citedh) that this diagram gives the same correction as that in QED at the order α if $R^{-1} \gg m_l$.

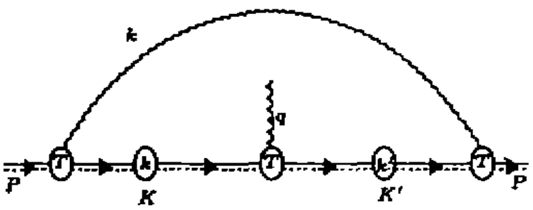

Fig. 4 The diagram corresponding the contribution of the bound state pole term. The circles with "T" and "$\mathcal{K}$" (or "$\mathcal{K}'$") inside it denote the electro-magnetic vertex of the bound state at the lowest order in α when K^2(or K'^2) $= m_l^2$ and the 4-point Green function respectively. $K = P + k$ and $K' = P' + k$ in the diagram.

4 Conclusions and Discussions

Apparently, the above results may be generalized to arbitrary orders in α and we have

$$a_\mu = a_\mu^{SM} + O\left(\frac{m_l}{R^{-1}}\right) \; , \qquad (25)$$

for the case A and

$$a_\mu = a_\mu^{SM} + O\left(\frac{m_l}{R^{-1}}\right) \; , \qquad (26)$$

or

$$a_\mu = a_\mu^{SM} + O\left(\frac{m_l m_F}{R^{-2}}\right) \; , \qquad (27)$$

for the case B. (26) is valid for theories without a chiral symmetry and (27) is valid for theories of the confining gauge interactions with an approximate global chiral symmetry. In Eqs. (25), (26) and (27) we have assumed $R^{-1} \gg m_Z$, which is necessary when including the electro-weak corrections.

Similar conclusions can be obtained if the scalar preon is charged. In contrast with the non-relativistic loose bound state, the magnetic moment of a relativistic tight bound state is not of the vectorial sum of the magnetic moments of its constituents. When the radius of the bound state is sufficiently small, the bound state behaves as a whole in electro-magnetic field, like a point particle does. Although our conclusions are obtained in a simple composite model, they depend only on the dimension analysis and the analytical property of the B-S wave function. So it is expected that the conclusions will stand also in other more complex models.

It is obvious from Eqs. (25), (26) and (27) that the observed deviation of a_μ can be a demonstration of the substructure of muon and give a constraint on the radius of muon or the masses of preons. From the theoretical and the experimental results, Eqs. (25, 26) and Eq. (27), we get

$$R^{-} \geqslant 10^{8} m_\mu \tag{28}$$

for theories without an approximate chiral symmetry,

$$R^{-1} \geqslant 10^{4} m_\mu \tag{29}$$

for composite leptons bound by gauge interactions with an approximate global chiral symmetry if m_{F} is of the same order of m_{I}.

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轻子复合模型中 μ 介子的反常磁矩*

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摘　要：由 Bethe-Salpeter 理论形式和一般的相对论协变量子场论出发，用一个简单的复合模型说明，实验观察到的 μ 介子反常磁矩对标准模型的偏离可能是轻子内部的结构效应，并且在不同的重组元和轻组元的情况下给出对轻子半径的限制.

关 键 词：复合模型；轻子；反常磁矩

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