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Closed Expression of B-S Interaction Kernel for Quark-antiquark Bound States*

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Abstract: The interaction kernel in the Bethe-Salpeter equation for quark-antiquark bound states is derived from the Bethe-Salpeter equations satisfied by the quark-antiquark four-point Green's function. The latter equations are established based on the equations of motion obeyed by the quark and antiquark propagators, the four-point Green's function and some other kinds of Green's functions which follow directly from the QCD generating function. The Bethe-Salpeter kernel derived is an exact and explicit expression which contains only a few types of Green's functions. This expression is not only convenient for perturbative calculations, but also suitable for nonperturbative investigations.

Key words: interaction kernel; Bethe-Salpeter equation; $q\bar{q}$ bound state

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In quantum field theory, the Bethe-Salpeter (B-S) equation, as a rigorous approach to the relativistic many-body problem, has been extensively investigated in the period of half century[1-5]. This equation is elegantly formulated in a Lorentz-covariant form in Minkowski space and the interaction kernel in it can perfectly be calculated by means of the technique of perturbation theory. Usually, the interaction kernel is defined as a perturbation series constructed by all the B-S irreducible (or say, two-particle-irreducible) graphs. In practical applications, the perturbation series has to be truncated in a ladder approximation. The ladder approximation has been proved to be successful in QED for studying the bound states formed by the electromagnetic interaction[6-9]. Nevertheless, it is not feasible in QCD for exploring multi-quark bound states because the confining force which must be taken into account in this case could not follow from a perturbative calculation. It was remarked in Ref. [10] that "The approach using the B-S equation has not led to a real breakthrough in our understanding of quark-quark force". The reason is mainly due to that we have not known the closed expression of the kernel which can be used to evaluate the confining force. In the previous application of the B-S equation to investigate the hadron structure, a phenomenological confining potential is necessarily introduced and added to the one-gluon exchange kernel so as to obtain reasonable theoretical results[10-11]. The confining potential was originally proposed by a nonperturbative computation, for example, the lattice gauge calculation of a special Wilson loop[12-14]. Since the B-S equation is an exact formalism for the bound state and the B-S kernel con-

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tains all the interactions responsible for the formation of bound states, certainly, the kernel appears to be the most suitable starting point of examining the quark confinement. For this examination, it is necessary at first to give a closed expression of the kernel which is not only convenient for perturbative calculations, but also tractable for nonperturbative investigations. In this paper, we are devoted to deriving a complete and explicit expression of the B-S kernel for the $q\bar{q}$ bound system. The procedure of derivation is similar to that proposed initially in Ref. [14] and subsequently demonstrated in Refs. [15, 16] where a closed expression of the interaction kernel appearing in the exact three-dimensional relativistic equation (or say, the Dirac-Schrödinger equation) for the $q\bar{q}$ bound state was derived. The B-S kernel derived in this paper is Lorentz-covariant and formally is rather different from the Lorentz-non-covariant one given in the three-dimensional equation. For brevity, we restrict ourself in this paper to discuss the bound system of the quark and antiquark which are of different flavors.

To derive the B-S interaction kernel appearing in the B-S equation for $q\bar{q}$ bound states, we first show how the kernel is defined through Green's functions. Before doing this, it is necessary to sketch the derivation of the B-S equation satisfied by the $q\bar{q}$ four-point Green's function which is defined as[5]

$$G(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = \langle 0^+|T\{\psi_\alpha(x_1)\psi^c_\beta(x_2)\bar{\psi}_\rho(y_1)\bar{\psi}^c_\sigma(y_2)\}|0^-\rangle , \tag{1}$$

where $\psi_\alpha(x_1)$ and $\psi^c_\beta(x_2)$ are the quark and antiquark field operators respectively, $\bar{\psi}_\rho(y_1)$ and $\bar{\psi}_\sigma(y_2)$ are their corresponding Dirac conjugates[5]

$$\psi^c(x) = C\bar{\psi}^T(x), \ \bar{\psi}^c(x) = -\psi^T(x)C^{-1} , \tag{2}$$

here C is the charge conjugation operator and T symbolizes the time-ordering product. For later convenience, we write here definitions of the Green's functions involved in this paper

$$\begin{aligned} &\Lambda^{a\cdots bc\cdots d}_{\mu\cdots\nu\kappa\cdots\theta}(x_i,\cdots,x_j;y_k,\cdots,y_l|x_1,y_1)_{\alpha\rho} \\ &= \frac{1}{i}\langle 0^+|T[A^a_\mu(x_i)\cdots A^b_\nu(x_j)A^c_\kappa(y_k)\cdots A^d_\theta(y_l)\psi_\alpha(x_1)\bar{\psi}_\rho(y_1)]|0^-\rangle , \end{aligned} \tag{3}$$

$$\begin{aligned} &\Lambda^{ca\cdots bc\cdots d}_{\mu\cdots\nu\kappa\cdots\theta}(x_i,\cdots,x_j;y_k,\cdots,y_l|x_2,y_2)_{\beta\sigma} \\ &= \frac{1}{i}\langle 0^+|T[A^a_\mu(x_i)\cdots A^b_\nu(x_j)A^c_\kappa(y_k)\cdots A^d_\theta(y_l)\psi^c_\beta(x_2)\bar{\psi}^c_\sigma(y_2)]|0^-\rangle , \end{aligned} \tag{4}$$

$$\begin{aligned} &G^{a\cdots bc\cdots d}_{\mu\cdots\nu\kappa\cdots\theta}(x_i,\cdots,x_j;y_k,\cdots,y_l|x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} \\ &= \langle 0^+|T[A^a_\mu(x_i)\cdots A^b_\nu(x_j)A^c_\kappa(y_k)\cdots A^d_\theta(y_l)\psi_\alpha(x_1)\psi^c_\beta(x_2)\bar{\psi}_\rho(y_1)\bar{\psi}^c_\sigma(y_2)]|0^-\rangle . \end{aligned} \tag{5}$$

where $i, j; k, l = 1,2$. Individual representations of the Green's functions encountered in later derivations can be read off from the above expressions.

The B-S equation for the $q\bar{q}$ bound system may be set up by acting on the Green's function $G(x_1,x_2;y_1,y_2)$ with the inverses of quark and antiquark propagators, i. e. the operator $(i\partial_{x_1} - m_1 + \Sigma)$ acting on the coordinate x_1 and the operator $(i\partial_{x_2} - m_2 + \Sigma^c)$ acting on the coordinate x_2 where $\partial_x = \gamma^\mu\partial^x_\mu$, m_1 and m_2 are the quark and antiquark masses, and Σ and Σ^c stand for the quark and antiquark proper self-energies which occur in the equations obeyed by the quark and antiquark propagators

$$[(i\partial_{x_1} - m_1 + \Sigma)S_F]_{\alpha\rho}(x_1,y_1) = \delta_{\alpha\rho}\delta^4(x_1 - y_1) \tag{6}$$

and

$$[(i\partial_{x_2} - m_2 + \Sigma^c)S^c_F]_{\beta\sigma}(x_2,y_2) = \delta_{\beta\sigma}\delta^4(x_2 - y_2) . \tag{7}$$

For deriving the B-S equation, it is necessary to use various equations of motion satisfied by the

$q\bar{q}$ four-point Green's functions and some other kinds of Green's functions. The latter equations of motion can easily be derived from the QCD generating functional (see the illustrations presented in Ref. [5]). According to this procedure of derivation, one may obtain

$$[(i\partial_{x_1} - m_1 + \Sigma)(i\partial_{x_2} - m_2 + \Sigma^c)G]_{\alpha\beta\rho\sigma}(x_1, x_2; y_1, y_2)$$
$$= \delta_{\alpha\rho}\delta_{\beta\sigma}\delta^4(x_1 - y_1)\delta^4(x_2 - y_2) + \mathscr{H}_1(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{8}$$

where
$$\mathscr{H}_1(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \sum_{i=1}^{4} H_1^{(i)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{9}$$

in which
$$H_1^{(1)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = -(\Gamma^{a\mu})_{\alpha\gamma}\int d^4z_2\Sigma^c(x_2, z_2)_{\beta\lambda}G^a_\mu(x_1 | x_1, z_2; y_1, y_2)_{\gamma\lambda\rho\sigma}, \tag{10}$$

$$H_1^{(2)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = -(\overline{\Gamma}^{b\nu})_{\beta\lambda}\int d^4z_1\Sigma(x_1, z_1)_{\alpha\gamma} G^b_\nu(x_2 | z_1, x_2; y_1, y_2)_{\gamma\lambda\rho\sigma}, \tag{11}$$

$$H_1^{(3)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = (\Gamma^{a\mu})_{\alpha\gamma}(\overline{\Gamma}^{b\nu})_{\beta\lambda}G^{ab}_{\mu\nu}(x_1, x_2 | x_1, x_2; y_1, y_2)_{\gamma\lambda\rho\sigma}, \tag{12}$$

$$H_1^{(4)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \int d^4z_1d^4z_2\Sigma(x_1, z_1)_{\alpha\gamma}\Sigma^c(x_2, z_2)_{\beta\lambda}G(z_1, z_2; y_1, y_2)_{\gamma\lambda\rho\sigma}. \tag{13}$$

In the above, $\Gamma^{a\mu} = g\gamma^\mu T^a$ and $\overline{\Gamma}^{b\nu} = g\gamma^\nu\overline{T}^b$ with $T^a = \lambda^a/2$ and $\overline{T}^b = -\lambda^{b*}/2$ being the quark and antiquark color matrices respectively.

By means of the technique of one- and two-particle-irreducible decompositions of Green's functions[5], it may be found that the function $\mathscr{H}_1(x_1, x_2; y_1, y_2)$ is B-S reducible and can be written in the form

$$\mathscr{H}_1(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \int d^4z_1d^4z_2K(x_1, x_2; z_1, z_2)_{\alpha\beta\gamma\lambda}G(z_1, z_2; y_1, y_2)_{\gamma\lambda\rho\sigma}, \tag{14}$$

where $K(x_1, x_2; z_1, z_2)$ is precisely the B-S irreducible kernel. With the above expression for the function $\mathscr{H}_1(x_1, x_2; y_1, y_2)$, the equation (8) has a closed form

$$[(i\partial_{x_1} - m_1 + \Sigma)(i\partial_{x_2} - m_2 + \Sigma^c)G]_{\alpha\beta\rho\sigma}(x_1, x_2; y_1, y_2)$$
$$= \delta_{\alpha\rho}\delta_{\beta\sigma}\delta^4(x_1 - y_1)\delta^4(x_2 - y_2) + \int d^4z_1d^4z_2K(x_1, x_2; z_1, z_2)_{\alpha\beta\gamma\lambda}G(z_1, z_2; y_1, y_2)_{\gamma\lambda\rho\sigma}. \tag{15}$$

This just is the B-S equation satisfied by the Green's function $G(x_1, x_2; y_1, y_2)$.

By making use of the Lehmann representation of the Green's function $G(x_1, x_2; y_1, y_2)$ or the well-known procedure proposed by Gell-Mann and Low[2], one may derive from Eq. (15) the B-S equation satisfied by B-S amplitudes describing the $q\bar{q}$ bound states[1-5]

$$[(i\partial_{x_1} - m_1 + \Sigma)(i\partial_{x_2} - m_2 + \Sigma^c)\chi_{P_\xi}](x_1, x_2) = \int d^4y_1d^4y_2K(x_1, x_2; y_1, y_2)\chi_{P_\xi}(y_1, y_2), \tag{16}$$

where
$$\chi_{P_\xi}(x_1, x_2) = \langle 0^+ | T[\psi(x_1)\psi^c(x_2)] | P_\xi\rangle \tag{17}$$

represents the B-S amplitude in which P denotes the total momentum of a $q\bar{q}$ bound state and ξ marks the other quantum numbers of the state.

Beyond the perturbation method, the B-S kernel may be derived by starting from its definition shown in Eq. (14). One method of the derivation is usage of B-S equations which describe variations of the Green's functions involved in the function $\mathscr{H}_1(x_1, x_2; y_1, y_2)$ with the coordinates y_1 and y_2. Let us act on the both sides of Eq. (14) with the operator $(i\overleftarrow{\partial}_{y_1} + m_1 - \Sigma)(i\overleftarrow{\partial}_{y_2} + m_2 - \Sigma^c)$

$$\int d^4z_1d^4z_2K(x_1, x_2; z_1, z_2)_{\alpha\beta\gamma\lambda}[G(i\overleftarrow{\partial}_{y_1} + m_1 - \Sigma)(i\overleftarrow{\partial}_{y_2} + m_2 - \Sigma^c)]_{\gamma\lambda\rho\sigma}(z_1, z_2; y_1, y_2)$$
$$= Q(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{18}$$

where
$$Q(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = [\mathscr{H}_1(i\overleftarrow{\partial}_{y_1} + m_1 - \Sigma)(i\overleftarrow{\partial}_{y_2} - m_2 - \Sigma^c)]_{\alpha\beta\rho\sigma}(x_1, x_2; y_1, y_2). \tag{19}$$

As seen from Eq. (18), to derive the B-S kernel, it is necessary to use a B-S equation with respect to y_1 and y_2 for the Green's function $G(x_1, x_2; y_1, y_2)$. This equation can be derived with the help of equations of motion with respect to y_1 and y_2 for the Green's functions. The latter equation may directly follow from the QCD generating functional. The result of the derivation is

$$[G(i\overleftarrow{\partial}_{y_1} + m_1 - \Sigma)(i\overleftarrow{\partial}_{y_2} + m_2 - \Sigma^c)]_{\alpha\beta\rho\sigma}(x_1, x_2; y_1, y_2) = \delta_{\alpha\rho}\delta_{\beta\sigma}\delta^4(x_1 - y_1)\delta^4(x_2 - y_2) + \mathscr{H}_2(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{20}$$

where

$$\mathscr{H}_2(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \sum_{i=1}^{4} H_2^{(i)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{21}$$

in which

$$H_2^{(1)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = -\int d^4z_2 G_\mu^a(y_1 | x_1, x_2; y_1, z_2)_{\alpha\beta\tau\delta}\Sigma^c(z_2, y_2)_{\delta\sigma}(\Gamma^{a\mu})_{\tau\rho}, \tag{22}$$

$$H_2^{(2)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = -\int d^4z_1 G_\nu^b(y_2 | x_1, x_2; z_1, y_2)_{\alpha\beta\tau\delta}\Sigma(z_1, y_1)_{\tau\rho}(\overline{\Gamma}^{b\nu})_{\delta\sigma}, \tag{23}$$

$$H_2^{(3)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = G_{\mu\nu}^{ab}(y_1, y_2 | x_1, x_2; y_1, y_2)_{\alpha\beta\tau\delta}(\Gamma^{a\mu})_{\tau\rho}(\overline{\Gamma}^{b\nu})_{\delta\sigma}, \tag{24}$$

$$H_2^{(4)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \int d^4z_1 d^4z_2 G(x_1, x_2; z_1, z_2)_{\alpha\beta\tau\delta}\Sigma(z_1, y_1)_{\tau\rho}\Sigma^c(z_2, y_2)_{\delta\sigma}. \tag{25}$$

Substituting Eq. (20) in Eq. (18), we have

$$K(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = Q(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} - \int d^4z_1 d^4z_2 K(x_1, x_2; z_1, z_2)_{\alpha\beta\tau\delta}\mathscr{H}_2(z_1, z_2; y_1, y_2)_{\tau\delta\rho\sigma}. \tag{26}$$

Clearly, to reach a closed expression of the B-S kernel, it is necessary to eliminate the kernel appearing in the second term on the RHS of Eq. (26). For this purpose, we operate on the both sides of Eq. (14) with the inverse of the Green's function $G(x_1, x_2; y_1, y_2)$. With this operation, noticing

$$\int d^4z_1 d^4z_2 G(x_1, x_2; z_1, z_2)_{\alpha\beta\gamma\lambda}G^{-1}(z_1, z_2; y_1, y_2)_{\gamma\lambda\rho\sigma} = \delta_{\alpha\rho}\delta_{\beta\sigma}\delta^4(x_1 - y_1)\delta^4(x_2 - y_2), \tag{27}$$

one can get

$$K(x_1, x_2; z_1, z_2)_{\alpha\beta\tau\delta} = \int d^4u_1 d^4u_2 \mathscr{H}_1(x_1, x_2; u_1, u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1, u_2; z_1, z_2)_{\gamma\lambda\tau\delta}. \tag{28}$$

Upon inserting the above expression into Eq. (26), we arrive at

$$K(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = Q(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} - S(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma}, \tag{29}$$

where

$$S(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \int d^4u_1 d^4u_2 d^4v_1 d^4v_2 \mathscr{H}_1(x_1, x_2; u_1, u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1, u_2; v_1, v_2)_{\gamma\lambda\tau\delta} \cdot \mathscr{H}_2(v_1, v_2; y_1, y_2)_{\tau\delta\rho\sigma}. \tag{30}$$

As we see, the second term in Eq. (29) has been explicitly written out. It contains a few types of the Green's functions as well as the self-energies appearing in the mutually conjugate functions $\mathscr{H}_1(x_1, x_2; y_1, y_2)$ and $\mathscr{H}_2(x_1, x_2; y_1, y_2)$ shown in Eqs. (9)—(13) and (21)—(25) respectively. Clearly, to give a final expression of the B-S kernel, according to the definition in Eq. (19), we need to calculate the function $Q(x_1, x_2; y_1, y_2)$ by means of the B-S equations with respect to y_1 and y_2 for the Green's functions involved in the function $\mathscr{H}_1(x_1, x_2; y_1, y_2)$. The result of the calculation is as follows

$$\begin{aligned} Q(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = {} & K_t^{(0)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} + \overline{K}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} + \\ & R(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} + M(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} + \\ & \int d^4u_1 d^4u_2 \Sigma(x_1, u_1)_{\alpha\gamma}\Sigma^c(x_2, u_2)_{\beta\lambda}\mathscr{H}_2(u_1, u_2; y_1, y_2)_{\gamma\lambda\rho\sigma} + \\ & \int d^4v_1 d^4v_2 H_1(x_1, x_2; v_1, v_2)_{\alpha\beta\tau\delta}\Sigma(v_1, y_1)_{\tau\rho}\Sigma^c(v_2, y_2)_{\delta\sigma}, \end{aligned} \tag{31}$$

where

$$K_t^{(0)}(x_1, x_2; y_1, y_2)_{\alpha\beta\rho\sigma} = \delta^4(x_1 - y_1)\delta^4(x_2 - y_2)(\Gamma^{a\mu})_{\alpha\rho}(\overline{\Gamma}^{b\nu})_{\beta\sigma} i\Delta_{\mu\nu}^{ab}(x_1 - x_2) \tag{32}$$

which is the t-channel one-gluon exchange kernel,

$$\overline{K}(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = (\Gamma^{a\mu})_{\alpha\gamma}(\overline{\Gamma}^{b\nu})_{\beta\lambda}G^{abcd}_{\mu\nu\kappa\theta}(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2)_{\gamma\lambda\tau\delta}(\Gamma^{c\kappa})_{\tau\rho}(\overline{\Gamma}^{d\theta})_{\delta\sigma}\ , \tag{33}$$

$$R(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = \sum_{i=1}^{4}R_i(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma}\ , \tag{34}$$

in which

$$\begin{aligned} R_1(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & \delta^4(x_1-y_1)(\Gamma^{a\mu})_{\alpha\rho}\int d^4u_2\Sigma^c(x_2,u_2)_{\beta\lambda}[\Lambda^{cab}_{\mu\nu}(x_1;y_2|u_2,y_2)_{\lambda\delta}(\overline{\Gamma}^{b\nu})_{\delta\sigma} - \\ & \int d^4v_2\Lambda^{ca}_{\mu}(x_1|u_2,v_2)_{\lambda\delta}\Sigma^c(v_2,y_2)_{\delta\sigma}] + \Sigma^c(x_2,y_2)_{\beta\sigma}(\Gamma^{a\mu})_{\alpha\gamma}[\Lambda^{ab}_{\mu\nu}(x_1;y_1|x_1,y_1)_{\gamma\tau}(\Gamma^{b\nu})_{\tau\rho} - \\ & \int d^4v_1\Lambda_{\mu}{}^{a}(x_1|x_1,v_1)_{\gamma\tau}\Sigma(v_1,y_1)_{\tau\rho}]\ , \end{aligned} \tag{35}$$

$$\begin{aligned} R_2(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & \delta^4(x_2-y_2)(\overline{\Gamma}^{a\mu})_{\beta\sigma}\int d^4u_1\Sigma(x_1,u_1)_{\alpha\gamma}[\Lambda^{ab}_{\mu\nu}(x_2;y_1|u_1,y_1)_{\gamma\tau}(\Gamma^{b\nu})_{\tau\rho} - \\ & \int d^4v_1\Lambda_{\mu}{}^{a}(x_2|u_1,v_1)_{\gamma\tau}\Sigma(v_1,y_1)_{\tau\rho}] + \Sigma(x_1,y_1)_{\alpha\rho}(\overline{\Gamma}^{a\mu})_{\beta\lambda}[\Lambda^{cab}_{\mu\nu}(x_2;y_2|x_2,y_2)_{\lambda\delta}(\overline{\Gamma}^{b\nu})_{\delta\sigma} - \\ & \int d^4v_2\Lambda^{ca}_{\mu}(x_2|x_2,v_2)_{\lambda\delta}\Sigma^c(v_2,y_2)_{\delta\sigma}]\ . \end{aligned} \tag{36}$$

$$\begin{aligned} R_3(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & -\delta^4(x_1-y_1)(\Gamma^{c\mu})_{\alpha\rho}(\overline{\Gamma}^{a\nu})_{\beta\lambda}[\Lambda^{cabc}_{\mu\nu\kappa}(x_1,x_2;y_2|x_2,y_2)_{\lambda\delta}(\overline{\Gamma}^{c\kappa})_{\delta\sigma} - \\ & \int d^4v_2\Lambda^{cab}_{\mu\nu}(x_1,x_2|x_2,v_2)_{\lambda\delta}\Sigma^c(v_2,y_2)_{\delta\sigma}] - \delta^4(x_2-y_2)(\overline{\Gamma}^{b\nu})_{\beta\sigma}(\Gamma^{a\mu})_{\alpha\gamma}\cdot \\ & [\Lambda^{abc}_{\mu\nu\kappa}(x_1,x_2;y_1|x_1,y_1)_{\gamma\tau}(\Gamma^{c\kappa})_{\tau\rho} - \int d^4v_1\Lambda^{ab}_{\mu\nu}(x_1,x_2|x_1,v_1)_{\gamma\tau}\Sigma(v_1,y_1)_{\tau\rho}]\ , \end{aligned} \tag{37}$$

$$R_4(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = -\Sigma(x_1,y_1)_{\alpha\rho}\Sigma^c(x_2,y_2)_{\beta\sigma}\ , \tag{38}$$

$$M(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = \sum_{i=1}^{3}M_i(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma}\ , \tag{39}$$

in which

$$\begin{aligned} M_1(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & -(\Gamma^{a\mu})_{\alpha\gamma}\int d^4u_2\Sigma^c(x_2,u_2)_{\beta\lambda}[G^{abc}_{\mu\nu\kappa}(x_1;y_1,y_2|x_1,u_2;y_1,y_2)_{\gamma\lambda\tau\delta}\cdot \\ & (\Gamma^{b\nu})_{\tau\rho}(\overline{\Gamma}^{c\kappa})_{\delta\sigma} - \int d^4v_2G^{ab}_{\mu\nu}(x_1;y_1|x_1,u_2;y_1,v_2)_{\gamma\lambda\tau\delta}\Sigma^c(v_2,y_2)_{\delta\sigma}(\Gamma^{b\nu})_{\tau\rho} - \\ & \int d^4v_1G^{ab}_{\mu\nu}(x_1;y_2|x_1,u_2;v_1,y_2)_{\gamma\lambda\tau\delta}\Sigma(v_1,y_1)_{\tau\rho}(\overline{\Gamma}^{b\nu})_{\delta\sigma}]\ , \end{aligned} \tag{40}$$

$$\begin{aligned} M_2(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & -(\overline{\Gamma}^{a\mu})_{\beta\lambda}\int d^4u_1\Sigma(x_1,u_1)_{\alpha\gamma}[G^{abc}_{\mu\nu\kappa}(x_2;y_1,y_2|u_1,x_2;y_1,y_2)_{\gamma\lambda\tau\delta}\cdot \\ & (\Gamma^{b\nu})_{\tau\rho}(\overline{\Gamma}^{c\kappa})_{\delta\sigma} - \int d^4v_2G^{ab}_{\mu\nu}(x_2;y_1|u_1,x_2;y_1,v_2)_{\gamma\lambda\tau\delta}\Sigma^c(v_2,y_2)_{\delta\sigma}(\Gamma^{b\nu})_{\tau\rho} - \\ & \int d^4v_1G^{ab}_{\mu\nu}(x_2;y_2|u_1,x_2;v_1,y_2)_{\gamma\lambda\tau\delta}\Sigma(v_1,y_1)_{\tau\rho}(\overline{\Gamma}^{b\nu})_{\delta\sigma}]\ , \end{aligned} \tag{41}$$

$$\begin{aligned} M_3(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = {} & -(\Gamma^{a\mu})_{\alpha\gamma}(\overline{\Gamma}^{b\nu})_{\beta\lambda}[\int d^4v_2G^{abc}_{\mu\nu\kappa}(x_1,x_2;y_1|x_1,x_2;y_1,v_2)_{\gamma\lambda\tau\delta}\cdot \\ & \Sigma^c(v_2,y_2)_{\delta\sigma}(\Gamma^{c\kappa})_{\tau\rho} + \int d^4v_1G^{abc}_{\mu\nu\kappa}(x_1,x_2;y_2|x_1,x_2;v_1,y_2)_{\gamma\lambda\tau\delta}\Sigma(v_1,y_1)_{\tau\rho}(\overline{\Gamma}^{c\kappa})_{\delta\sigma}] \end{aligned} \tag{42}$$

and

$$H_j(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = \sum_{i=1}^{1}H_j^{(i)}(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma}\ , \tag{43}$$

here $j=1, 2$. It is noted here that from the irreducible decomposition of the Green's functions included in the B-S kernel, it can be seen that the self-energy-related terms of the function $M(x_1,x_2,y_1,y_2)$ defined in Eqs. (39)—(42) are responsible for eliminating the corresponding terms contained in the function $K(x_1,x_2;y_1,y_2)$ defined in Eq. (33). Similarly, the self-energy-related terms in the function $R(x_1,x_2;y_1,y_2)$ defined in Eqs. (34)—(38) are responsible for cancelling the corresponding terms contained in the two self-energy-irrelavant terms in the function $R_3(x_1,x_2;y_1,y_2)$ shown in Eq. (37). Due to this cancellation, the

B-S kernel is, actually, irrelevant to the self-energy corrections of the external quark and antiquark lines (by the external quark and antiquark, we mean the ones occurring in the B-S amplitude). Particularly, the function $R(x_1,x_2;y_1,y_2)$ gives all the higher order corrections to the bare vertices in the kernel $K_1^{(0)}(x_1,x_2;y_1,y_2)$ so as to make the one-gluon-exchange kernel to be exact.

Now let us turn to the function $S(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma}$ expressed in Eq. (30). According to the definitions denoted in Eqs. (9), (21) and (43), we can write

$$\mathscr{H}_j(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = H_j(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} + H_j^{(4)}(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} \ . \tag{44}$$

Thus, Eq. (30) may be rewritten in the form

$$\begin{aligned} S(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = &\int d^4u_1d^4u_2d^4v_1d^4v_2 H_1(x_1,x_2;u_1,u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1,u_2;v_1,v_2)_{\gamma\lambda\tau\delta}H_2(v_1,v_2;y_1,y_2)_{\tau\delta\rho\sigma} + \\ &\int d^4u_1d^4u_2d^4v_1d^4v_2 H_1^{(4)}(x_1,x_2;u_1,u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1,u_2;v_1,v_2)_{\gamma\lambda\tau\delta}\mathscr{H}_2(v_1,v_2;y_1,y_2)_{\tau\delta\rho\sigma} + \\ &\int d^4u_1d^4u_2d^4v_1d^4v_2 H_1(x_1,x_2;u_1,u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1,u_2;v_1,v_2)_{\gamma\lambda\tau\delta}H_2^{(4)}(v_1,v_2;y_1,y_2)_{\tau\delta\rho\sigma} \ . \end{aligned} \tag{45}$$

When the functions $H_1^{(4)}(x_1,x_2;u_1,u_2)_{\alpha\beta\gamma\lambda}$ and $H_2^{(4)}(v_1,v_2;y_1,y_2)_{\tau\delta\rho\sigma}$ in the second and third terms In Eq. (45) are respectively replaced by the expressions given in Eqs. (13) and (25) and employing the relation in Eq. (27), it is easy to see that the last two terms in Eq. (45) exactly equal to the last two terms in Eq. (31) respectively. Therefore, on substituting Eqs. (31) and (45) into Eq. (29), the B-S kernel will be finally expressed by the following closed form

$$K(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = Q(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} - S(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} \ , \tag{46}$$

where

$$\begin{aligned} Q(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} = &K_1^{(0)}(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} + \overline{K}(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} + \\ &R(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} + M(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} \end{aligned} \tag{47}$$

and

$$\begin{aligned} &S(x_1,x_2;y_1,y_2)_{\alpha\beta\rho\sigma} \\ &= \int d^4u_1d^4u_2d^4v_1d^4v_2 H_1(x_1,x_2;u_1,u_2)_{\alpha\beta\gamma\lambda}G^{-1}(u_1,u_2;v_1,v_2)_{\gamma\lambda\tau\delta}H_2(v_1,v_2;y_1,y_2)_{\tau\delta\rho\sigma} \ . \end{aligned} \tag{48}$$

The B-S kernel shown above is manifestly Lorentz-covariant even in the space-like region of Minkowski space where the bound states exist. Particularly, the kernel is represented through only a few types of Green's functions contained in the functions $R(x_1,x_2;y_1,y_2)$, $M(x_1,x_2;y_1,y_2)$, $H_1(x_1,x_2;u_1,u_2)$ and $H_2(v_1,v_2;y_1,y_2)$. Therefore, it is not only easily calculated by the perturbation method, but also provides a proper basis for studying the QCD nonperturbative effect.

It is well-known that the B-S equation is invariant with respect to renorrmalization. In other words, the equation (16) keeps the same form before and after renormalimation. Therefore, the renormalized B-S kernel is still represented by the formulas shown before provided that all the quantities are replaced by the renormalized ones. Next, we note that in the derivation of the B-S kernel, we acted on the Green's functions with the operators $(i\partial_{x_1} - m_1 + \Sigma)$ and $(i\partial_{x_1} - m_1 + \Sigma^c)$ other than the operators $(i\partial_{x_1} - m_1)$ and $(i\partial_{x_2} - m_2)$. This is because the B-S equation established in this way is conveniently renormalized. Certainly, the B-S equation can be set up by using the operators $(i\partial_{x_1} - m_1)$ and $(i\partial_{x_2} - m_2)$ to act on the Green's function $G(x_1,x_2;y_1,y_2)$. In this case, the B-S kernel has a formally simple expression which may be written out from the kernel derived before by deleting out the self-energy-related terms, as shown in the following

$$K(x_1,x_2;y_1,y_2)$$
$$= K_r^{(0)}(x_1,x_2;y_1,y_2) + R'(x_1,x_2;y_1,y_2) + G(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2) - S(x_1,x_2;y_1,y_2) \quad (49)$$

where $K_r^{(0)}(x_1,x_2;y_1,y_2)$ was shown in Eq. (32), $R'(x_1,x_2;y_1,y_2)$ is given by the two terms without the self-energies in Eq. (37),

$$G(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2) = \langle 0^+|T\{A(x_1)A(x_2)\psi(x_1)\psi^c(x_2)\bar{\psi}(y_1)\bar{\psi}^c(y_2)A(y_1)A(y_2)\}|0^-\rangle \quad (50)$$

in which $A(x_1) = \Gamma^{a\mu}A_\mu^a(x_1)$, $A(x_2) = \bar{\Gamma}^{b\nu}A_\nu^b(x_2)$, $A(y_1) = \Gamma^{a\mu}A_\mu^a(y_1)$, $A(y_2) = \bar{\Gamma}^{b\nu}A_\nu^b(x_2)$ (51)

and
$$S(x_1,x_2;y_1,y_2) = \int d^4u_1d^4u_2d^4v_1d^4v_2G(x_1,x_2|x_1,x_2;u_1,u_2)\cdot$$
$$G^{-1}(u_1,u_2;v_1,v_2)G(y_1,y_2|v_1,v_2;y_1,y_2) \quad (52)$$

in which
$$G(x_1,x_2|x_1,x_2;u_1,u_2) = \langle 0^+|T\{A(x_1)A(x_2)\psi(x_1)\psi^c(x_2)\bar{\psi}(u_1)\bar{\psi}^c(u_2)\}|0^-\rangle \quad (53)$$
$$G(y_1,y_2|v_1,v_2;y_1,y_2) = \langle 0^+|T\{\psi(v_1)\psi^c(v_2)\bar{\psi}(y_1)\bar{\psi}^c(y_2)A(y_1)A(y_2)\}|0^-\rangle \quad (54)$$

At last, it is necessary to mention the role played by the last term in Eq. (49). As pointed out before, the Green's function $G(x_1,x_2|x_1,x_2;u_1,u_2)$ with two-gluon fields at the positions x_1 and x_2 is B-S reducible and can be represented as

$$G(x_1,x_2|x_1,x_2;u_1,u_2)$$
$$= \int d^4z_1d^4z_2K_1(x_1,x_2;z_1,z_2)\cdot$$
$$G(z_1,z_2;u_1,u_2) \quad (55)$$

where $K_1(x_1,x_2;z_1,z_2)$ is a part of the kernel $K(x_1,x_2;z_1,z_2)$ which is generated from the Green's function $G(x_1,x_2|x_1,x_2;u_1,u_2)$. Similarly, the Green's function $G(y_1,y_2|v_1,v_2;y_1,y_2)$ with two gluon fields at y_1 and y_2 is also B-S reducible and can be expressed in the form

$$G(y_1,y_2|v_1,v_2;y_1,y_2)$$
$$= \int d^4z_1'd^4z_2'G(v_1,v_2;z_1',z_2')\cdot$$
$$K_2(z_1',z_2';y_1,y_2) \quad (56)$$

where the kernel K_2, as can be proved, is conjugate to the kernel K_1. Substituting Eqs. (55) and (56) into Eq. (52) and using the identity denoted in Eq. (27), it is found that

$$S(x_1,x_2;y_1,y_2)$$
$$= \int d^4u_1d^4u_2d^4v_1d^4v_2K_1(x_1,x_2;u_1,u_2)\cdot$$
$$G(u_1,u_2;v_1,v_2)K_2(v_1,v_2;y_1,y_2) \quad (57)$$

This expression shows the typical structure of the two-particle reducible part of the B-S kernel. On the other hand, the Green's function $G(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2)$ for which at every position a gluon field is nested, in general, can be split into a B-S irreducible part $G_{IR}(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2)$ and a B-S reducible part $G_{RE}(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2)$ and the B-S reducible part just equals to the function $S(x_1,x_2;y_1,y_2)$. Therefore, both of the functions $S(x_1,x_2;y_1,y_2)$ and $G_{RE}(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2)$ are cancelled with each other in Eq. (49). Thus, we have

$$K(x_1,x_2;y_1,y_2)$$
$$= K_r(x_1,x_2;y_1,y_2) + R'(x_1,x_2;y_1,y_2) +$$
$$G_{IR}(x_1,x_2;y_1,y_2|x_1,x_2;y_1,y_2) \quad (58)$$

in which the B-S irreducible part of the Green's function represents two and more than two-gluon exchange interactions in the sense of perturbation theory. A similar cancellation takes place in Eq. (29) where the function $S(x_1,x_2;y_1,y_2)$ just plays the role of eliminating the B-S reducible part contained in the function $Q(x_1,x_2;y_1,y_2)$.

For the purpose of exploring the QCD nonperturbative effect existing in the bound state, it is appropriate to start from the kernel derived in this paper. In the ordinary quark potential model, the Last term in Eq. (58) is usually simulated by a linear potential kernel or some other modified confinement one. Certainly, This simulation is oversimplified. To search for a sophisticated confining potential including not only its spatial form, but also its spin and color structures, it is necessary to call for a suitable nonperturbation method to calculate the Green's functions contained in the B-S kernel derived in Eqs. (46)—(48) or (49)—(54).

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夸克-反夸克束缚态的B-S相互作用核的封闭表示式*

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摘　要：根据从QCD生成泛函所建立的夸克和反夸克的传播子、四点格林函数及其它类型的格林函数所满足的运动方程，推导出了夸克-反夸克束缚态的Bethe-Salpeter方程中相互作用核的明显且封闭的表示式，给出了这个表示式的未重整化和重整化了的形式．这个表示式不仅易于进行微扰计算，而且适于进行非微扰的计算，特别是它提供了求解夸克禁闭问题一个恰当的理论出发点．

关键词：B-S方程；相互作用核；夸克-反夸克束缚态

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