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Dynamical Fluctuation in High Energy Heavy Ion Collision and Quantum Chaos*

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Abstract: In high energy heavy ion collisions, a possible measure for a certain irregularity of the spectra of produced particles is proposed by using the Wigner-Dyson statistical analysis method. The preliminary results from EMU01 experiments are given by using this statistical analysis method. The analysis shows that the dominant effect is random emission in available high energy nucleus-nucleus collisions at CERN/SPS and BNL/AGS regions.

Key words: random matrix theory; statistical analysis method; high energy heavy-ion collision

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1 Introduction

The occurrence of large rapidity density fluctuations and some short-rang correlations of charged particles and events of very large multiplicity fluctuation have been observed in high energy heavy-ion collisions, which has been suggested as a signature of collective effects of hadronization or the dynamics of multiparticle production[1]. The most frequently used methods for carrying out such studies have been to perform the normalized factorial moment analysis[2] for fluctuations.

A large number of particle produced in a finite volume of the collision hints a fairly large density of energy released during colliding and thus a possibility of appearance of exotic phenomena. In recent years, one of main tools of studying the quantum manifestation of chaos, the analysis of spectral properties, has been successfully applied to a variety of quantum systems[3]. We mention the many examples of the Wigner-Dyson type random matrix theories (RMT)[4-7], for instance, in nuclear physics, atomic physics, molecule physics and chemistry etc[8-10]. Therefore, it is not surprising to expect that the behavior of correlations and fluctuations of multi-particle production in high energy heavy-ion physics should become the focus of keen interest as precise new data can be available in present and future relativistic heavy-ion collision experiments.

This paper is organized as follows: The Wigner-Dyson spectral analysis method for quantum systems is preliminary introduced in Section 2. The preliminary results from the EMU01 experiment data by using the Wigner-Dyson method are shown in Section 3. Wigner's random matrix theory and Wigner-Dyson statistic method are given in Appendix A and B.

2 The Approach to Wigner-Dyson Spectral Statistics

In general assumptions and results of the Wigner-Dyson type random matrix theories (RMT) are relevant with the context of chaos dynamics and quantum physics. By using RMT one should

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keep in mind: (1) simplicity may emerge in situations with initially appear as desperately complicated and random; (2) probabilistic and statistical arguments may be fruitfully applied to systems where no external randomness nor stochasticity has been introduced from outside. However as emphasized by Bohigas[3], even though the beauty and depth of RMT has been recognized and appreciated almost since the theories birth: it is the one-dimensional theory par excellence! Not only does it have immediate usefulness and validity for real physical systems but it has given rise to profound results and makes use of the deepest theorems of analysis. Until some years ago, it has remained the almost secret garden of a few theoretical physicists with high mathematical skill. The situation now is changing and a broader community is interested in RMT.

As well known, the quantum mechanics for a dynamical system in a compact space yields a set of discrete eigenvalues, called the spectrum, rather than a continuous possible range as in classical mechanics. It is obvious that the spectrum can give us many more characteristic features and much more information than the classical continuous range. The details of the spectrum can change from system (or event to event), but some certain properties should be quite general (some general discussion are given in Appendix A and B).

It is often taken for granted that one results to statistical studies of such systems only because detailed properties of the spectrum are not really open to calculation. This is indeed one of the reasons but not really the main one. Such studies, whose nature is essentially different from the study of individual density, are of interest because they reveal new features of the system. The situation is analogous to that in statistical mechanics, where properties such as temperature and entropy are exhibited best by systems of many particles. Even in system where the individual properties and their quantum descriptions are better known than the corresponding phase-space or related descriptions statistical method are often essential for a more complete understanding.

The study of many particle spectra are usually concentrated with properties of short sequences of related density. In contrast to that, our present purpose will be paid attention to the density fluctuations and correlation in general case.

Considering an event which has a finite sequence of n particles with positions $\xi_1<\xi_2< \cdots \xi_n$ in one-dimension axis of some certain argument (for instance, energy, transverse energy, rapidity etc.). The particle density function of this single event is

$$\rho(\xi) = \sum_{i=1}^{n}\delta(\xi - \xi_i) . \qquad (1)$$

Assume that the considered range or window from $-L/2$ or $L/2$ in which there n particles are distributed. Then, the average value of the nearest neighboring spacing of this event is

$$\bar{s} = \frac{L}{n} . \qquad (2)$$

To make reasonable comparison of the detail structure of the nearest neighboring spacing among the different events, we need to re-scale the positions of the considered axis:

$$x_i = \frac{\xi_i}{\bar{s}} , \; i = 1, 2, \cdots, n . \qquad (3)$$

So that the values of rescaled arguments of particles range by $|x|\leq n/2$. Let us denote by

$$s = x_{i-1+k} - x_i ,$$

the k-order spacings between two particles with scaled arguments x_i and x_{i+1+k} having k particle in between $k = 0, 1, \cdots$. The nearest neighboring spacing is to set $k=0$, which we shall focus in the following.

Now we introduce a staircase function of x for the sequence of new arguments x_i, $i=1, 2, \cdots, n$ as

$$N(x) = \int_{-n/2}^{x} \mathrm{d}x \, \rho(x') , \qquad (4)$$

where $\rho(x)=\bar{s}\rho(\eta)$.

The quantity, $N(x)$ is just the number of particles with positions from $-n/2$ to x. For a sequence of ξ with constant average spacing, $\bar{s}$, the event-average of the staircase function, $\langle N(x)\rangle$, will lie on a straight line, with slope $1/\bar{s}$. Therefore, it is convenient to introduce a measure of the intensity of the fluctuations of the staircase function around a straight line. A possible measure is given by the least square deviation (the minimized variance):

$$D(n) = \min \frac{1}{n}\int_{-n/2}^{n/2} dx[N(x) - a - bx]^2 , \quad (5)$$

in which parameters a and b are chosen by the minimized requirement that

$$\frac{\partial D}{\partial a} = \frac{\partial D}{\partial b} = 0 .$$

This will give a description of the fluctuations of the considered spectrum around a straight line $a + bx$ for multi-particle production event by event. This straight line is corresponding to the uniform distribution of particles in the considered argument window. For events which have the same fixed multiplicity n, we can get the event-averaged value of the minimized variance of the spectrum, $\langle D(n)\rangle$.

Theoretically, it is always possible to give a n-particle joint probability density, $P(x_1, \cdots, x_n)$. We shall denote the ensemble average, the average with respect to the distribution $P(x_1, \cdots, x_n)$, by bracket$\langle \cdots \rangle$. Thus the semi-inclusive single-particle density of rescaled argument is

$$\langle \rho(x)\rangle = \int_{-\infty}^{+\infty} dx_1 \cdots dx_n \rho(x) P(x_1, \cdots, x_n) \quad (6)$$

and the two-particle correlation function is defined

$$\langle \rho(x)\rho(x')\rangle = \int_{-\infty}^{-\infty} dx_1 \cdots dx_n \rho(x)\rho(x) \cdot P(x_1, \cdots, x_n) . \quad (7)$$

The Wigner-Dyson statistics is the average of $D(n)$ over this ensemble of sequence (experimentally corresponding to the average over events). That is

$$\langle D(n) \rangle = \langle \overline{N^2} - \overline{N}^2 - \frac{12}{n^2} \overline{xN} \rangle , \quad (8)$$

where we introduced the horizontal average of single event,

$$\bar{f} = \frac{1}{n} \int_{-n/2}^{n/2} dx f(x) ,$$

and the averages in Eq. (8) can be written

$$\langle \overline{N^2}\rangle = \frac{1}{n} \int_{-n/2}^{n/2} dx \int_{-n/2}^{x} dx_1 \cdot \int_{-n/2}^{x} dx_2 \langle \rho(x_1)\rho(x_2)\rangle , \quad (9)$$

$$\langle \overline{N}^2\rangle = \frac{1}{n^2} \int_{-n/2}^{n/2} dx \int_{-n/2}^{n/2} dx' \int_{-n/2}^{x} dx_1 \cdot \int_{-n/2}^{x'} dx_2 \langle \rho(x_1)\rho(x_2)\rangle \quad (10)$$

and

$$\langle \overline{xN}^2\rangle = \frac{1}{n^2} \int_{-n/2}^{n/2} x dx \int_{-n/2}^{n/2} x' dx' \int_{-n/2}^{x} dx_1 \cdot \int_{-n/2}^{x'} dx_2 \langle \rho(x_1)\rho(x_2)\rangle . \quad (11)$$

For a purely random sequence, a sequence with constant density and without correlation between particles, the distributions of the rescaled argument spacing between two nearest neighboring particles is the Poisson type (see Appendix B) with

$$P_{\text{Poisson}}(s) = \exp(-s) . \quad (12)$$

Take the average of $D(n)$ over the Poisson ensembles, we get a simple expression

$$\langle D(n)\rangle_{\text{Poisson}} = \frac{n}{15} \quad (13)$$

which we shall use to make comparison with data of the EMU01 experiment in the following section.

As pointed out by Reichl[9], conservative system can be divided into two types, integrable and non-integrable. The latter may themselves be divided into two classes. One classes contains the completely chaotic systems with non-smooth Hamiltonians. Non-integrable systems with smooth Hamiltonians comprise the second class. The vast majority of physical systems that we deal with belong to this second class. All non-integrable systems exhibit chaotic behavior to various degrees. However, we must careful in studying quantum systems because of the Pauli uncertainty principle. It appears that quantum systems are of two types, they are either integrable or non-integrable. The spectral properties of these two types of quantum system can be quite different. Integrable quantum systems have a random spectrum. Non-integrable quantum systems, whose classical counterparts are chaotic, have a spectrum which exhibits eigenvalue repulsion and is fit quite well by random matrix theory.

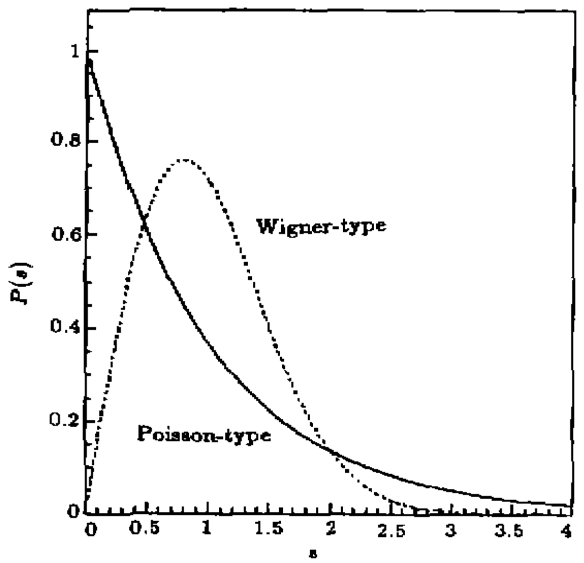

Fig. 1 A plot of the Wigner distribution $P_{\rm Wigner}(s)$ and the Poisson distribution $P_{\rm Poisson}(s)$ as a function of the spacing s, for $\bar{s}=1$.

The random matrix theory is based on the precondition that we know very little about the dynamics of the system we are considering except for certain symmetry properties. For our purpose the system formed from high energy heavy-ion collisions is extremely complicated and we know really very little about its dynamics. So one can consider the colliding system as a "black box" in which all particles are interacting according to some unknown laws and pay intensive attention on deviations of some quantities from their mean values. Such deviations are normally thought to be less important and seldom studied carefully in normal statistics because they are very small. Even though the multiplicity can be several hundreds or thousands as in JACEE cosmic-ray event or in future ALICE events on the LHC, the thermodynamic limit cannot be satisfied and deviations from mean values should be much more important than before.

As shown in Appendix A, the basic problem of random matrix theory is to set up a probability distribution on the elements of the Hamiltonian matrix. There are a number of statistical properties of random matrices that are commonly used in analyzing the spectral properties of the considered systems. The simplest of these is the density of eigenvalues. In Fig. 1, we compare the Wigner distribution.

$$P_{\rm Wigner}(s) = \frac{\pi s}{2\bar{s}^2}\exp\left(-\frac{\pi s^2}{4\bar{s}^2}\right) \tag{14}$$

with the Poisson distribution, Eq. [12], for the case $\bar{s}=1$.

3 Preliminary Results from EMU01 Experiments

In this section the preliminary results from EMU01 experiments[11] will be given to show that the dominant effect is random emission in available high energy nucleus-nucleus collisions at CERN/SPS and BNL/AGS by using the Wigner-Dyson statistical analysis method.

The EMU01 collaboration has measured nucleus-nucleus collisions for various projectiles and targets at different incident energies taken with nuclear emulsion detector. In Ref. [10], they attempt to extend the statistical analysis method proposed by Wigner, Dyson and Mehta[4–6] to present their data on fluctuations and correlations in ^{16}O, ^{28}Si, ^{32}S and ^{197}Au induced heavy-ion interactions at 10.7 — 200 AGeV. Measure of the size of the fluctuations is made by using the probability of the nearest-neighboring rapidity-spacings between produced particles event by event.

The EMU01 experiment[11,12] used two sorts of detectors including both conventional emulsion stacks exposed horizontally and special emulsion chambers exposed vertically. The exposures took place during the period 1986—1989 at BNL/AGS for ^{16}O and ^{28}Si beam at 14.6 AGeV and at CERN/SPS for ^{16}O at 60 and 200 AGeV and for ^{32}S beam at 200 AGeV. One of the highlight during 1992 was the realization of ^{197}Au beam running at the BNL/AGS for 10.7 AGeV. The experimental data used in that investigation is collected only horizontally exposed emulsion stacks with minimum bias. The measured shower particles are singly charged particles with $\beta > 0.7$ which are mainly produced hadrons for the study.

Their analysis was in the selected parts of full phase space with limited size. It is obvious that the analysis in a limited rapidity window, $|\eta-\eta_0|\leq L/2$, where η_0 is the rapidity position of the center of mass of nucleon-nucleon system corresponding to different incident energies, should be more detail for the study than that in a full longitudinal phase space[12]. They have chosen the windows with not large size only in the central rapidity region in which the single-particle rapidity distribution should be rather even.

In Fig. 2—4 are shown the relevance of data to the window cuts, the incident energies and the projectile nuclei respectively. The curves of theoretical calculation for comparison are also followed by the shift. It is obvious from the figures that data lie practically mostly around the Poisson-type curve $\langle D(n)\rangle_{Poisson}=n/15$. It looks like that the data points from more narrow window ($L=0.2$) to wider window ($L=1.8$) lie on the same curve although the fluctuation is large for higher multiplicities from Fig. 2. That is to say the random emission does dominate over the present selected windows which are all belong to central rapidity region. For comparison of data from different energies each other we selected the window with size $L=0.6$ for oxygen induced interactions. Considering the present beam energy region from 14.6 to 200 A GeV, the independence of data on the energy is obvious, which can be seen from Fig. 3. The BNL/AGS gave us the possibility to get events from different projectile nuclei (oxygen, silicon and gold) at nearly same energies. The data comparison is shown in Fig. 4 where there is no evidence for projectile dependence.

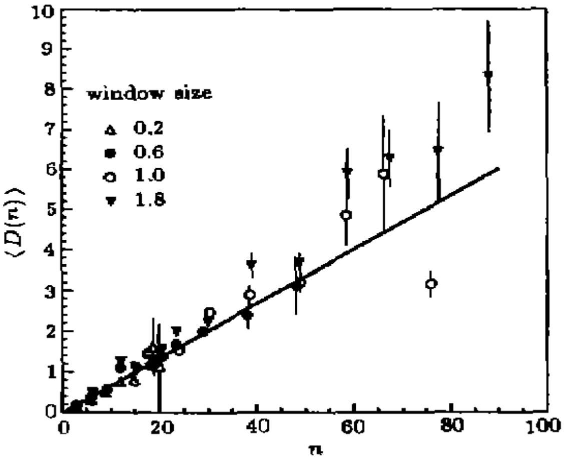

Fig. 2 Preliminary results from the EMU01 experiment[11]. Event-averaged values $\langle D(n)\rangle$ vs multiplicities n in selected rapidity regions with sizes $L=0.2$, 0.6, 1.0 and 1.8 for ^{16}O+Em interactions at 200 AGeV. Solid curve is the theoretical calculation from Poisson type.

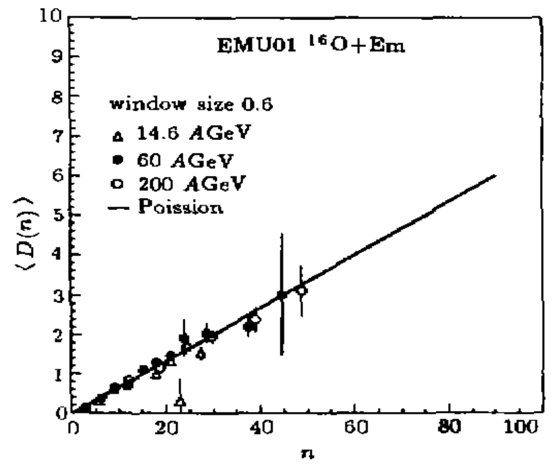

Fig. 3 Preliminary results from the EMU01 experiment[11]. Event-averaged value $\langle D(n)\rangle$ vs multiplicities n in selected rapidity region with size $L=0.6$ for ^{16}O+Em interactions at 14.6, 60 and 200 A GeV. Solid curve is the theoretical calculation from Poisson type.

In summary the main points from the analysis of the EMU01 experimental data are as following.

(1) The dominant effect for events of nuclear interactions induced by high energy heavy-ions especially for those with larger multiplicities might be random emission according to the available EMU01 data. Although there should be some ef-

fects on correlation between produced particles from sub-processes, which is given rise to by some reaction mechanism, but they might be submerged in the dominant information of random environment of nucleus-nucleus collisions. This appears from the above analysis that the $\langle D(n) \rangle$ values lie on the estimation from random sequence of Poisson-type.

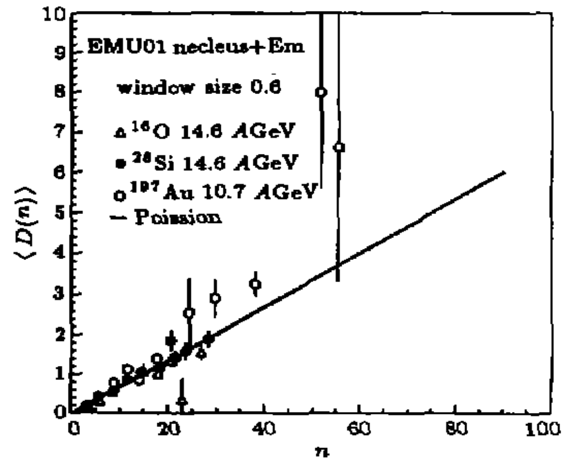

Fig. 4 Preliminary results from the EMU01 experiment[11]. Event-averaged values $\langle D(n) \rangle$ vs multiplicities n in selected rapidity region with size $L=0.6$ for $^{16}O+Em$ and $^{28}Si+Em$ interactions at 14.6 A GeV and $^{197}Au+Em$ at 10.7 A GeV. Solid curve is the theoretical calculation from Poisson type.

(2) The data show that this feature is window-cut independence in the central rapidity region.

(3) Energy scaling of such effect is also in existence at the present analysis.

(4) There is no obvious difference among the data from different projectile nuclei.

As pointed out in Ref. [12] that the geometry of nuclear collisions and the number of participating nucleons play an important role in the particle production, which result to larger fluctuations and random emissions. So that for future experiments with higher energies and heavier nuclei we have to face the dominant effect of random emission from nuclear geometrical fluctuation and intra-nuclear rescattering due to the enough high energy.

Another point we have to emphasise is that even though the present analysis have been made by quoting the Wigner-Dyson statistics, we do know it is more sensitive for the measurement of correlation among the one-dimension sequence and can provide a new and better proof to supplement the measurement from other methods. And dynamical mechanism in detail has not been paid attention to. It is so early to say anything about that. At moment the experimental data must be so lack far from drawing any conclusion.

Appendix A: Wigner's Argument of Random Matrix

The random matrix theory of quantum systems is base on the assumption that we know very little about the dynamics of the considered system except for certain symmetry properties. These symmetry properties impose restrictions on the from of the Hamiltonian matrix of the system, where the matrix elements were unknown and unknowable.

Consider a 2×2 dimensional real symmetric Hamiltonian matrix,

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}, \qquad \text{(A-1)}$$

with independent random matrix elements, h_{11}, h_{22} and h_{12}. The eigenvalues are

$$\varepsilon_{\pm} = \frac{1}{2}[(h_{11} + h_{22}) \pm \sqrt{(h_{11} - h_{22})^2 + 4h_{12}^2}\,]. \qquad \text{(A-2)}$$

Thus, the spacing between these eigenvalues,

$$s = \sqrt{(h_{11} - h_{22})^2 + 4h_{12}^2}, \qquad \text{(A-3)}$$

will be random, if h_{11}, h_{22} and h_{12} are random.

Now we consider the distribution of nearest neighboring spacing for a random sequence in one-dimension axis. The axis may be the energy, the points corresponding to the discrete energy levels of a quantum system (e.g., a complex atomic nucleus, an atom, or a molecule); or it may be the mass axis, the points corresponding to the nuclear fragments in nucleus-nucleus collisions; or it may be the frequency axis, the points corresponding to the normal frequencies of a vibrating membrane, or the eigenfrequencies of a microwave cavity, or the eigenfrequencies of a small metallic block; or it may be the energy axis, the points corresponding to the measured energies of particles produced in high energy heavy-ion collisions, or the components of energy, $E_\perp$ and $E_{/\!/}$, or rapidity.

The probability that a position will be occupied by a particle in the small interval $(\varepsilon+s, \varepsilon+s+d\varepsilon)$, proportional to ds, which will be dependent of whether or not there is a particle at ε.

Given a particle at ε, let the probability the next position at which another particle will be occupied be in $(\varepsilon+s, \varepsilon+s+ds)$ be $P(s)ds$ where $s \geq 0$. We call $P(s)$ the nearest-neighbor spacing distribution which can be determined by

$$P(s)\mathrm{d}s = P(I \in \mathrm{d}s | 0 \in s)\ P(0 \in s)\ , \tag{A-4}$$

where $P(n \in s)$ is the probability that the interval of length s contain n particles and $P(m \in ds | n \in s)$ the conditional probability that the interval of length ds contain m particles, when that of length s contains n particles.

So we have the probability that the spacing is larger than s

$$P(0 \in s) = \int_s^\infty P(x)\mathrm{d}x\ . \tag{A-5}$$

Define

$$R_{mn}(s)\mathrm{d}s = P(m \in \mathrm{d}s | n \in s)\ . \tag{A-6}$$

The Poisson law: if one takes

$$R_{10}(s) = \frac{1}{\bar{s}}\ , \tag{A-7}$$

where $\bar{s}$ is the mean local spacing, so that $1/\bar{s}$ is the density of particles, one obtains

$$P_{\mathrm{Poisson}}(s) = \frac{1}{\bar{s}}\exp\left(\frac{s}{\bar{s}}\right)\ , \tag{A-8}$$

The Wigner law: from the assumption of a linear repulsion

$$R_{10}(s) = \alpha s\ , \tag{A-9}$$

where α is a constant, one gets

$$P_{\mathrm{Wigner}}(s) = \frac{\pi s}{2\bar{s}^2}\exp(-\frac{\pi s^2}{4\bar{s}^2})\ . \tag{A-10}$$

Normalization conditions are

$$\int_0^\infty P(s)\mathrm{d}s = 1 \tag{A-11}$$

and

$$\int_0^\infty sP(s)\mathrm{d}s = \bar{s}\ . \tag{A-12}$$

For purely random sequences, the spacing distribution is Poisson type in Eq. A-1. In Fig. 1 we compare the Wigner distribution, $P_{\mathrm{Wigner}}(s)$, with the Poisson distribution, $P_{\mathrm{Poisson}}(s)$, for the case $\bar{s} = 1$. For a random sequence there is a high probability of finding very small spacings between the near neighboring eigenvalues.

Appendix B: Wigner-Dyson Statistics

Let $x_1, x_2, \cdots, x_N$ be the rescaled positions of N particles on an one-dimensional real axis, with average density unity and with the joint probability regardless of ordering

$$P'_N(x_1, \cdots, x_N)\mathrm{d}x_1 \cdots \mathrm{d}x_N\ .$$

The statistical properties of this sequence can be characterized by a set of n-particle correlation function

$$R_n(x_1, \cdots, x_n) = \frac{N!}{(N-n)!}\ \cdot \int_{-\infty}^{+\infty}\mathrm{d}x_{n+1}\cdots\mathrm{d}x_N P_N(x_1, \cdots, x_N)\ . \tag{B-1}$$

It is convenient to introduce a set of n-particle cluster functions by subtracting out the lower-order correlation terms from R_n

$$T_1(x_1) = R_1(x_1)\ , \tag{B-2}$$

$$T_2(x_1, x_2) = -R_2(x_1, x_2) + R_1(x_1)R_1(x_2)\ , \tag{B-3}$$

$$\begin{aligned} T_3(x_1, x_2, x_3) &= R_3(x_1, x_2, x_3) - [R_1(x_1)R_2(x_2, x_3) + \\ &R_1(x_2)R_2(x_3, x_1) + R_1(x_3)R_2(x_1, x_2)] + \\ &2R_1(x_1)R_1(x_2)R_1(x_3)\ , \end{aligned} \tag{B-4}$$

etc. The advantage of the cluster functions is that they have the property of vanishing when any one (or several) of the gaps $r = |x_i - x_j|$ becomes large. One also uses the Fourier transform for two-particle cluster function T_2, $b(k)$, called the two-particle from factor.

$$b(k) = \int_{-\infty}^{+\infty} T_2(r)\exp(i2\pi kr)dr . \quad (B\text{-}5)$$

The semi-inclusive single-particle density and the two-particle correlation function can be expressed as

$$\langle\rho(x)\rangle = T_1(x) . \quad (B\text{-}6)$$

$$\langle\rho(x)\rho(x')\rangle = \delta(x - x')T_1(x) + T_1(x)T_1(x') + T_2(x, x') . \quad (B\text{-}7)$$

Thus the average of $D(n)$ can be expressed in terms of T_2

$$\langle D(n)\rangle = \frac{n}{15} - \frac{1}{15n^4}\int_{-n/2}^{n/2}\left(\frac{n}{2} - x\right)^3\left[2n^2 - 9n\left(\frac{n}{2} + x\right) - 3\left(\frac{n}{2} + x\right)^2\right] \cdot T_2\left(\frac{n}{2} + x\right)dx , \quad (B\text{-}8)$$

where the two-particle cluster function T_2 depends only on the relative distance $x = x_1 - x_2$.

The following is cited two typical ensembles which can be obtained from available references. The Poisson type ensemble with $T_2(x_1, x_2) = 0$ simply give Eq. (13) for random emission processes.

Another well known one is the Wigner type, a sequence of particles with joint rescaled rapidity distribution

$$P(x_1, \cdots, x_n) \propto \prod_{i \neq j = 1}^{n} |x_i - x_j| \cdot \exp\left(-\frac{1}{2}\sum_{k=1}^{n} x_k^2\right) , \quad (B\text{-}9)$$

which might be corresponding to some kind of strong negative correlation between particle and gives the distribution of rescaled spacing between two nearest neighboring particles:

$$P_{\text{Wigner}}(s) = \frac{\pi}{2}s \exp\left(-\frac{\pi s^2}{4}\right) . \quad (B\text{-}10)$$

The average of $D(n)$ over Wigner ensemble is

$$\langle D(n)\rangle_{\text{Wigner}} = \frac{1}{\pi^2}\left[\ln(2\pi n) + \gamma - \frac{\pi^2}{8} - \frac{5}{4}\right] , \quad (B\text{-}11)$$

where $\gamma = 0.577\ 216$ is Euler's constant.

One can consider other types of ensembles, e.g. with some other types of correlation, but it is difficult to deduce an obvious analytical expression for $\langle D(n)\rangle$.

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高能重离子碰撞中的动力学涨落与量子混沌*

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摘　要：Wigner-Dyson 型的随机矩阵理论作为一种可能的统计分析方法用来分析高能重离子碰撞中所产生的粒子谱的某些无规性，此统计分析方法被用来初步分析了EMU01的实验结果，通过分析可以看出在CERN/SPS和BNL/AGS能区粒子主要是随机产生的。

关 键 词：随机矩阵理论；Wigner-Dyson 型统计分析方法；高能重离子碰撞

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